Mechatronic Instrumentations

004.0

Introduction

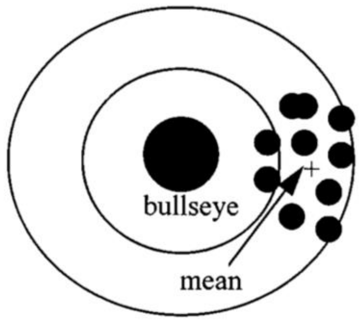
Dr Ezideen A Hasso

Characteristics of instruments

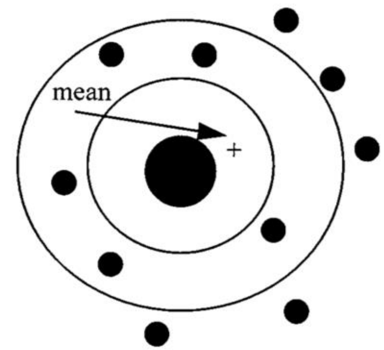
Measurement is a process of mapping actually occurring variables into equivalent values. Deviations from perfect measurement mappings are called errors:

Characteristics of Instrument

- Static Characteristics of Instrument Systems Output/Input Relationship
- Drift
- Hysteresis and Backlash
- Saturation
- Bias
- Error of Nonlinearity



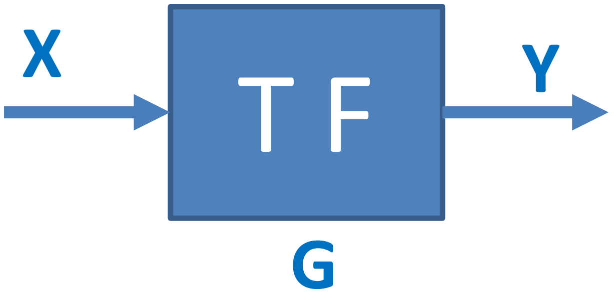
(a)



(b)

