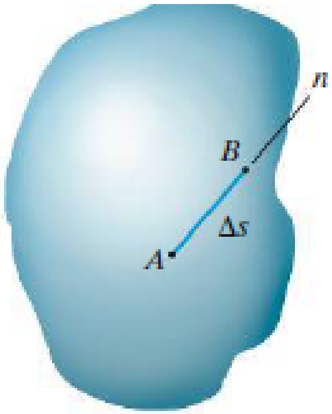
Strain

Lecture 4

2.2 Strain

In order to describe the deformation of a body by changes in length of line segments and the changes in the angles between them, we will develop the concept of strain. Strain is actually measured by experiments, and once the strain is obtained, it will be shown in the next chapter how it can be related to the stress acting within the body.

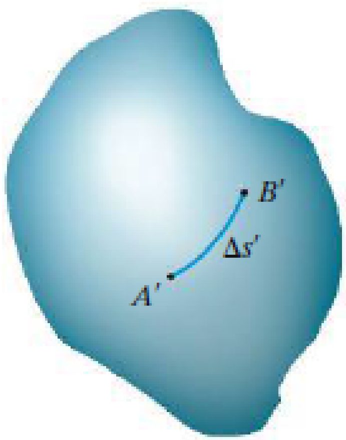


Undeformed body

(a)

Normal Strain. If we define the normal strain as the change in length of a line per unit length, then we will not have to specify the *actual length* of any particular line segment. Consider, for example, the line AB, which is contained within the undeformed body shown in Fig. 2–1*a*. This line lies along the n axis and has an original length of Δs. After deformation, points A and B are displaced to A' and B', and the line becomes a curve having a length of $\Delta s'$, Fig. 2–1*b*. The change in length of the line is therefore $\Delta s' - \Delta s$. If we define the *average normal strain* using the symbol ϵ_{avg} (epsilon), then

$$\boxed{\epsilon_{\text{avg}} = \frac{\Delta s' - \Delta s}{\Delta s}} \tag{2–1}$$

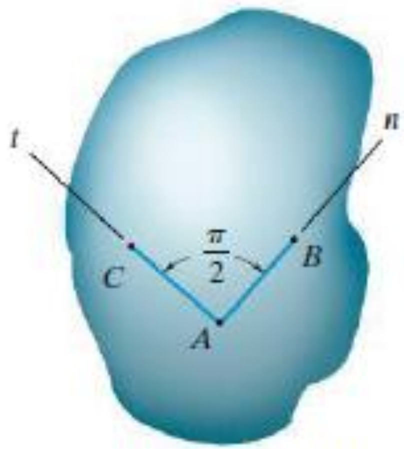


Deformed body

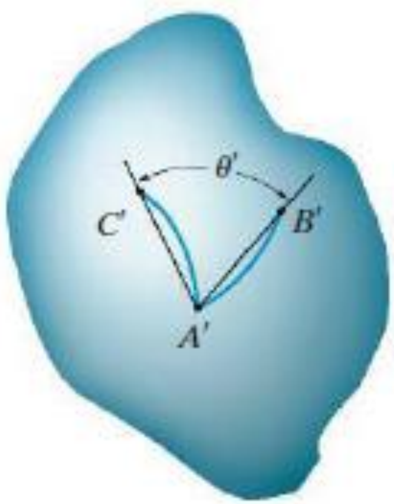
(b)

As point B is chosen closer and closer to point A, the length of the line will become shorter and shorter, such that $\Delta s \rightarrow 0$. Also, this causes B' to approach A', such that $\Delta s' \rightarrow 0$. Consequently, in the limit the normal strain at *point* A and in the direction of n is

$$\boxed{\epsilon = \lim_{B \rightarrow A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}} \tag{2–2}$$



Undeformed body
(a)

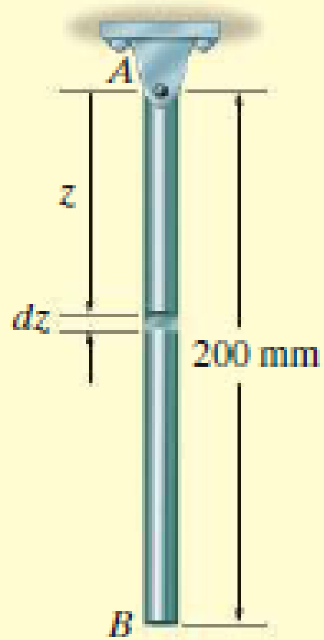


Deformed body
(b)

