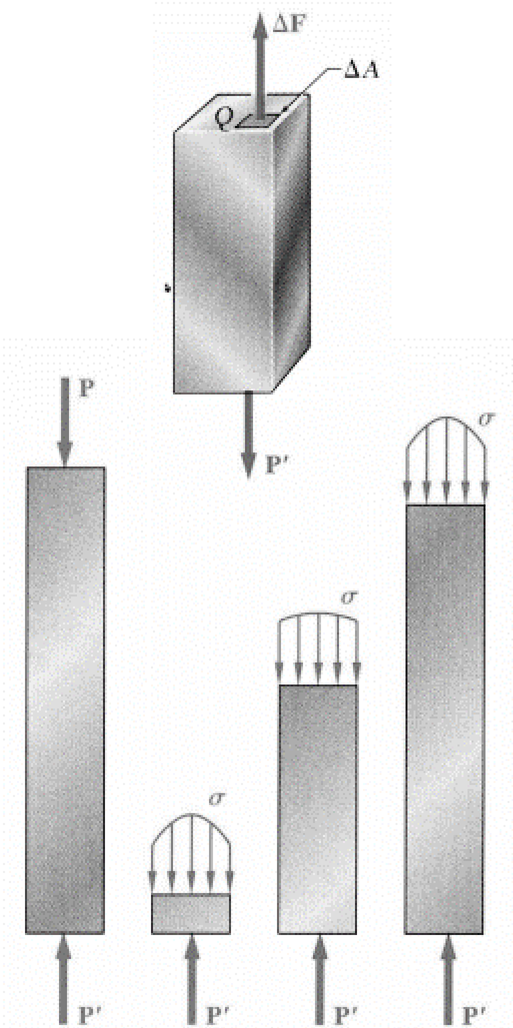
Lecture 2

Introduction – Concept of Stress

Axial Loading: Normal Stress



- The resultant of the internal forces for an axially loaded member is *normal* to a section cut perpendicular to the member axis.

- The force intensity on that section is defined as the normal stress.

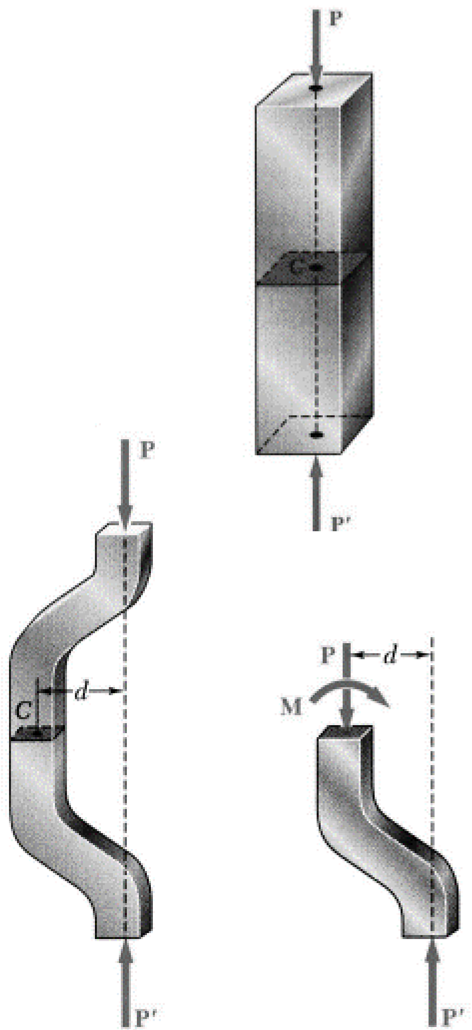
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \qquad \sigma_{ave} = \frac{P}{A}$$

- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

- The detailed distribution of stress is statically indeterminate, i.e., can not be found from statics alone.

Centric & Eccentric Loading



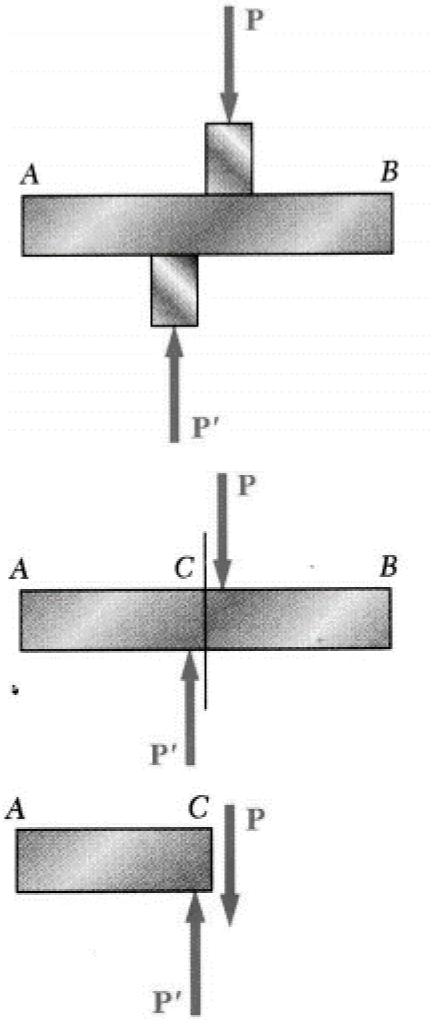
- A uniform distribution of stress in a section implies that the line of action of the resultant of the internal forces passes through the centroid of the section.

