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SAMPLE PROBLEM 1.1

In the hanger shown, the upper portion of link ABC is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of $\frac{3}{8}$ -in. diameter while a $\frac{1}{4}$ -in.-diameter pin is used at C. Determine (a) the shearing stress in pin A, (b) the shearing stress in pin C, (c) the largest normal stress in link ABC, (d) the average shearing stress on the bonded surfaces at B, (e) the bearing stress in the link at C.

Solution

(a) shearing stress in pin A

$$\tau_A = \frac{F_{AC}}{A} = \frac{F_{AC}}{\frac{\pi}{4} d_A^2} = \frac{F_{AC}}{\frac{\pi}{4} (\frac{3}{8})^2} \quad (1)$$

we need to calculate $F_{AC} \Rightarrow$ we draw the free body diagram (graph I)

$$\Rightarrow \sum M_D = -F_{AC} \times 10 + 500 \times 15 = 0$$

$$\Rightarrow \boxed{F_{AC} = 750 \text{ lb}}$$

We put the value in (1)

$$\tau_A = \frac{750}{\frac{\pi}{4} (\frac{3}{8})^2} = \underline{\underline{6790 \text{ psi}}}$$

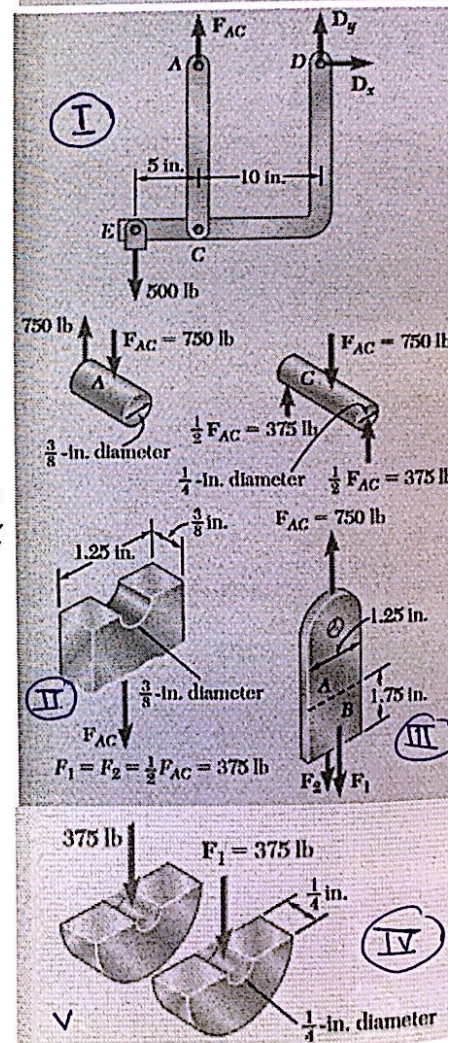
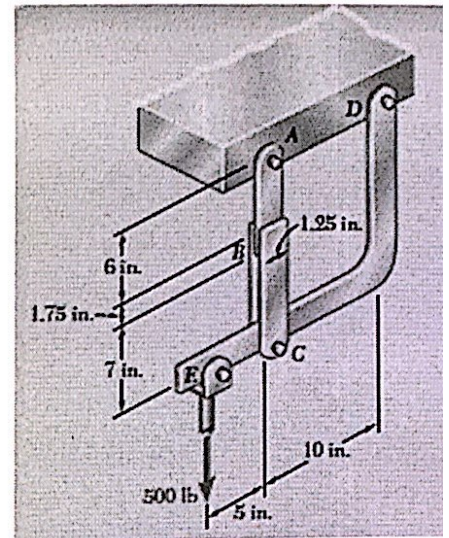
(b) $\tau_C = \frac{F_C}{A_C} = \frac{\frac{1}{2} F_{AC}}{\frac{\pi}{4} (d_C)^2} = \frac{\frac{1}{2} \cdot 750}{\frac{\pi}{4} (\frac{1}{4})^2} = \underline{\underline{7640 \text{ psi}}}$