Shear Strain. Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between them is referred to as ***shear strain***. This angle is denoted by γ (gamma) and is always measured in radians (rad), which are dimensionless. For example, consider the line segments AB and AC originating from the same point A in a body, and directed along the perpendicular n and t axes, Fig. 2–2*a*. After deformation, the ends of both lines are displaced, and the lines themselves become curves, such that the angle between them at A is θ', Fig. 2–2*b*. Hence the shear strain at point A associated with the n and t axes becomes

$$\boxed{\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'} \tag{2–3}$$

The slender rod shown in Fig. 2–4 is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\epsilon_z = 40(10^{-3})z^{1/2}$, where z is measured in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.



SOLUTION

Part (a). Since the normal strain is reported at each point along the rod, a differential segment dz, located at position z, Fig. 2–4, has a deformed length that can be determined from Eq. 2–1; that is,

$$dz' = dz + \epsilon_z\, dz$$
$$dz' = \left[1 + 40(10^{-3})z^{1/2}\right]dz$$

The sum of these segments along the axis yields the *deformed length* of the rod, i.e.,

$$\begin{aligned} z' &= \int_0^{0.2\text{ m}} \left[1 + 40(10^{-3})z^{1/2}\right]dz \\ &= \left[z + 40(10^{-3})\tfrac{2}{3}z^{3/2}\right]\Big|_0^{0.2\text{ m}} \\ &= 0.20239\text{ m} \end{aligned}$$

The displacement of the end of the rod is therefore

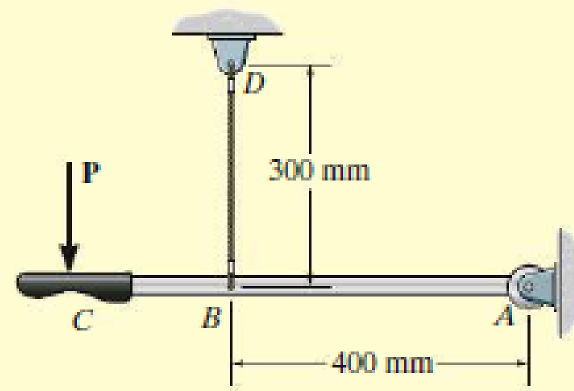
$$\Delta_B = 0.20239\text{ m} - 0.2\text{ m} = 0.00239\text{ m} = 2.39\text{ mm}\downarrow \qquad \textit{Ans.}$$

Part (b). The average normal strain in the rod is determined from Eq. 2–1, which assumes that the rod or "line segment" has an original length of 200 mm and a change in length of 2.39 mm. Hence,

$$\epsilon_{\text{avg}} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{2.39\text{ mm}}{200\text{ mm}} = 0.0119\text{ mm/mm} \qquad \textit{Ans.}$$

This strain is called a thermal strain, caused by temperature, *not* by any load.

When force **P** is applied to the rigid lever arm ABC in Fig. 2–5a, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain developed in wire BD.



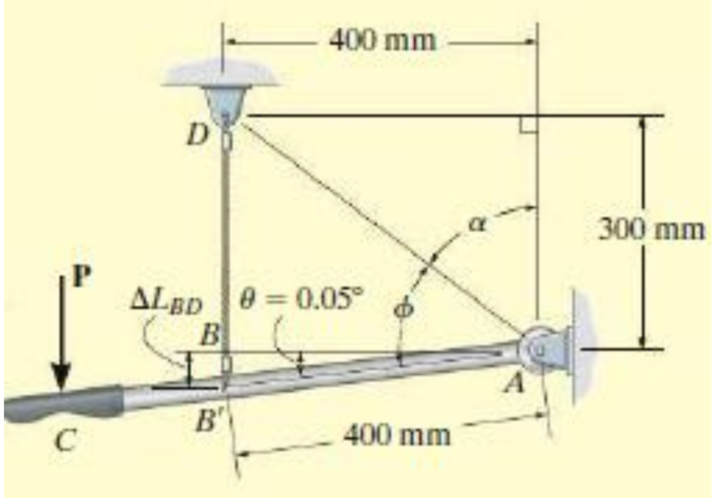
SOLUTION II

Since the strain is small, this same result can be obtained by approximating the elongation of wire BD as ΔL_{BD}, shown in Fig. 2–5b. Here,