- A uniform distribution of stress is only possible if the concentrated loads on the end sections of two-force members are applied at the section centroids. This is referred to as *centric loading*.

- If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.

- The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

Shearing Stress



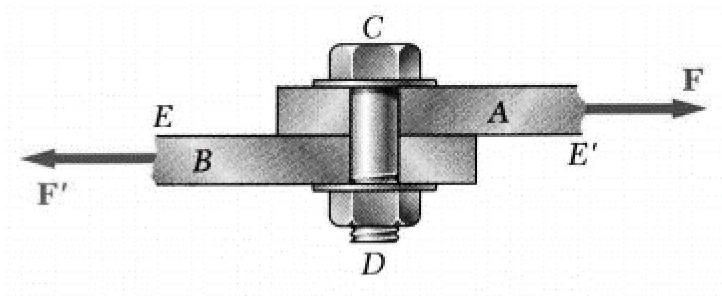
- Forces P and P' are applied transversely to the member AB.
- Corresponding internal forces act in the plane of section C and are called *shearing* forces.
- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load P.
- The corresponding average shear stress is,

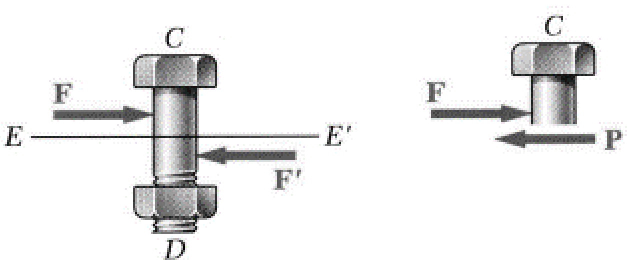
$$\tau_{\text{ave}} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