(c) The largest stress is found where the area is the smallest \Rightarrow this occurs at the cross section at A (look at figure II)

$$\sigma = \frac{F_{AC}}{A} = \frac{750}{\frac{3}{8} (1.25 - \frac{3}{8})} = \underline{\underline{2290 \text{ psi}}}$$

(d) $\tau_B = \frac{\frac{1}{2} F_{AC}}{A} = \frac{\frac{1}{2} \cdot 750}{1.25 \times 1.75} = \underline{\underline{171.4 \text{ psi}}}$
look at (III)

(e) $\sigma_b = \frac{\frac{1}{2} F_{AC}}{A} = \frac{\frac{1}{2} \cdot 750}{\frac{1}{4} \cdot \frac{1}{4}} = \underline{\underline{6000 \text{ psi}}}$
look at (IV)



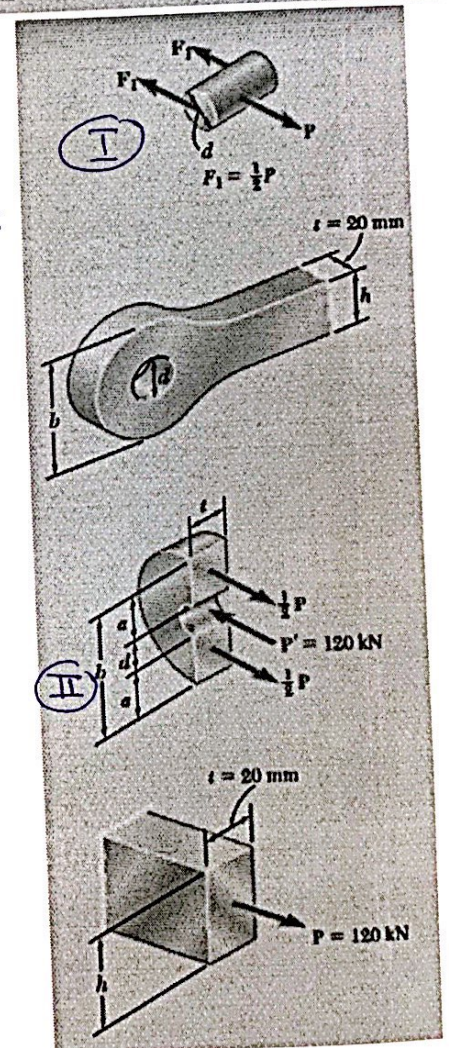
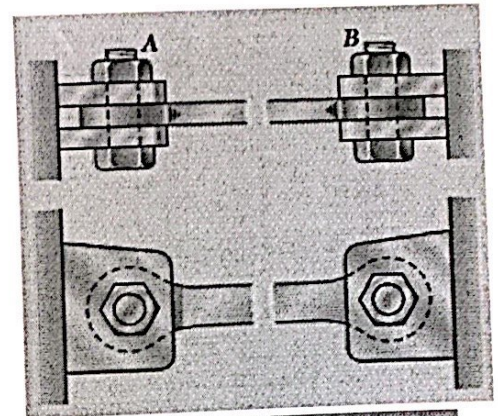
SAMPLE PROBLEM 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120 \text{ kN}$ when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175 \text{ MPa}$, $\tau = 100 \text{ MPa}$, $\sigma_b = 350 \text{ MPa}$. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, (c) the dimension h of the bar.