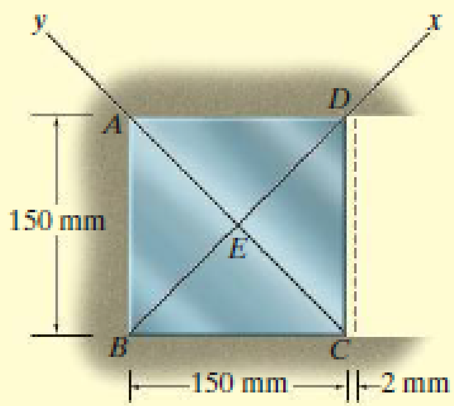
$$\Delta L_{BD} = \theta L_{AB} = \left[\left(\frac{0.05^\circ}{180^\circ}\right)(\pi \text{ rad})\right](400 \text{ mm}) = 0.3491 \text{ mm}$$

Therefore,

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.3491 \text{ mm}}{300 \text{ mm}} = 0.00116 \text{ mm/mm} \qquad \textit{Ans.}$$



(b)



(a)

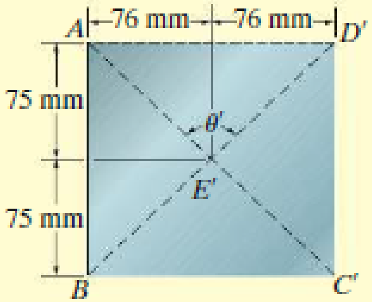
The plate shown in Fig. 2–6*a* is fixed connected along AB and held in the horizontal guides at its top and bottom, AD and BC. If its right side CD is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal AC, and (b) the shear strain at E relative to the x, y axes.

SOLUTION

Part (a). When the plate is deformed, the diagonal AC becomes AC', Fig. 2–6*b*. The lengths of diagonals AC and AC' can be found from the Pythagorean theorem. We have

$$AC = \sqrt{(0.150\text{ m})^2 + (0.150\text{ m})^2} = 0.21213\text{ m}$$

$$AC' = \sqrt{(0.150\text{ m})^2 + (0.152\text{ m})^2} = 0.21355\text{ m}$$



(b)

Fig. 2–6

Therefore the average normal strain along the diagonal is

$$(\epsilon_{AC})_{\text{avg}} = \frac{AC' - AC}{AC} = \frac{0.21355\text{ m} - 0.21213\text{ m}}{0.21213\text{ m}}$$

$$= 0.00669\text{ mm/mm} \qquad \textit{Ans.}$$

Part (b). To find the shear strain at E relative to the x and y axes, it is first necessary to find the angle θ' after deformation, Fig. 2–6*b*. We have

$$\tan\left(\frac{\theta'}{2}\right) = \frac{76\text{ mm}}{75\text{ mm}}$$

$$\theta' = 90.759^\circ = \left(\frac{\pi}{180^\circ}\right)(90.759^\circ) = 1.58404\text{ rad}$$

Applying Eq. 2–3, the shear strain at E is therefore

$$\gamma_{xy} = \frac{\pi}{2} - 1.58404\text{ rad} = -0.0132\text{ rad} \qquad \textit{Ans.}$$

