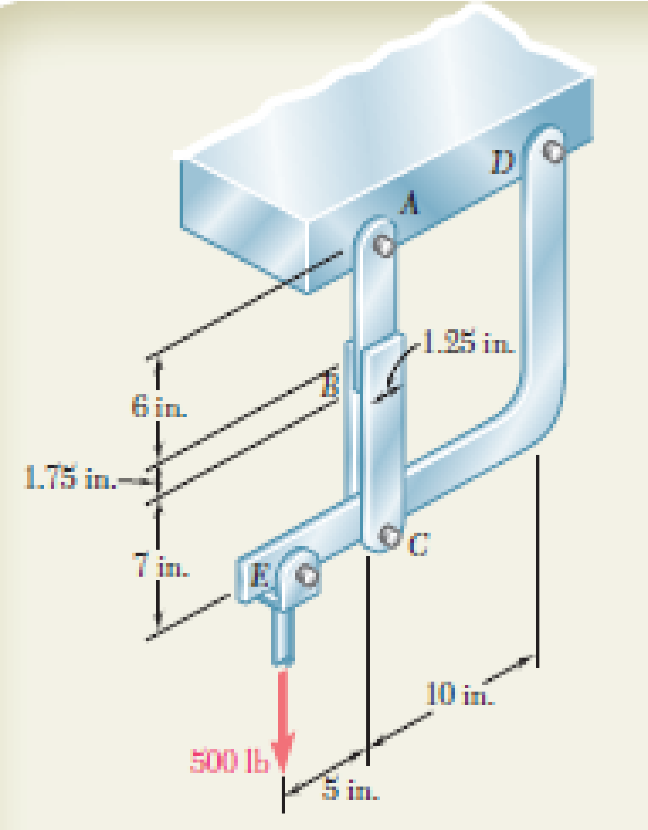
Lecture 3

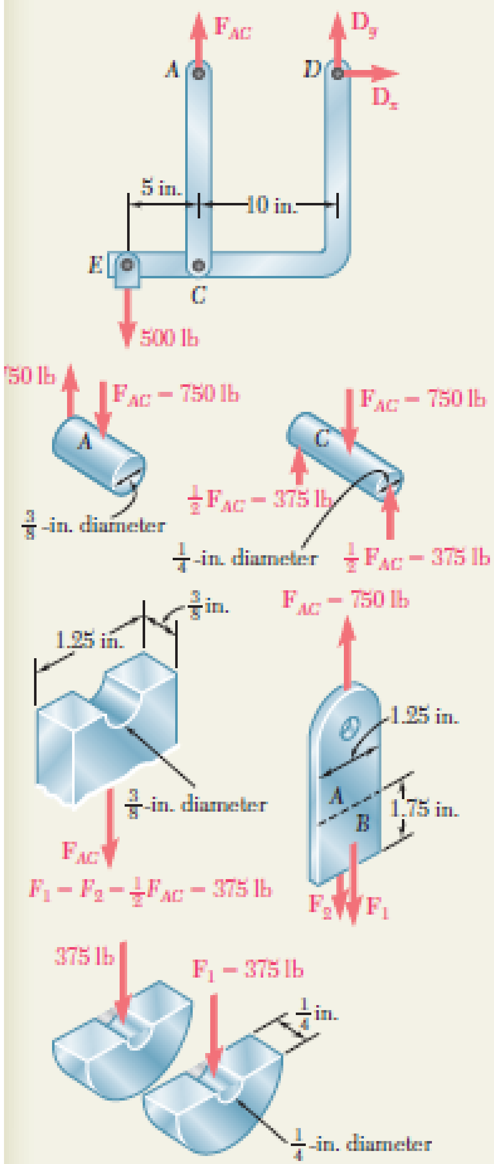
SAMPLE PROBLEM 1.1

In the hanger shown, the upper portion of link *ABC* is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at *B*. The pin at *A* is of $\frac{3}{8}$-in. diameter while a $\frac{1}{4}$-in.-diameter pin is used at *C*. Determine (*a*) the shearing stress in pin *A*, (*b*) the shearing stress in pin *C*, (*c*) the largest normal stress in link *ABC*, (*d*) the average shearing stress on the bonded surfaces at *B*, (*e*) the bearing stress in the link at *C*.