Shearing Stress Examples

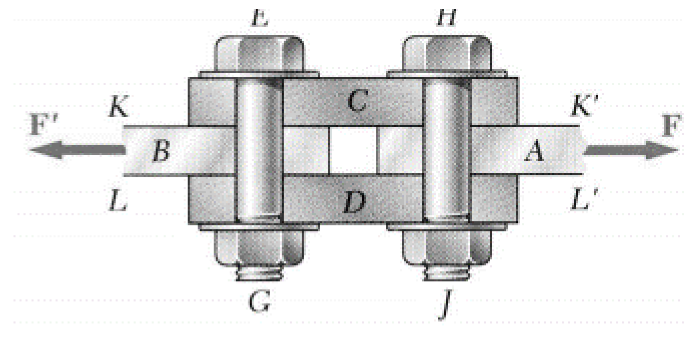
Single Shear

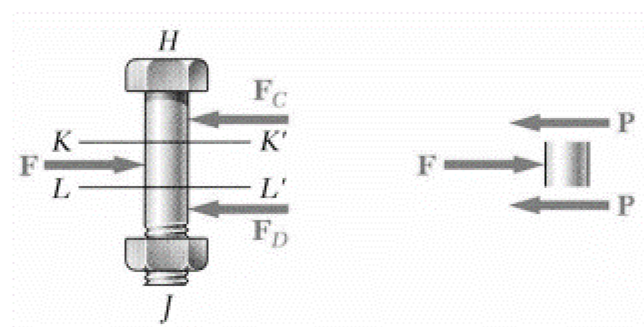




$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

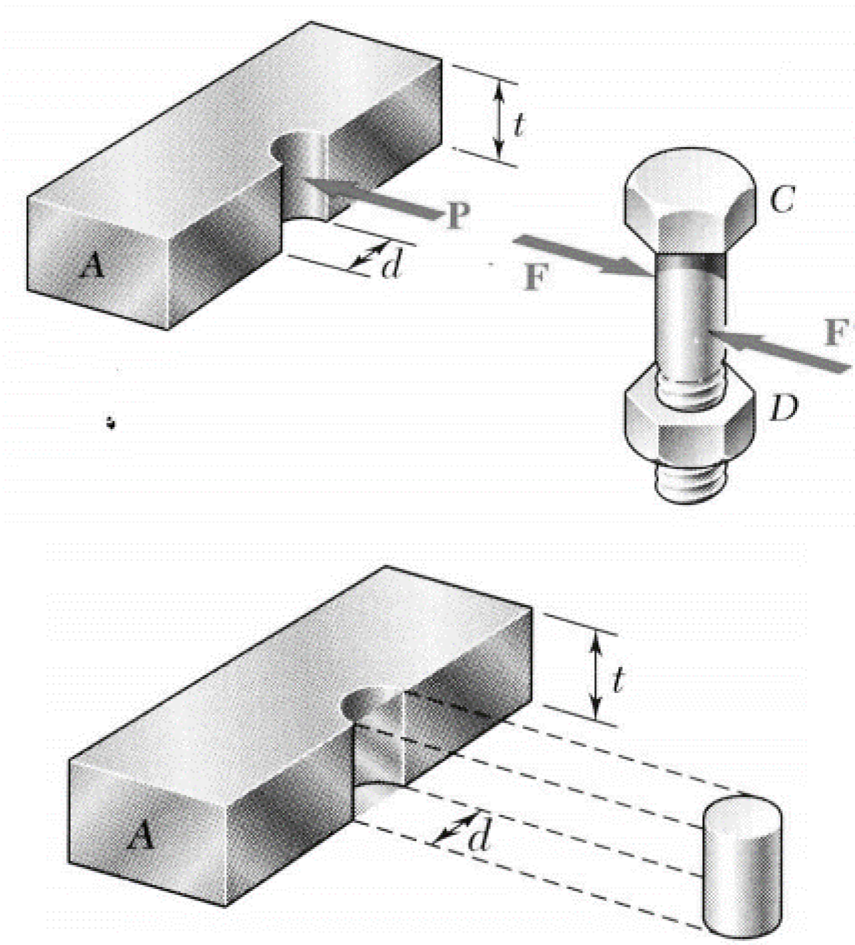
Double Shear





$$\tau_{ave} = \frac{P}{A} = \frac{F}{2A}$$

Bearing Stress in Connections



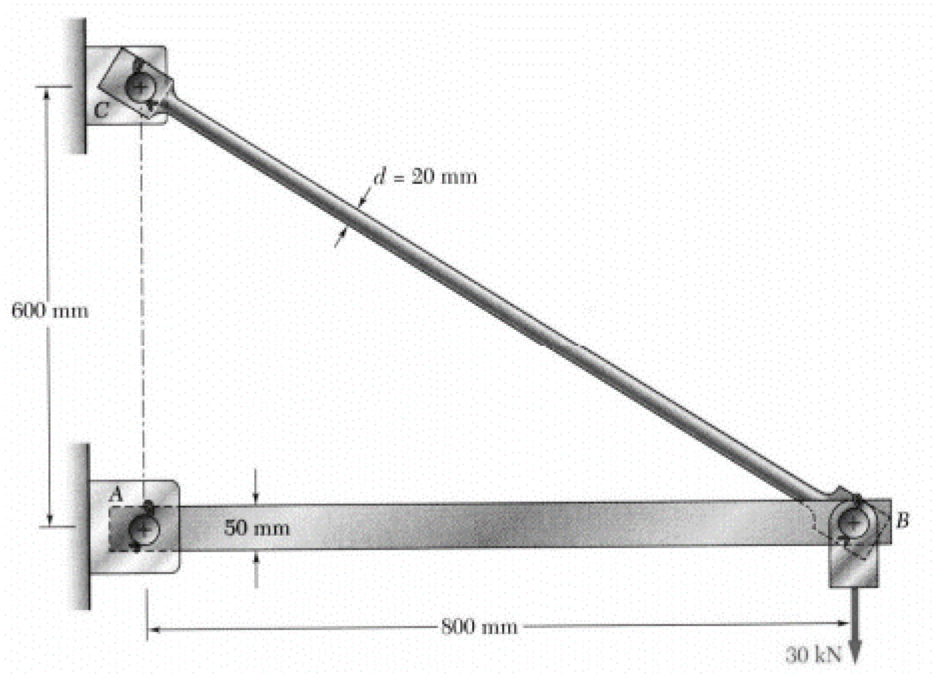
- Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.

- The resultant of the force distribution on the surface is equal and opposite to the force exerted on the pin.