(a) $\tau = \frac{\frac{1}{2}P}{A} = \frac{\frac{1}{2} \cdot 120 \times 10^3}{\frac{\pi}{4} \cdot d^2} = 100 \times 10^6$
 $\Rightarrow \underline{d = 28 \times 10^{-3} \text{ m}}$

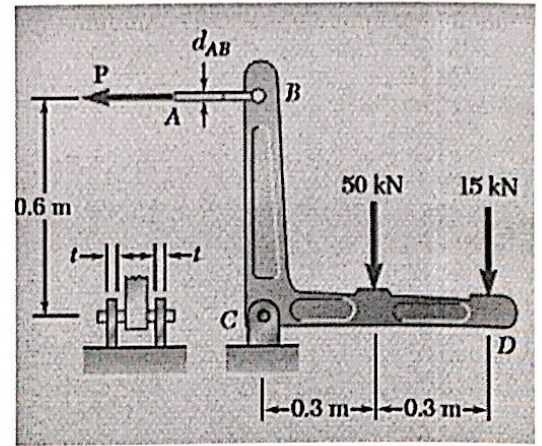
(b) $\sigma = \frac{\frac{1}{2}P}{A} = \frac{\frac{1}{2} \cdot 120}{20 \times 10^{-3} \times (b - 28 \times 10^{-3})} = 175 \times 10^6$
 100% at (II) $\Rightarrow \underline{b = 62.3 \times 10^{-3} \text{ m}}$

(c) $\sigma = \frac{P}{A} = \frac{120}{h \times 20 \times 10^{-3}} = 175 \times 10^6$
 $\Rightarrow \underline{h = 35 \times 10^{-3} \text{ m}}$



3

3-Two forces are applied to the bracket BCD as shown. (a) Knowing that the control rod AB is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (b) The pin at C is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin C for which the factor of safety with respect to shear will also be 3.3. (c) Determine the required thickness of the bracket supports at C knowing that the allowable bearing stress of the steel used is 300 MPa.



$$\textcircled{a} \quad F.S. = \frac{\sigma_u}{\sigma_{all}} \Rightarrow \sigma_{all} = \frac{\sigma_u}{F.S.} = \frac{600}{3.3} = 181.8 \text{ MPa}$$

$$\sigma_{all} = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2} \rightarrow \text{lets calculate } P \text{ from the FBD}$$

$$\textcircled{1} \quad \sum F_y = 50 + 15 = 0 \quad \dots \textcircled{1}$$

$$\textcircled{2} \quad \sum F_x = P = 0 \quad \dots \textcircled{2}$$

$$\textcircled{3} \quad \sum M_C = P \cdot 0.6 - 50 \cdot 0.3 - 15 \cdot 0.6 = 0 \quad \dots \textcircled{3}$$

by solving $\textcircled{1}$ and $\textcircled{2}$ and $\textcircled{3}$

$$\Rightarrow \boxed{P = 40 \text{ kN}}, \quad \boxed{C_x = 40 \text{ kN}}, \quad \boxed{C_y = 65 \text{ kN}}$$

$$C = \sqrt{(40)^2 + (65)^2} = 76.3 \text{ kN}$$

$$\Rightarrow \sigma_{all} = \frac{P}{A} = \frac{40 \times 10^3}{\frac{\pi}{4} d_{AB}^2} = 181.8 \times 10^6$$

$$\Rightarrow \underline{d_{AB} = 16.74 \times 10^{-3} \text{ m}}$$

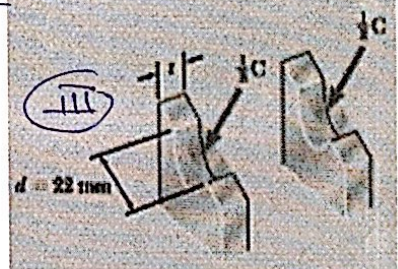
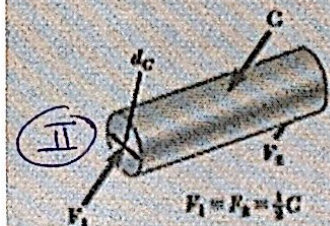
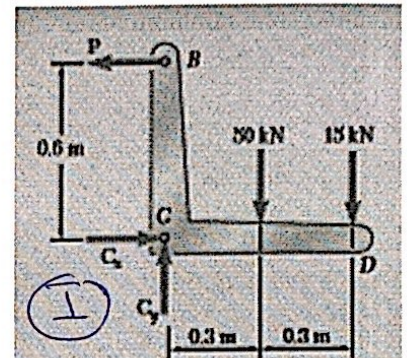
$$\textcircled{b} \quad F.S. = \frac{\tau_u}{\tau_{all}} \Rightarrow \tau_{all} = \frac{\tau_u}{F.S.} = \frac{350}{3.3} = 106.1 \text{ MPa}$$