SOLUTION

Free Body: Entire Hanger. Since the link ABC is a two-force member, the reaction at A is vertical; the reaction at D is represented by its components $\mathbf{D}_x$ and $\mathbf{D}_y$. We write



$$+\circlearrowleft \Sigma M_D = 0: \qquad (500 \text{ lb})(15 \text{ in.}) - F_{AC}(10 \text{ in.}) = 0$$

$$F_{AC} = +750 \text{ lb} \qquad F_{AC} = 750 \text{ lb} \quad \textit{tension}$$

a. Shearing Stress in Pin *A*. Since this $\frac{3}{8}$-in.-diameter pin is in single shear, we write

$$\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ lb}}{\frac{1}{4}\pi(0.375 \text{ in.})^2} \qquad \tau_A = 6790 \text{ psi} \blacktriangleleft$$

b. Shearing Stress in Pin *C*. Since this $\frac{1}{4}$-in.-diameter pin is in double shear, we write

$$\tau_C = \frac{\frac{1}{2}F_{AC}}{A} = \frac{375 \text{ lb}}{\frac{1}{4}\pi(0.25 \text{ in.})^2} \qquad \tau_C = 7640 \text{ psi} \blacktriangleleft$$

c. Largest Normal Stress in Link *ABC*. The largest stress is found where the area is smallest; this occurs at the cross section at A where the $\frac{3}{8}$-in. hole is located. We have

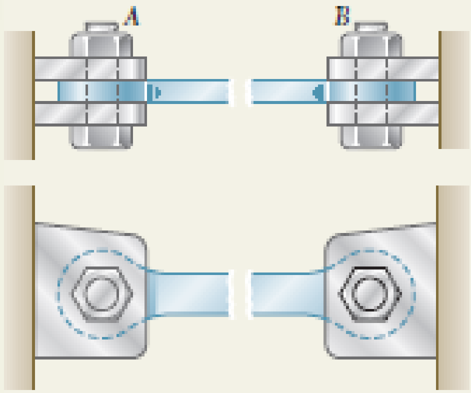
$$\sigma_A = \frac{F_{AC}}{A_{net}} = \frac{750 \text{ lb}}{(\frac{3}{8} \text{ in.})(1.25 \text{ in.} - 0.375 \text{ in.})} = \frac{750 \text{ lb}}{0.328 \text{ in}^2} \qquad \sigma_A = 2290 \text{ psi} \blacktriangleleft$$

d. Average Shearing Stress at *B*. We note that bonding exists on both sides of the upper portion of the link and that the shear force on each side is $F_1 = (750 \text{ lb})/2 = 375$ lb. The average shearing stress on each surface is thus

$$\tau_B = \frac{F_1}{A} = \frac{375 \text{ lb}}{(1.25 \text{ in.})(1.75 \text{ in.})} \qquad \tau_B = 171.4 \text{ psi} \blacktriangleleft$$

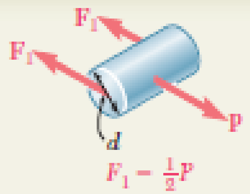
e. Bearing Stress in Link at *C*. For each portion of the link, $F_1 = 375$ lb and the nominal bearing area is $(0.25 \text{ in.})(0.25 \text{ in.}) = 0.0625 \text{ in}^2$.