- Corresponding average force intensity is called the bearing stress,

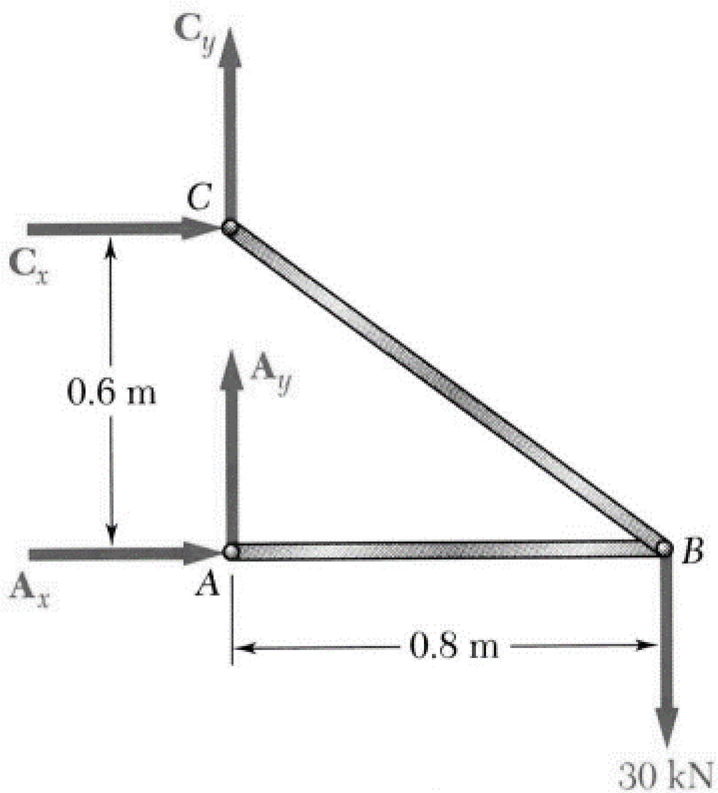
$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$

Analysis and Design example



- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

Structure Free-Body Diagram



- Structure is detached from supports and the loads and reaction forces are indicated
- Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$A_x = 40\,\mathrm{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

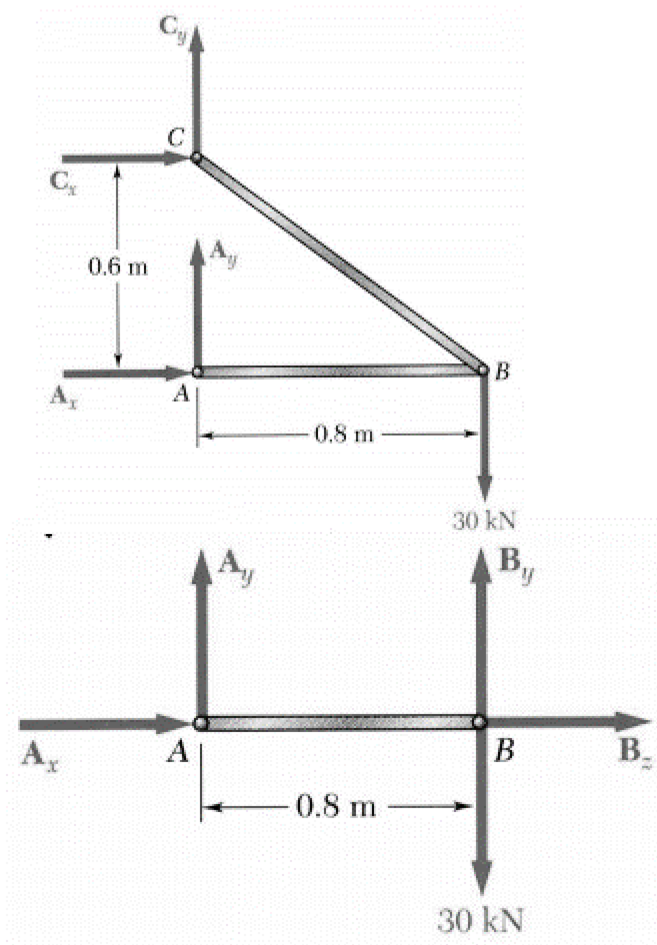
$$C_x = -A_x = -40\,\mathrm{kN}$$

$$\sum F_y = 0 = A_y + C_y - 30\,\mathrm{kN} = 0$$

$$A_y + C_y = 30\,\mathrm{kN}$$

- A_y and C_y can not be determined from these equations

Component Free-Body Diagram



- In addition to the complete structure, each component must satisfy the conditions for static equilibrium

- Consider a free-body diagram for the boom:

  $$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

  $$A_y = 0$$

  substitute into the structure equilibrium equation

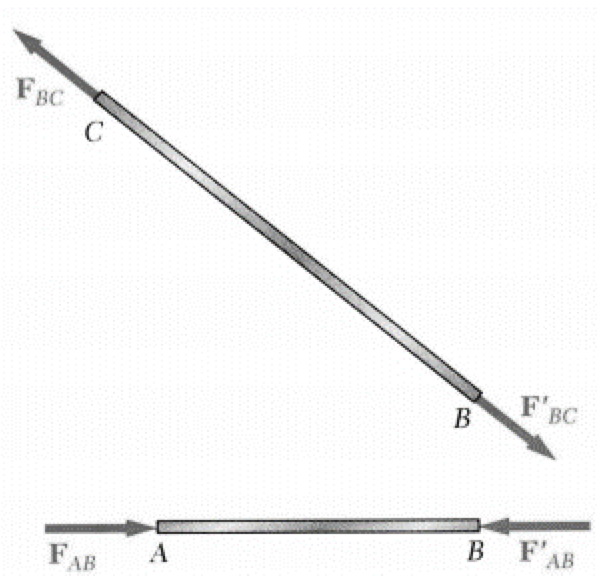
  $$C_y = 30\,\text{kN}$$

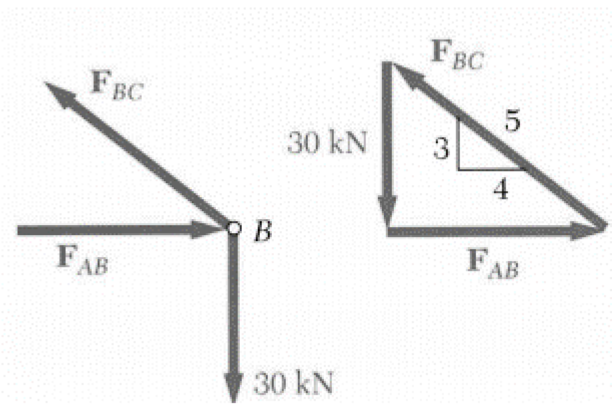
- Results:

  $$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

  Reaction forces are directed along boom and rod

Method of Joints





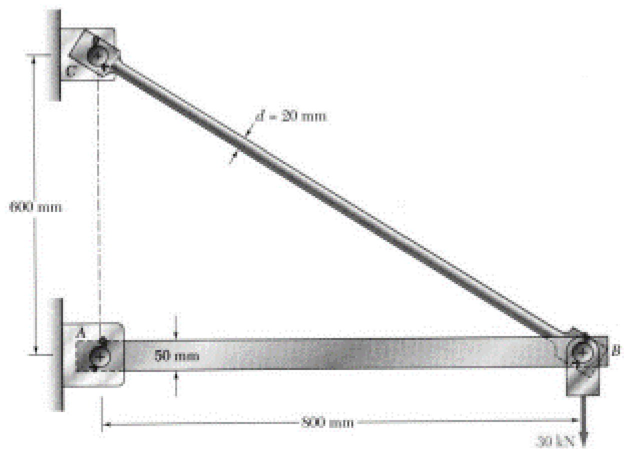
- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

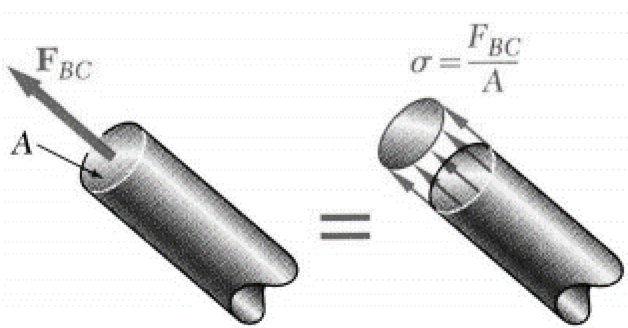
$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

Stress Analysis



$d_{BC} = 20$ mm



Can the structure safely support the 30 kN load?

- From a statics analysis

  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or <u>stress</u> of

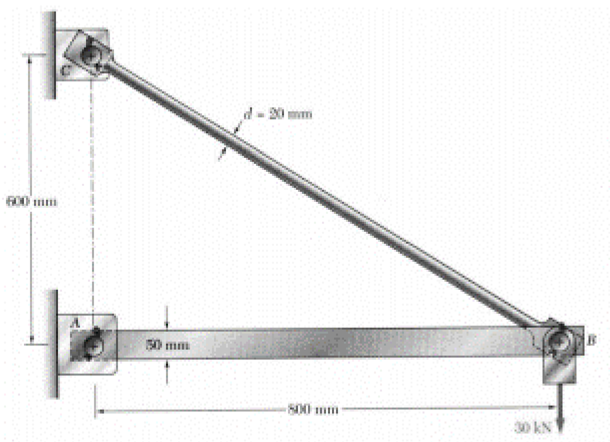
$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^{3}\,\mathrm{N}}{314\times10^{-6}\,\mathrm{m}^{2}} = 159\ \mathrm{MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\mathrm{all}} = 165\ \mathrm{MPa}$$

- Conclusion: the strength of member *BC* is adequate

Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements
- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