Mechanical Properties of Materials

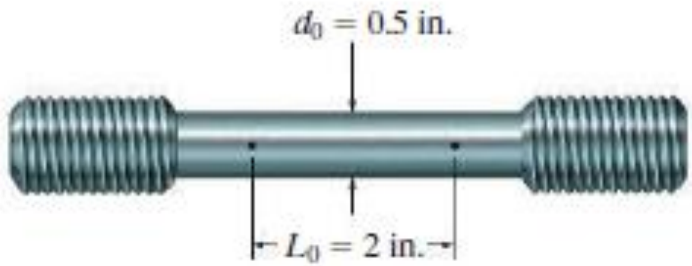
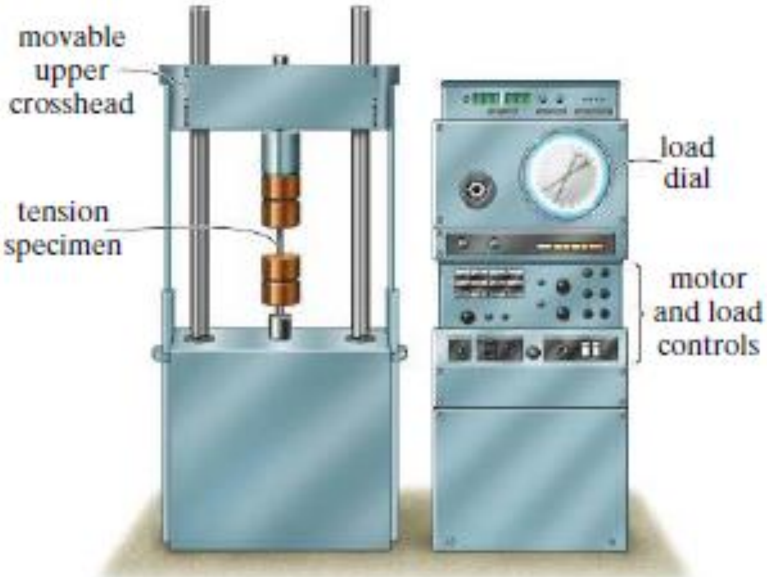
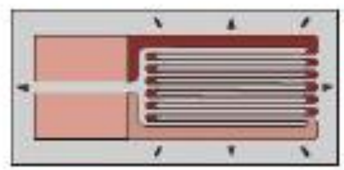


Fig. 3–1

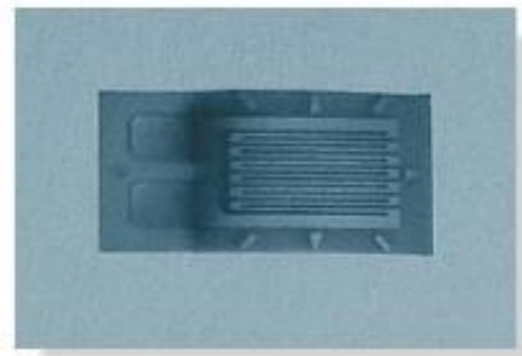
3

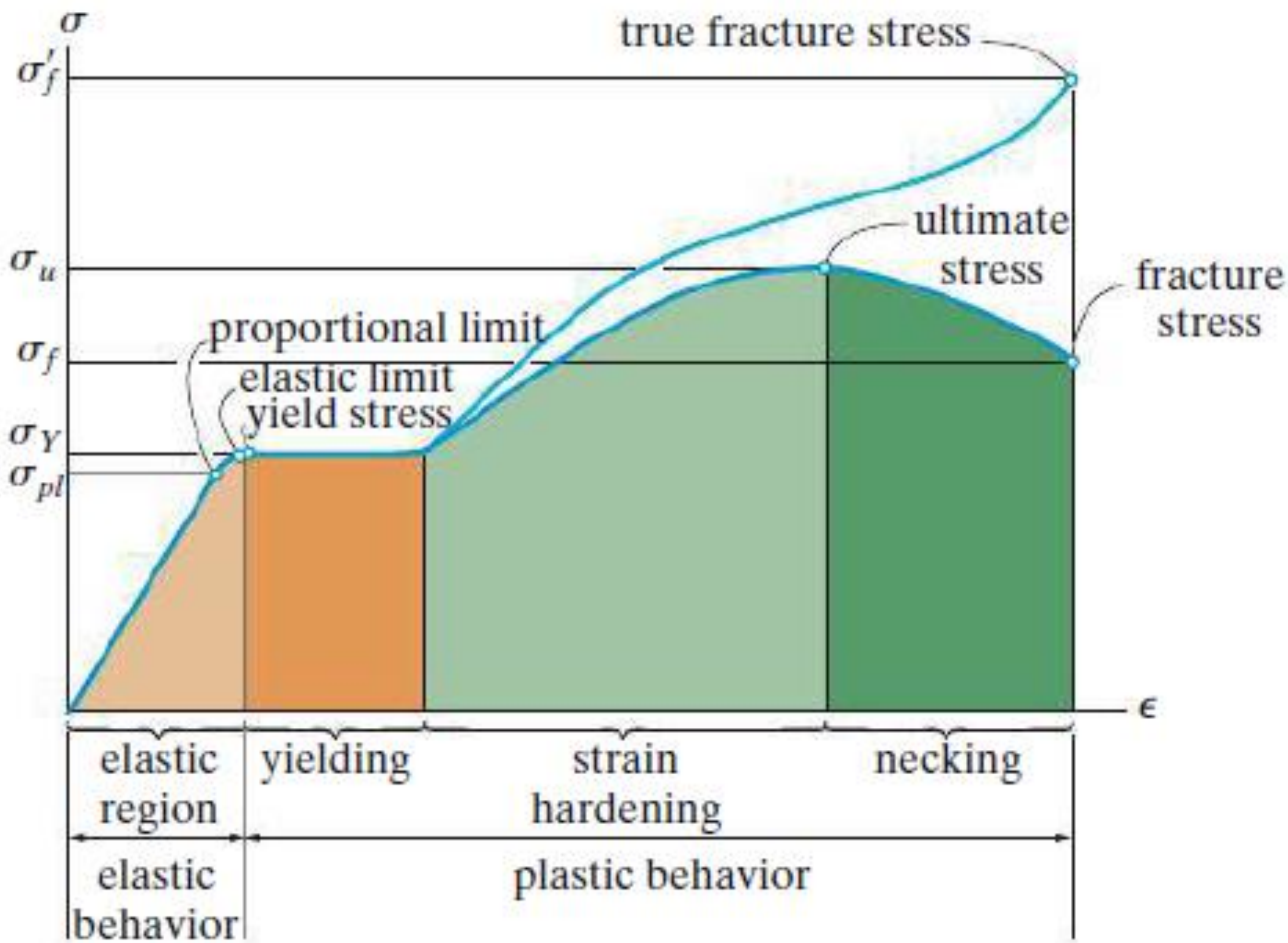
Typical steel specimen with attached strain gauge.