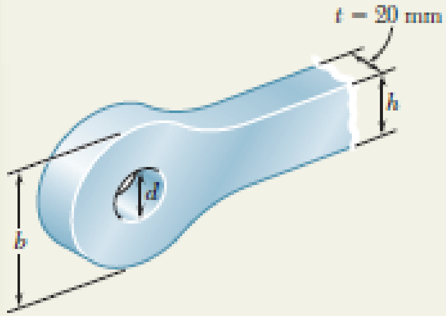
$$\sigma_b = \frac{F_1}{A} = \frac{375 \text{ lb}}{0.0625 \text{ in}^2} \qquad \sigma_b = 6000 \text{ psi} \blacktriangleleft$$

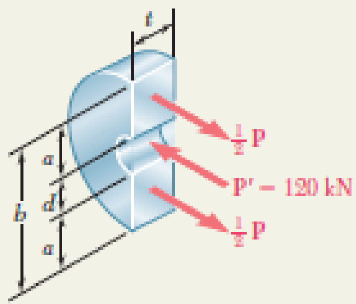


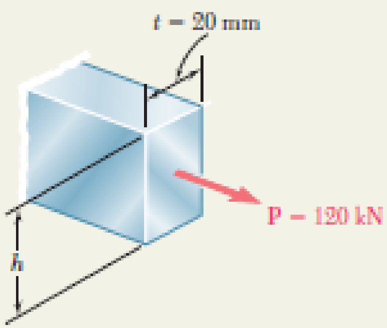
SAMPLE PROBLEM 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (*a*) the diameter d of the bolt, (*b*) the dimension b at each end of the bar, (*c*) the dimension h of the bar.









SOLUTION

a. Diameter of the Bolt. Since the bolt is in double shear, $F_1 = \frac{1}{2}P = 60\ \text{kN}$.

$$\tau = \frac{F_1}{A} = \frac{60\ \text{kN}}{\frac{1}{4}\pi\, d^2} \qquad 100\ \text{MPa} = \frac{60\ \text{kN}}{\frac{1}{4}\pi\, d^2} \qquad d = 27.6\ \text{mm}$$

We will use $\quad d = 28\ \text{mm}$ ◀

At this point we check the bearing stress between the 20-mm-thick plate and the 28-mm-diameter bolt.

$$\tau_b = \frac{P}{td} = \frac{120\ \text{kN}}{(0.020\ \text{m})(0.028\ \text{m})} = 214\ \text{MPa} < 350\ \text{MPa} \qquad \text{OK}$$

b. Dimension *b* at Each End of the Bar. We consider one of the end portions of the bar. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, we write

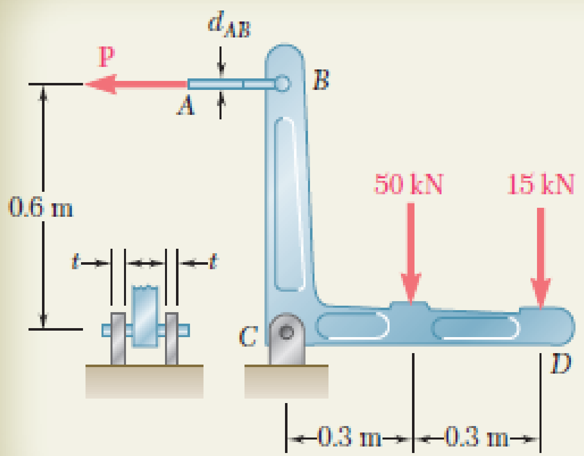
$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175\ \text{MPa} = \frac{60\ \text{kN}}{(0.02\ \text{m})a} \qquad a = 17.14\ \text{mm}$$

$$b = d + 2a = 28\ \text{mm} + 2(17.14\ \text{mm}) \qquad b = 62.3\ \text{mm}$$ ◀

c. Dimension *h* of the Bar. Recalling that the thickness of the steel plate is $t = 20$ mm, we have