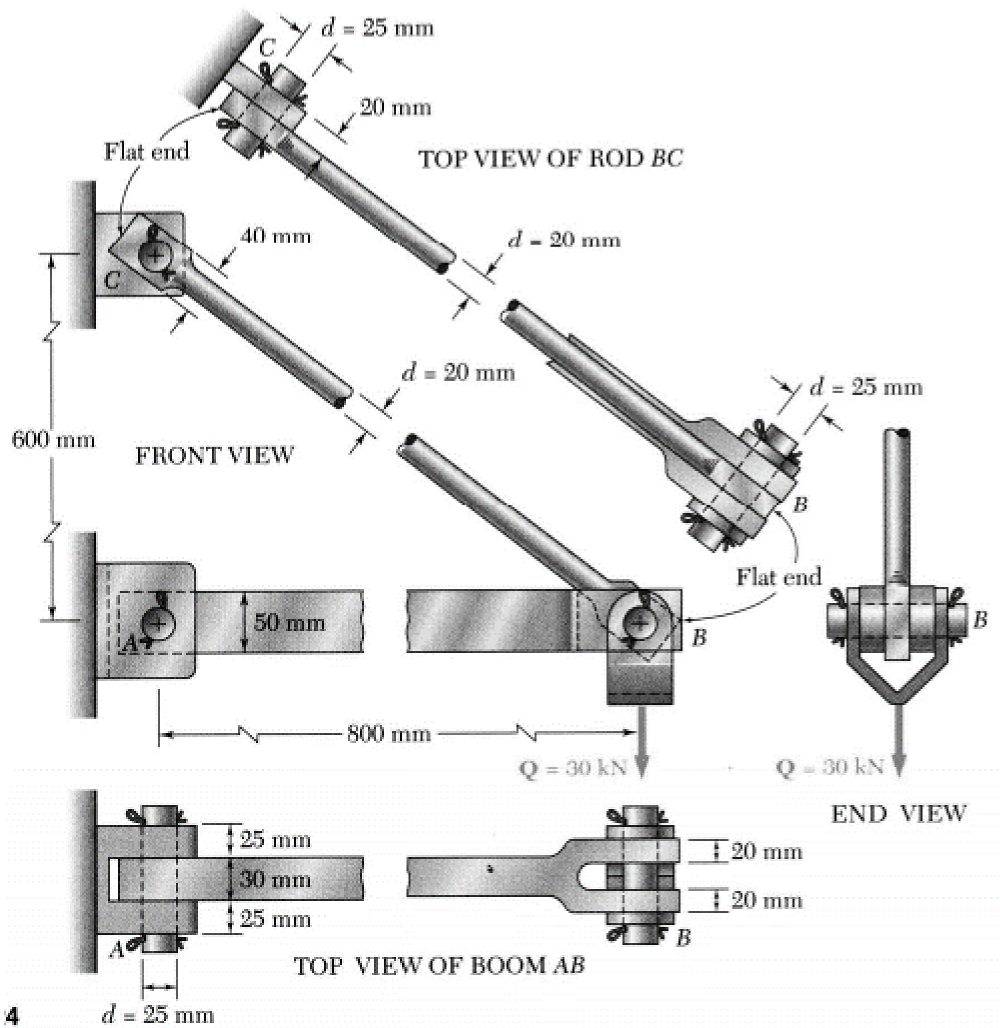
$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^3\,\mathrm{N}}{100\times10^6\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

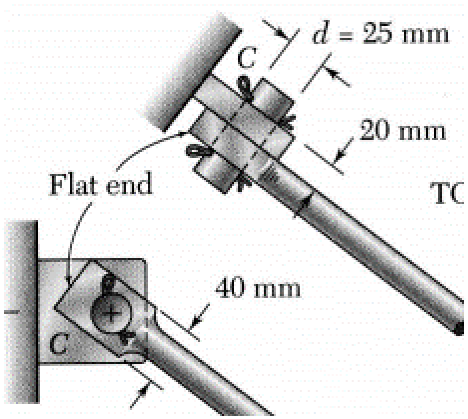
- An aluminum rod 26 mm or more in diameter is adequate

Stress Analysis & Design Example



- Would like to determine the stresses in the members and connections of the structure shown.
- From a statics analysis:
  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)
- Must consider maximum normal stresses in *AB* and *BC*, and the shearing stress and bearing stress at each pinned connection

Rod & Boom Normal Stresses



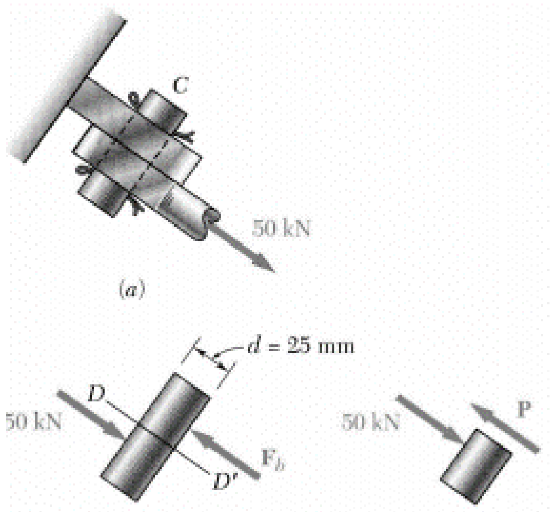
- The rod is in tension with an axial force of 50 kN.
- At the rod center, the average normal stress in the circular cross-section ($A = 314\text{x}10^{-6}\text{m}^2$) is $\sigma_{BC} = +159$ MPa.
- At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline,