Electrical–resistance
strain gauge





Conventional and true stress-strain diagrams
for ductile material (steel) (not to scale)

Necking

(a)

Failure of a
ductile material

(b)

Elastic Behavior. Elastic behavior of the material occurs when the strains in the specimen are within the light orange region shown in Fig. 3–4 . Here the curve is actually a *straight line* throughout most of this region, so that the stress is *proportional* to the strain. The material in this region is said to be *linear elastic* . The upper stress limit to this linear relationship is called the ***proportional limit*** , s_{pl} . If the stress slightly exceeds the proportional limit, the curve tends to bend and flatten out as shown. This continues until the stress reaches the ***elastic limit*** . Upon reaching this point, if the load is removed the specimen will still return back to its original shape. Normally for steel, however, the elastic limit is seldom determined, since it is very close to the proportional limit and therefore rather difficult to detect.

Yielding. A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently* . This behavior is called ***yielding*** , and it is indicated by the rectangular dark orange region of the curve. The stress that causes yielding is called the ***yield stress*** or ***yield point*** , s_Y , and the deformation that occurs is called ***plastic deformation*** . Although not shown in Fig. 3–4 , for low-carbon steels or those that are hot rolled, the yield point is often distinguished by two values. The ***upper yield point*** occurs first, followed by a sudden decrease in load-carrying capacity to a ***lower yield point*** . Notice that once the yield point is reached, then as shown in Fig. 3–4 , the specimen will continue to elongate (strain) *without any increase in load* . When the material is in this state, it is often referred to as being ***perfectly plastic***

Strain Hardening. When yielding has ended, an increase in load can be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ***ultimate stress*** , s*u* . The rise in the curve in this manner is called ***strain hardening*** , and it is identified in Fig. 3–4 as the region in light green.

Necking. Up to the ultimate stress, as the specimen elongates, its cross-sectional area will decrease. This decrease is fairly *uniform* over the specimen's entire gauge length; however, just after, at the ultimate stress, the cross-sectional area will begin to decrease in a *localized* region of the specimen. As a result, a constriction or "neck" tends to form in this region as the specimen elongates further, Fig. 3–5 *a* . This region of the curve due to necking is indicated in dark green in Fig. 3–4 . Here the stress–strain diagram tends to curve downward until the specimen breaks at the ***fracture stress*** , s*f* , Fig. 3–5 *b* .