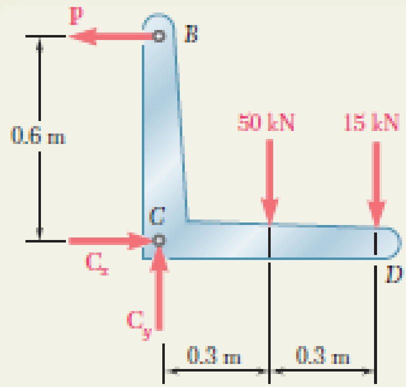
$$\sigma = \frac{P}{th} \qquad 175\ \text{MPa} = \frac{120\ \text{kN}}{(0.020\ \text{m})h} \qquad h = 34.3\ \text{mm}$$

We will use $\quad h = 35\ \text{mm}$ ◀



SAMPLE PROBLEM 1.3

Two forces are applied to the bracket *BCD* as shown. (*a*) Knowing that the control rod *AB* is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at *C* is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin *C* for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at *C* knowing that the allowable bearing stress of the steel used is 300 MPa.



SOLUTION

Free Body: Entire Bracket. The reaction at C is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.6 \text{ m}) - (50 \text{ kN})(0.3 \text{ m}) - (15 \text{ kN})(0.6 \text{ m}) = 0 \quad P = 40 \text{ kN}$$

$$\Sigma F_x = 0: \qquad C_x = 40 \text{ k}$$

$$\Sigma F_y = 0: \qquad C_y = 65 \text{ kN} \qquad C = \sqrt{C_x^2 + C_y^2} = 76.3 \text{ kN}$$

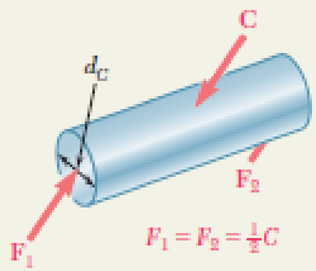
a. Control Rod AB. Since the factor of safety is to be 3.3, the allowable stress is

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600 \text{ MPa}}{3.3} = 181.8 \text{ MPa}$$

For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40 \text{ kN}}{181.8 \text{ MPa}} = 220 \times 10^{-6} \text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6} \text{ m}^2 \qquad d_{AB} = 16.74 \text{ mm} \quad \blacktriangleleft$$



b. Shear in Pin C. For a factor of safety of 3.3, we have

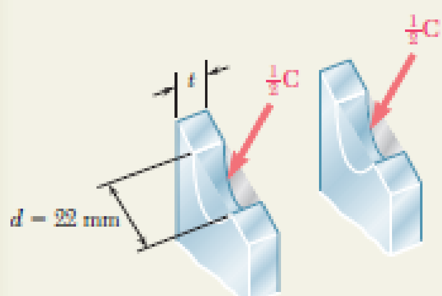
$$\tau_{\text{all}} = \frac{\tau_U}{F.S.} = \frac{350 \text{ MPa}}{3.3} = 106.1 \text{ MPa}$$

Since the pin is in double shear, we write

$$A_{\text{req}} = \frac{C/2}{\tau_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{106.1 \text{ MPa}} = 360 \text{ mm}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_C^2 = 360 \text{ mm}^2 \qquad d_C = 21.4 \text{ mm} \qquad \text{Use: } d_C = 22 \text{ mm} \quad \blacktriangleleft$$

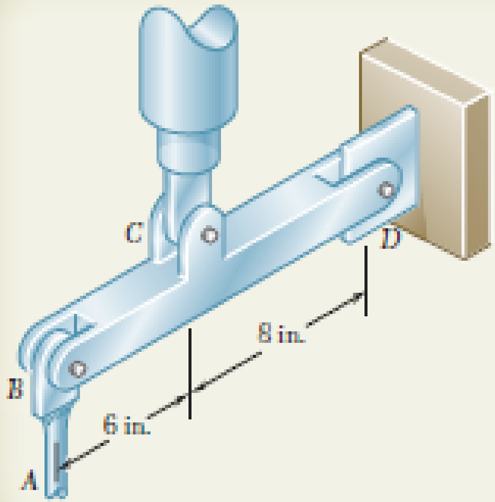
The next larger size pin available is of 22-mm diameter and should be used.