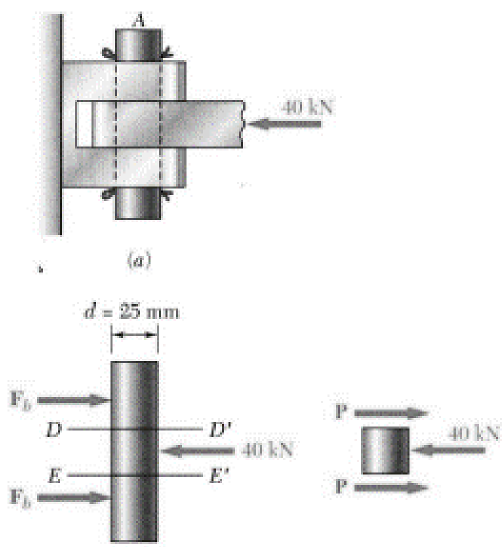
$$A = (20\,\text{mm})(40\,\text{mm} - 25\,\text{mm}) = 300\times10^{-6}\,\text{m}^2$$

$$\sigma_{BC,end} = \frac{P}{A} = \frac{50\times10^3\,N}{300\times10^{-6}\,\text{m}^2} = 167\,\text{MPa}$$

- The boom is in compression with an axial force of 40 kN and average normal stress of –26.7 MPa.
- Since *AB* in compression, minimum section remain unstressed at A in the boom. Only bearing stress takes place.

Pin Shearing Stresses





- The cross-sectional area for pins at A, B, and C,

$$A = \pi r^2 = \pi\left(\frac{25\,\text{mm}}{2}\right)^2 = 491\times10^{-6}\,\text{m}^2$$

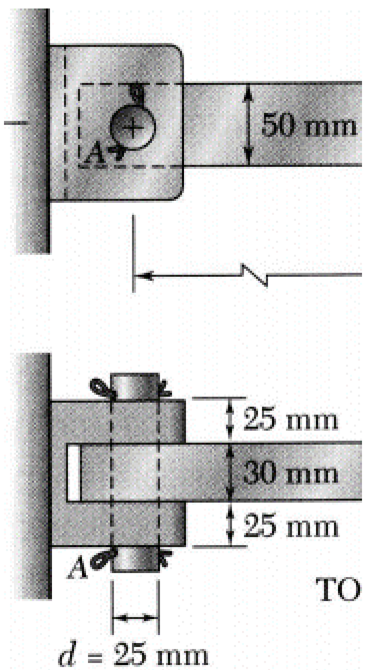
- The force on the pin at C is equal to the force exerted by the rod BC,

$$\tau_{C,ave} = \frac{P}{A} = \frac{50\times10^3\,\text{N}}{491\times10^{-6}\,\text{m}^2} = 102\,\text{MPa}$$

- The pin at A is in double shear with a total force equal to the force exerted by the boom AB,

$$\tau_{A,ave} = \frac{P}{A} = \frac{20\,\text{kN}}{491\times10^{-6}\,\text{m}^2} = 40.7\,\text{MPa}$$

Pin Bearing Stresses



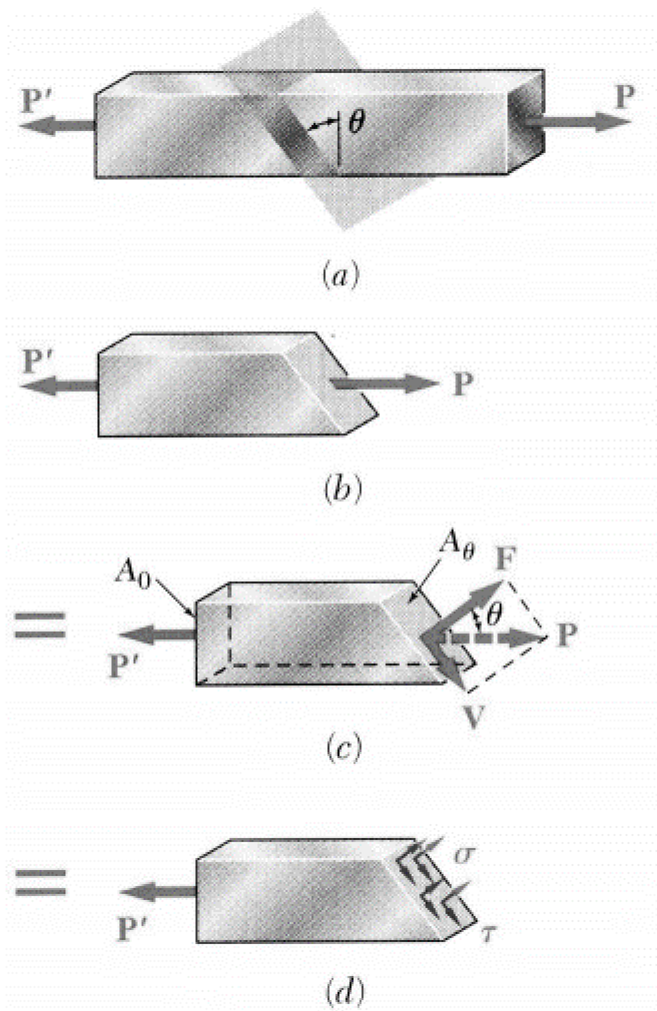
- To determine the bearing stress at A in the boom AB, we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(30\,\mathrm{mm})(25\,\mathrm{mm})} = 53.3\,\mathrm{MPa}$$