$$\tau_{all} = \frac{C}{A} = \frac{76.3 \times 10^3}{\frac{\pi}{4} (d_c)^2} = 106.1 \times 10^6$$

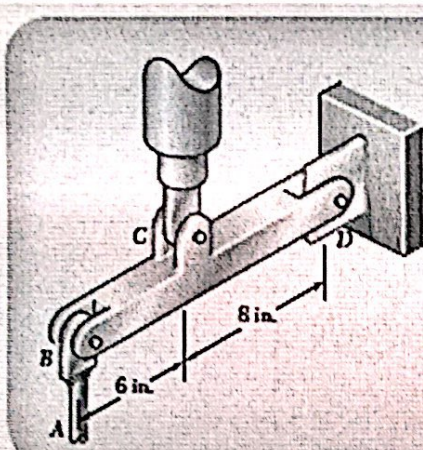
$$\Rightarrow \underline{d_c = 22 \times 10^{-3} \text{ m}}$$

$$\textcircled{c} \quad \sigma_b = \frac{C/2}{A} = \frac{\frac{1}{2} \cdot 76.3 \times 10^3}{t \cdot 22 \times 10^{-3}} = 300 \times 10^6$$

$$\text{look at } \textcircled{III} \Rightarrow \underline{t = 6 \times 10^{-3} \text{ m}}$$



(4)



SAMPLE PROBLEM 1.4

The rigid beam BCD is attached by bolts to a control rod at B , to a hydraulic cylinder at C , and to a fixed support at D . The diameters of the bolts used are: $d_B = d_D = \frac{3}{4}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod AB has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C .

From the Free body Diagram (I) \Rightarrow

$$\textcircled{a} \sum M_D = 0 \Rightarrow B \cdot 14 - C \cdot 8 = 0 \Rightarrow C = \frac{B \cdot 14}{8} \quad \textcircled{1}$$

$$\textcircled{b} \sum M_B = 0 \Rightarrow -D \cdot 14 + C \cdot 6 = 0 \Rightarrow C = \frac{D \cdot 14}{6} \quad \textcircled{2}$$

* in the control rod AB

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{60}{3} = 20 \text{ Ksi}$$

$$\sigma_{all} = \frac{B}{A} = \frac{B}{\frac{\pi}{4} d_{AB}^2} = \frac{B}{\frac{\pi}{4} \left(\frac{7}{16}\right)^2} = 20 \Rightarrow$$

$$B = 3.01 \text{ Kips}$$

lets put the value in $\textcircled{1} \Rightarrow C = 5.27 \text{ Kips}$

* in the Bolt B

$$\tau_{all} = \frac{\tau_U}{F.S.} = \frac{40}{3} = 13.33 \text{ Ksi}$$

$$\tau_{all} = \frac{B/2}{A} = \frac{B/2}{\frac{\pi}{4} d_B^2} = \frac{B/2}{\frac{\pi}{4} \left(\frac{3}{4}\right)^2} = 13.33 \Rightarrow$$

$$B = 2.94 \text{ Kips}$$

lets put the value in $\textcircled{1} \Rightarrow C = 5.15 \text{ Kips}$

* in the Bolt D

$$\tau_{all} = \frac{D/2}{A} = \frac{D/2}{\frac{\pi}{4} d_D^2} = \frac{D/2}{\frac{\pi}{4} \left(\frac{3}{4}\right)^2} = 13.33$$