c. Bearing at C. Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write

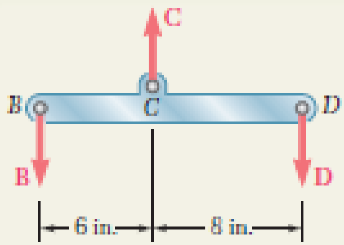
$$A_{\text{req}} = \frac{C/2}{\sigma_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{300 \text{ MPa}} = 127.2 \text{ mm}^2$$

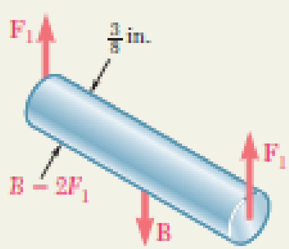
Thus $22t = 127.2 \qquad t = 5.78 \text{ mm} \qquad \text{Use: } t = 6 \text{ mm} \quad \blacktriangleleft$

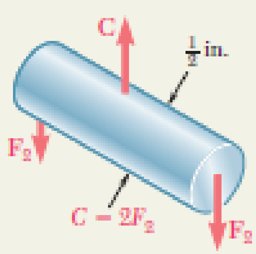


SAMPLE PROBLEM 1.4

The rigid beam BCD is attached by bolts to a control rod at B, to a hydraulic cylinder at C, and to a fixed support at D. The diameters of the bolts used are: $d_B = d_D = \frac{3}{8}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod AB has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C.







SOLUTION

The factor of safety with respect to failure must be 3.0 or more in each of the three bolts and in the control rod. These four independent criteria will be considered separately.

Free Body: Beam *BCD*. We first determine the force at C in terms of the force at B and in terms of the force at D.

$$+\circlearrowleft \Sigma M_D = 0: \quad B(14 \text{ in.}) - C(8 \text{ in.}) = 0 \qquad C = 1.750B \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -D(14 \text{ in.}) + C(6 \text{ in.}) = 0 \qquad C = 2.33D \qquad (2)$$

Control Rod. For a factor of safety of 3.0 we have

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{3.0} = 20 \text{ ksi}$$

The allowable force in the control rod is

$$B = \sigma_{\text{all}}(A) = (20 \text{ ksi})\tfrac{1}{4}\pi(\tfrac{7}{16} \text{ in.})^2 = 3.01 \text{ kips}$$

Using Eq. (1) we find the largest permitted value of C:

$$C = 1.750B = 1.750(3.01 \text{ kips}) \qquad C = 5.27 \text{ kips} \blacktriangleleft$$

Bolt at *B*. $\tau_{\text{all}} = \tau_U/F.S. = (40 \text{ ksi})/3 = 13.33$ ksi. Since the bolt is in double shear, the allowable magnitude of the force **B** exerted on the bolt is

$$B = 2F_1 = 2(\tau_{\text{all}}A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{3}{8} \text{ in.})^2 = 2.94 \text{ kips}$$

From Eq. (1): $\quad C = 1.750B = 1.750(2.94 \text{ kips}) \qquad C = 5.15 \text{ kips} \blacktriangleleft$

Bolt at *D*. Since this bolt is the same as bolt B, the allowable force is $D = B = 2.94$ kips. From Eq. (2):

$$C = 2.33D = 2.33(2.94 \text{ kips}) \qquad C = 6.85 \text{ kips} \blacktriangleleft$$

Bolt at *C*. We again have $\tau_{\text{all}} = 13.33$ ksi and write

$$C = 2F_2 = 2(\tau_{\text{all}}A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{1}{2} \text{ in.})^2 \qquad C = 5.23 \text{ kips} \blacktriangleleft$$

Summary. We have found separately four maximum allowable values of the force C. In order to satisfy all these criteria we must choose the smallest value, namely: $\qquad C = 5.15 \text{ kips} \blacktriangleleft$

1.37 Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What is the safety factor used if the structure shown was designed to support a 16-kN load **P**?