- To determine the bearing stress at A in the bracket, we have t = 2(25 mm) = 50 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(50\,\mathrm{mm})(25\,\mathrm{mm})} = 32.0\,\mathrm{MPa}$$

Stress on an Oblique Plane



- Pass a section through the member forming an angle θ with the normal plane.

- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force *P*.

- Resolve *P* into components normal and tangential to the oblique section,

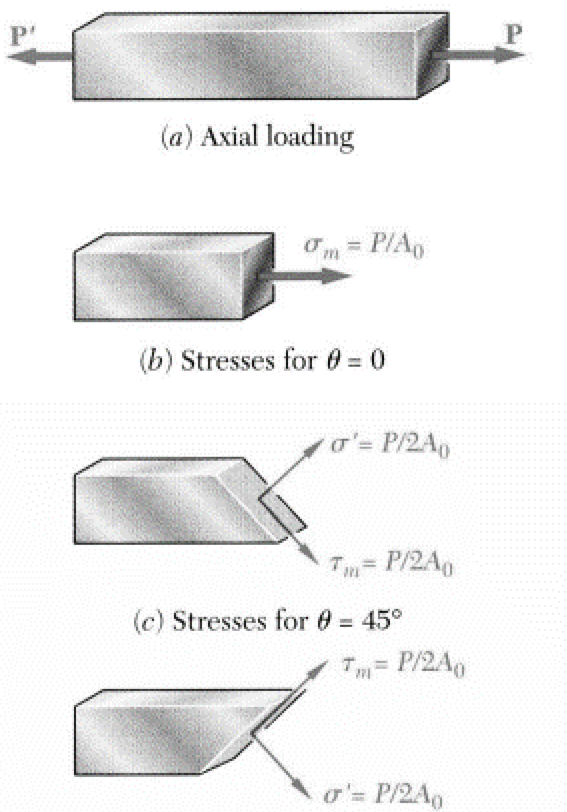
$$F = P\cos\theta \qquad V = P\sin\theta$$

- The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

Maximum Stresses



(*a*) Axial loading

(*b*) Stresses for $\theta = 0$

(*c*) Stresses for $\theta = 45°$

(*d*) Stresses for $\theta = -45°$

- Normal and shearing stresses on an oblique plane

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

- The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

$$\sigma_m = \frac{P}{A_0} \qquad \tau' = 0$$

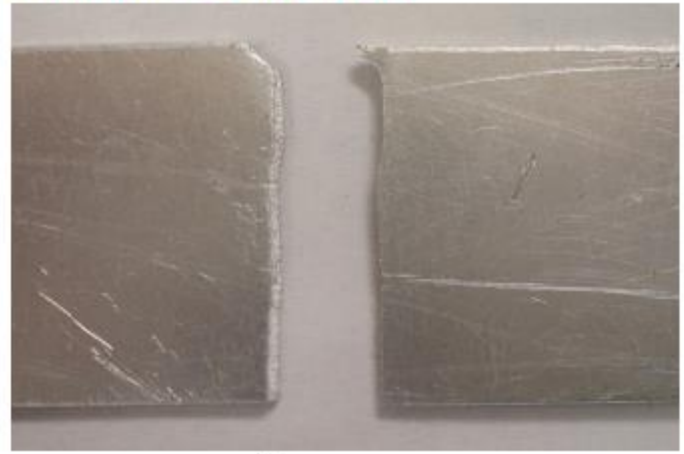
- The maximum shear stress occurs for a plane at $\pm$ 45° with respect to the axis,

$$\tau_m = \frac{P}{A_0}\sin 45\cos 45 = \frac{P}{2A_0} = \sigma'$$

Maximum Normal Stress occurs when θ=0° and $\sigma_{max} = \sigma_x$

Maximum Shear Stress occurs when θ=±45° and $|\tau_{max}| = \sigma_x / 2$

Failure of Aluminum in Tensile Test



Failure at 45° plane from a simple tensile test

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

$$FS = \text{Factor of safety}$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to structures integrity
- risk to life and property
- influence on machine function