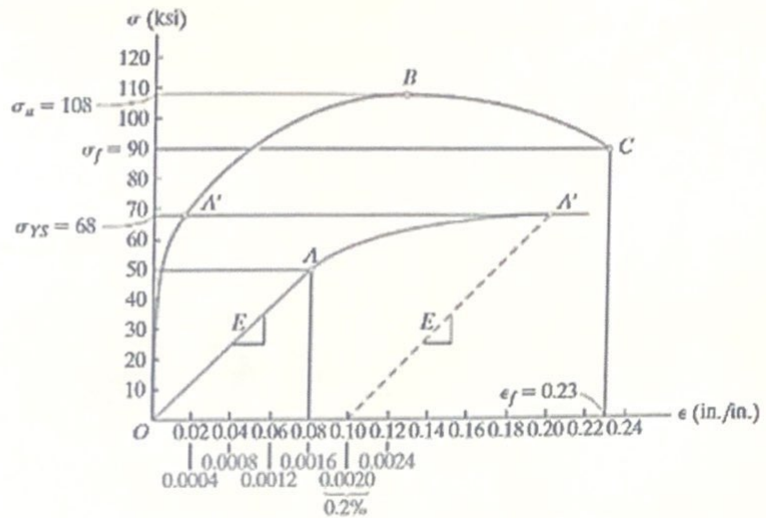


A tension test for a steel alloy results in the stress-strain diagram shown in Fig. 3-18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.



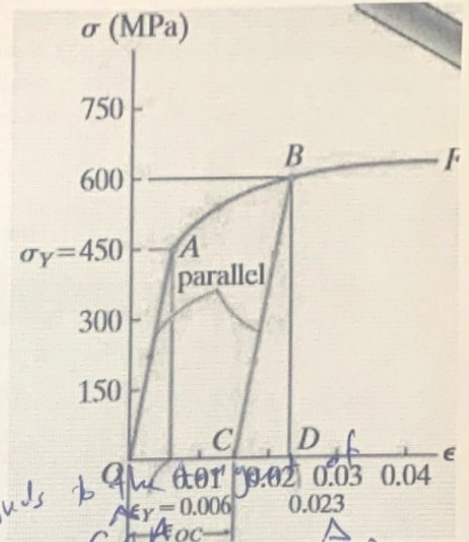
• The Modulus of Elasticity E

$$E = \frac{\sigma}{\epsilon} = \frac{345 \times 10^6}{0.0016} = \boxed{215 \times 10^9 \text{ Pa}}$$

• The yield strength:
on the ϵ axis, we specify the point $\epsilon_0 = 0.002$ then we draw from this point a parallel to the line of the elastic behaviour area. the line is intersecting the ~~graph~~ curve in A'. the stress at A' is 469 MPa $\Rightarrow \boxed{\sigma_{ys} = 469 \text{ MPa}}$

• From the graph, the ultimate stress is 745,2 MPa and the fracture stress is 621 MPa

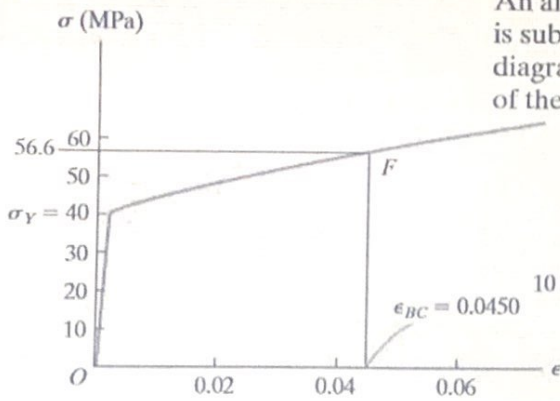
The stress-strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3-19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.



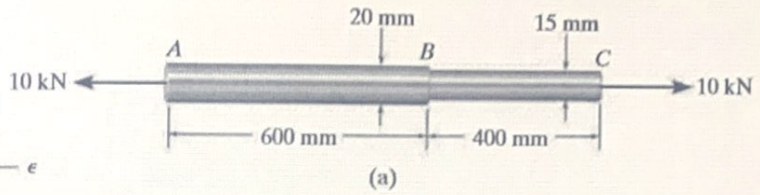
Solution:

- $E = \frac{\sigma_y}{\epsilon_y} = \frac{450 \times 10^6}{0.006} = 75 \times 10^9 \text{ Pa} \Rightarrow \text{Equals to } E$
 We draw from B a line $\vec{BC} \parallel \vec{AO}$ then from the triangle CBD
 $\Rightarrow \tan \angle DCB = E = \frac{BD}{CD} \Rightarrow 75 \times 10^9 = \frac{600 \times 10^6}{CD} \Rightarrow \boxed{CD = 0.008}$
 The permanent strain = $OC = OD - CD = 0.023 - 0.008$
 $= \underline{\underline{0.015}}$