$$\Rightarrow D = 2.94 \text{ Kips}$$

lets put the value in $\textcircled{2} \Rightarrow$

$$C = 6.85 \text{ Kips}$$

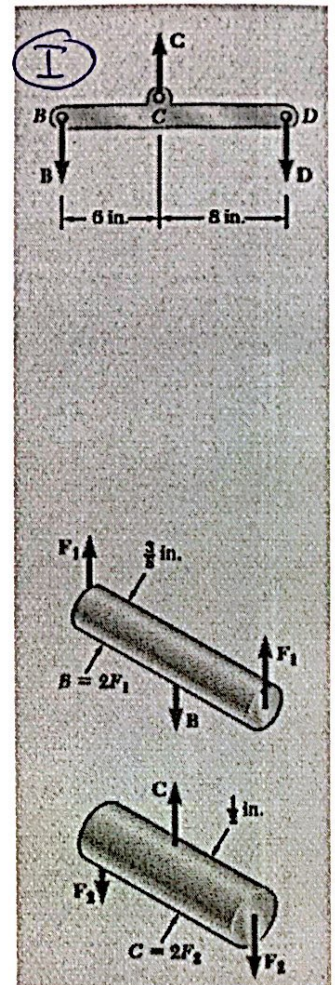
* in the Bolt C

$$\tau_{all} = \frac{C/2}{A} = \frac{C/2}{\frac{\pi}{4} d_C^2} = \frac{C/2}{\frac{\pi}{4} \left(\frac{1}{2}\right)^2} = 13.33$$

$$\Rightarrow C = 5.23$$

in order to satisfy all the criteria
we take the smallest C value

$$\Rightarrow C = 5.15 \text{ Kips}$$



5

Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What is the safety factor used if the structure shown was designed to support a 16-kN load P ?

Solution

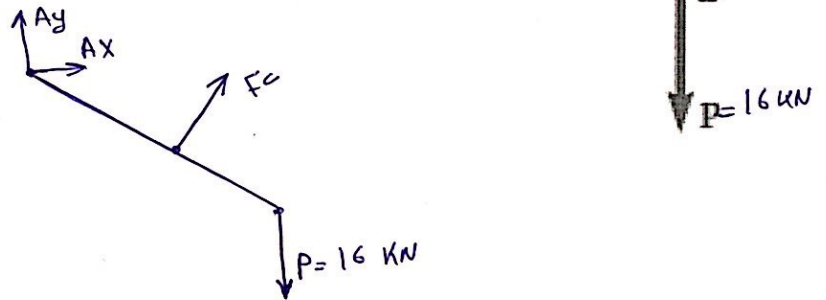
$$F.S. = \frac{\sigma_u}{\sigma_{all}} = \frac{480 \times 10^6}{\sigma_{all}}$$

$$\sigma_{all} = \frac{F_c}{w \cdot \text{thickness}} = \frac{F_c}{25 \times 10^{-3} \times 6 \times 10^{-3}}$$

lets calculate F_c

$$F_c = ?$$

From the FBD \Rightarrow



$$\begin{aligned} \sum M_A &= 0 \Rightarrow \\ -16 \times 0.6 + F_c \cdot 0.48 &= 0 \\ \Rightarrow F_c &= 25 \text{ kN} \end{aligned}$$

$$\Rightarrow \sigma_{all} = \frac{F_c}{25 \times 10^{-3} \times 6 \times 10^{-3}} = \frac{25 \times 10^3}{25 \times 10^{-3} \times 6 \times 10^{-3}} = 166 \times 10^6 \text{ Pa}$$

$$F.S. = \frac{480 \times 10^6}{166 \times 10^6} = \boxed{2.896}$$