- The modulus of resilience before load application
 $U_{r \text{ initial}} = \frac{1}{2} \sigma_{pl} \cdot \epsilon_{pl} = \frac{1}{2} (450) (0.006)$
 $= \underline{\underline{1.35 \text{ MJ/m}^3}}$
- The modulus of resilience after load application
 $U_{r \text{ final}} = \frac{1}{2} \sigma_{pl} \cdot \epsilon_{pl} = \frac{1}{2} (600) (0.008)$
 $= \underline{\underline{2.40 \text{ MJ/m}^3}}$



An aluminum rod, shown in Fig. 3-20a, has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress-strain diagram is shown in Fig. 3-20b, determine the approximate elongation of the rod when the load is applied. Take $E_{al} = 70 \text{ GPa}$.



Solution

• let's find the stresses:

$$\sigma_{AB} = \frac{P}{A} = \frac{10 \times 10^3}{\frac{\pi}{4} (20 \times 10^{-3})^2} = 31,83 \times 10^6 \text{ Pa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10 \times 10^3}{\frac{\pi}{4} (15 \times 10^{-3})^2} = 56,59 \times 10^6 \text{ Pa}$$

• we will calculate the elongations from the strains at AB, the applied σ_{AB} is less than $\sigma_Y = 40 \times 10^6 \Rightarrow$ AB is within the elastic behaviour area \Rightarrow we can apply Hooke's law $\Rightarrow E = \frac{\sigma}{\epsilon} = \frac{31,83 \times 10^6}{\epsilon} = 70 \times 10^9$

$$\Rightarrow \epsilon_{AB} = \frac{31,83 \times 10^6}{70 \times 10^9} = 0,0004547$$

$$\Rightarrow \delta_{AB} = \epsilon_{AB} \cdot l_{AB} = 0,0004547 \times 600 = \boxed{0,272 \text{ mm}}$$

* ~~the~~ the stress in BC is greater than $\sigma_Y = 40 \times 10^6 \Rightarrow$ we can't apply Hooke's law \Rightarrow from the graph, when $\sigma = 56,6 \text{ MPa} \Rightarrow \epsilon_{BC} = 0,045$

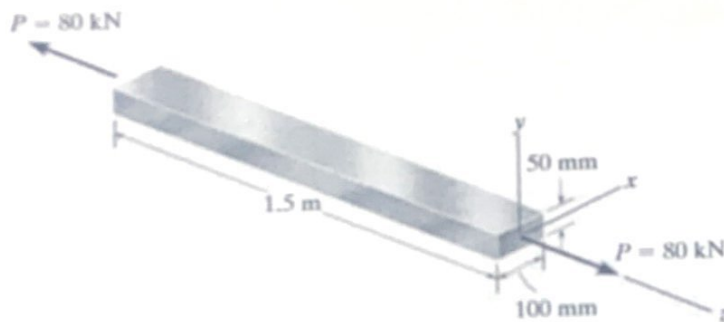
$$\delta_{BC} = \epsilon_{BC} \cdot l_{BC} = 0,045 \times 400 = \boxed{18 \text{ mm}}$$

$$\delta_{BC} = \epsilon_{BC} \cdot l_{BC} = 0,045 \times 400 = \boxed{18 \text{ mm}}$$

$$\Rightarrow \delta_{\text{total}} = 0,272 + 18 = \underline{\underline{18,272 \text{ mm}}}$$

(3)

A bar made of A-36 steel has the dimensions shown in Fig. 5-22. If an axial force of $P = 80 \text{ kN}$ is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.



Solution

- the change in the length:

$$\sigma = \frac{P}{A} = \frac{80 \times 10^3}{50 \times 100 \times 10^{-6}} = 16 \times 10^6 \text{ Pa}$$

From the tables $\Rightarrow E = 200 \text{ GPa}$

$$\sigma = E \cdot \epsilon \Rightarrow \epsilon = \frac{\sigma}{E} = \frac{16 \times 10^6}{200 \times 10^9} = 8 \times 10^{-5}$$

$$\Rightarrow \epsilon = \frac{\delta}{L_0} \Rightarrow \delta = \epsilon \cdot L_0 = 8 \times 10^{-5} \times 1.5 = \boxed{120 \times 10^{-6} \text{ m}}$$

- the change in the cross section:

$$\nu = - \frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

ν from the tables is 0,32

$$\Rightarrow 0,32 = - \frac{\epsilon_{\text{lat}}}{8 \times 10^{-5}} \Rightarrow \epsilon_{\text{lat}} = \epsilon_x = \epsilon_y = -25,6 \cdot 10^{-6}$$

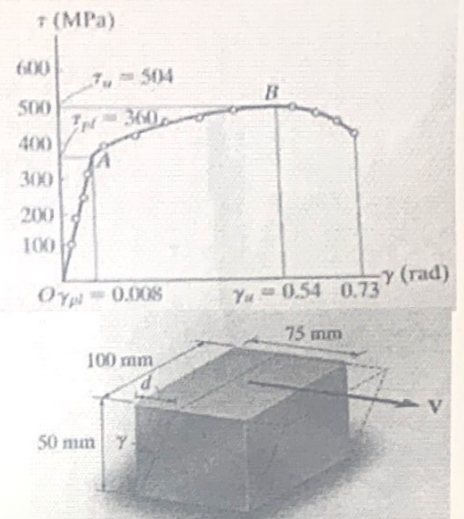
$$\delta_x = \epsilon_x \cdot l_x = -25,6 \times 10^{-6} \times 100 \times 10^{-3} = \underline{\underline{-2,56 \times 10^{-6} \text{ m}}}$$

$$\delta_y = \epsilon_y \cdot l_y = -25,6 \times 10^{-6} \times 50 \times 10^{-3} = \underline{\underline{-1,28 \times 10^{-6} \text{ m}}}$$

(4)

A specimen of titanium alloy is tested in torsion and the shear stress-strain diagram is shown in Fig. 3-25a. Determine the shear modulus G , the proportional limit, and the ultimate shear stress. Also, determine the maximum distance d that the top of a block of this material, shown in Fig. 3-25b, could be displaced horizontally if the material behaves elastically when acted upon by a shear force V . What is the magnitude of V necessary to cause this displacement?

SOLUTION



Solution

- the shear modulus

$$G = \frac{360 \times 10^6}{0.008} = 45 \times 10^9 \text{ Pa}$$

- The proportional limit from the graph

$$\tau = 360 \text{ MPa}$$

- the ultimate shear stress from the graph

$$\tau_u = 504 \text{ MPa}$$

- maximum d ,

Maximum d happens when the shear strain is the maximum within the Elastic Behavior area which means when

$$\gamma = \gamma_{pl} = 0.008$$

$$\Rightarrow \tan \gamma \approx \gamma = \frac{d}{50}$$

$$\Rightarrow \boxed{d = 0.4 \text{ mm}}$$

- the magnitude of V .

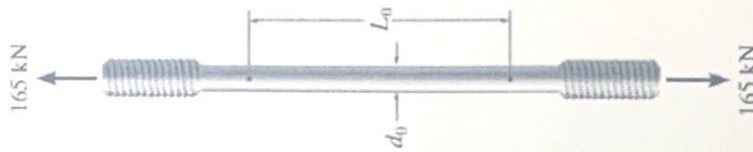
$$\tau = \frac{V}{A} \Rightarrow V = \tau \cdot A$$

$$V = 360 \times 10^6 \times 75 \times 10^{-3} \times 50 \times 10^{-3}$$

$$\boxed{V = 2700 \times 10^3 \text{ N}}$$

(5)

An aluminum specimen shown in Fig. 3-26 has a diameter of $d_0 = 25 \text{ mm}$ and a gauge length of $L_0 = 250 \text{ mm}$. If a force of 165 kN elongates the gauge length 1.20 mm , determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take $G_{al} = 26 \text{ GPa}$ and $\sigma_Y = 440 \text{ MPa}$.



Solution

• the modulus of Elasticity

$$E = \frac{\sigma}{\epsilon}$$

$$\sigma = \frac{P}{A} = \frac{165 \times 10^3}{\frac{\pi}{4} (25 \times 10^{-3})^2} = 336,1 \times 10^6 \text{ Pa}$$

$$\epsilon = \frac{\delta}{L_0} = \frac{1,2 \times 10^{-3}}{250 \times 10^{-3}} = 0,00480$$

$$\Rightarrow E = \frac{336,1 \times 10^6}{0,00480}$$

$$E = 70 \times 10^9 \text{ Pa}$$

• the contraction of the Diameter

$$\nu = - \frac{\epsilon_{lat}}{\epsilon_{long}} = - \frac{\epsilon_{lat}}{0,00480}$$

We need ν .

$$G = \frac{E}{2(1+\nu)}$$

$$26 \times 10^9 = \frac{70 \times 10^9}{2(1+\nu)}$$

$$\Rightarrow \nu = 0,3471$$

$$\Rightarrow \frac{-\epsilon_{lat}}{0,00480} = 0,347$$

$$\Rightarrow \epsilon_{lat} = -0,001661$$

$$\epsilon_{lat} = \frac{\delta}{d}$$

$$0,00166 = \frac{\delta}{25}$$

$$\Rightarrow \delta = 0,0416 \text{ mm}$$

(6)