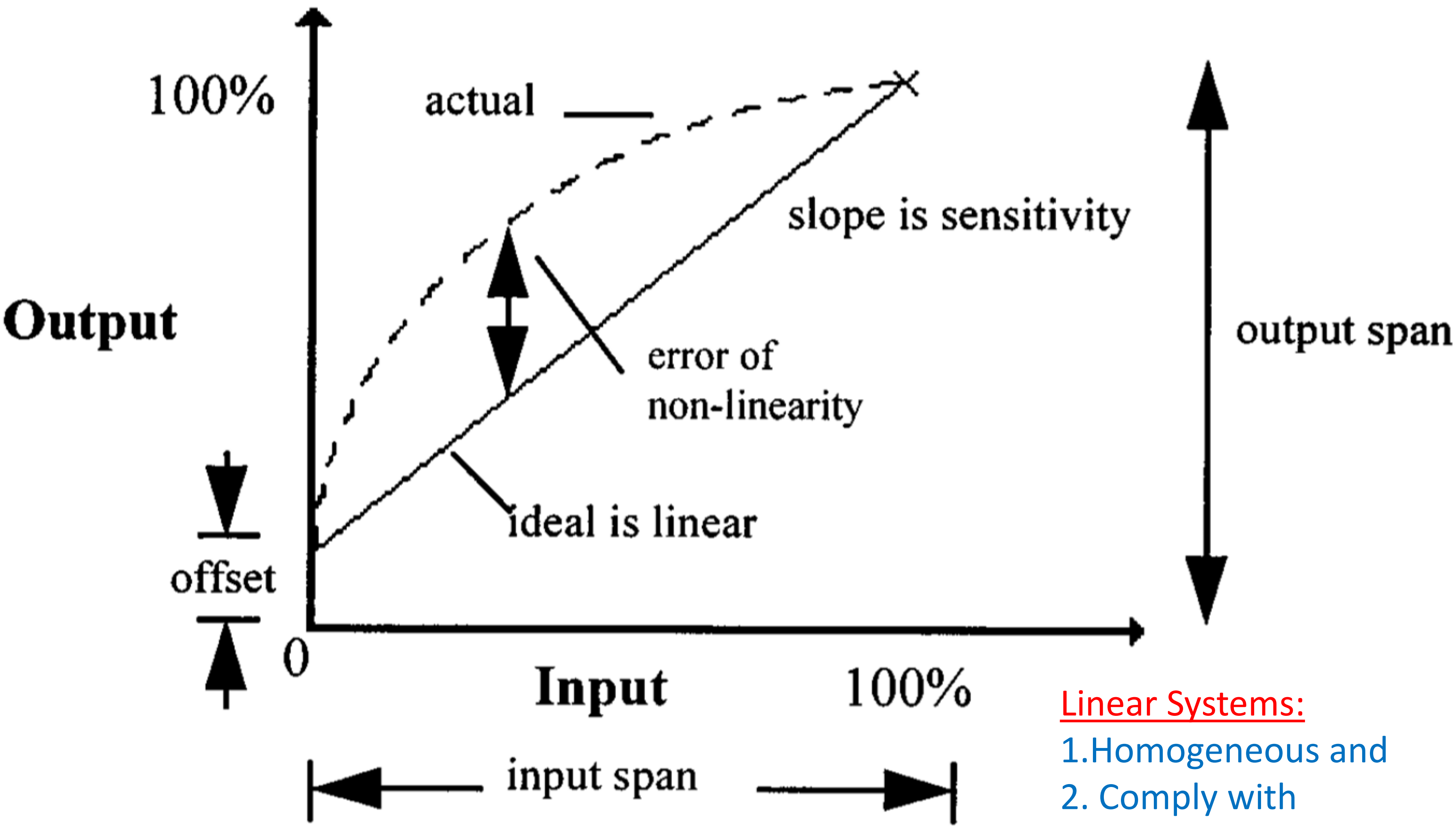
Transfer Function

- T F = Ratio of Output to Input
- $TF = \frac{Output}{Input} = \frac{Y}{X} = G$



- Systems that are consisting of subsystems $TF_{total} = G_1 x G_2 x G_3 \cdots$

Typical input-output characteristic curve of an instrument



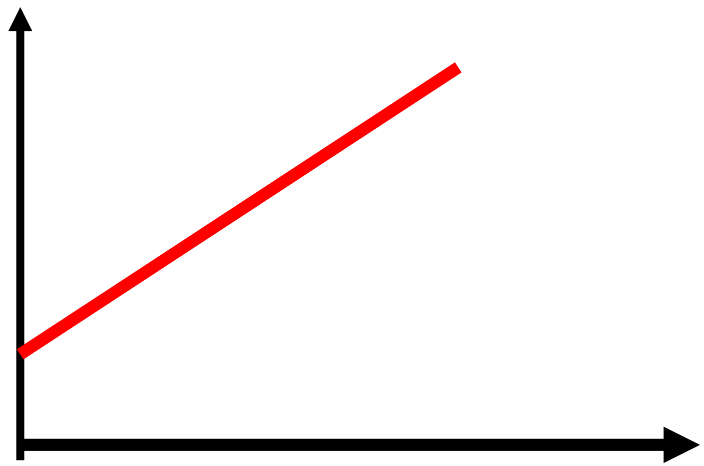
Linear Systems:

1.Homogeneous and

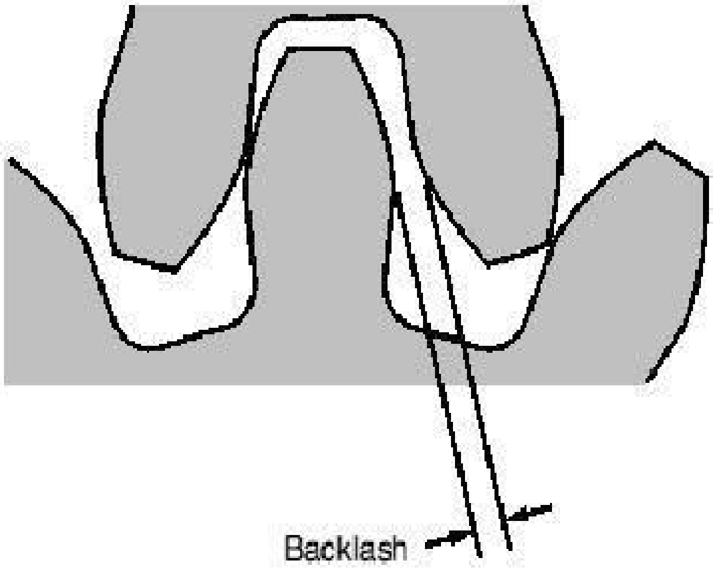
2. Comply with Superposition

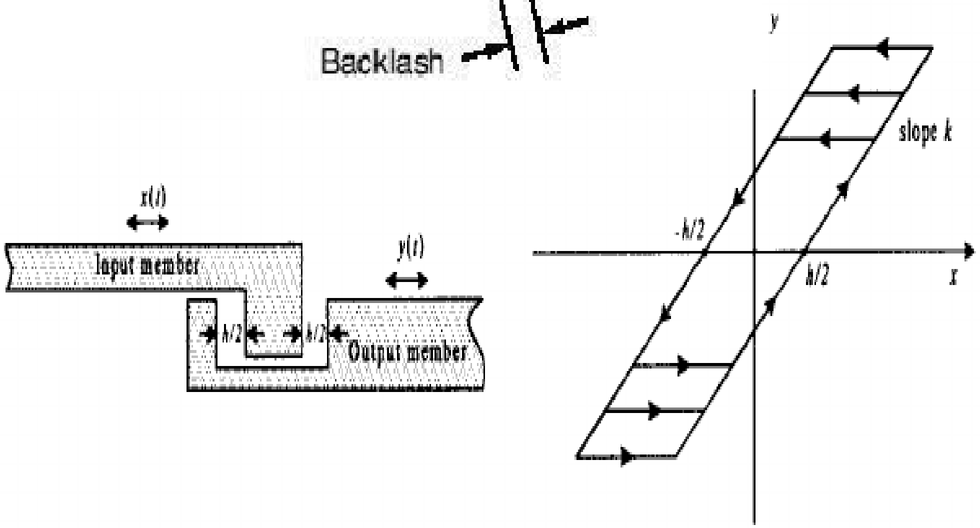
Drift

- Slackening of spring and stretchir
- DC voltage drift in amplifier



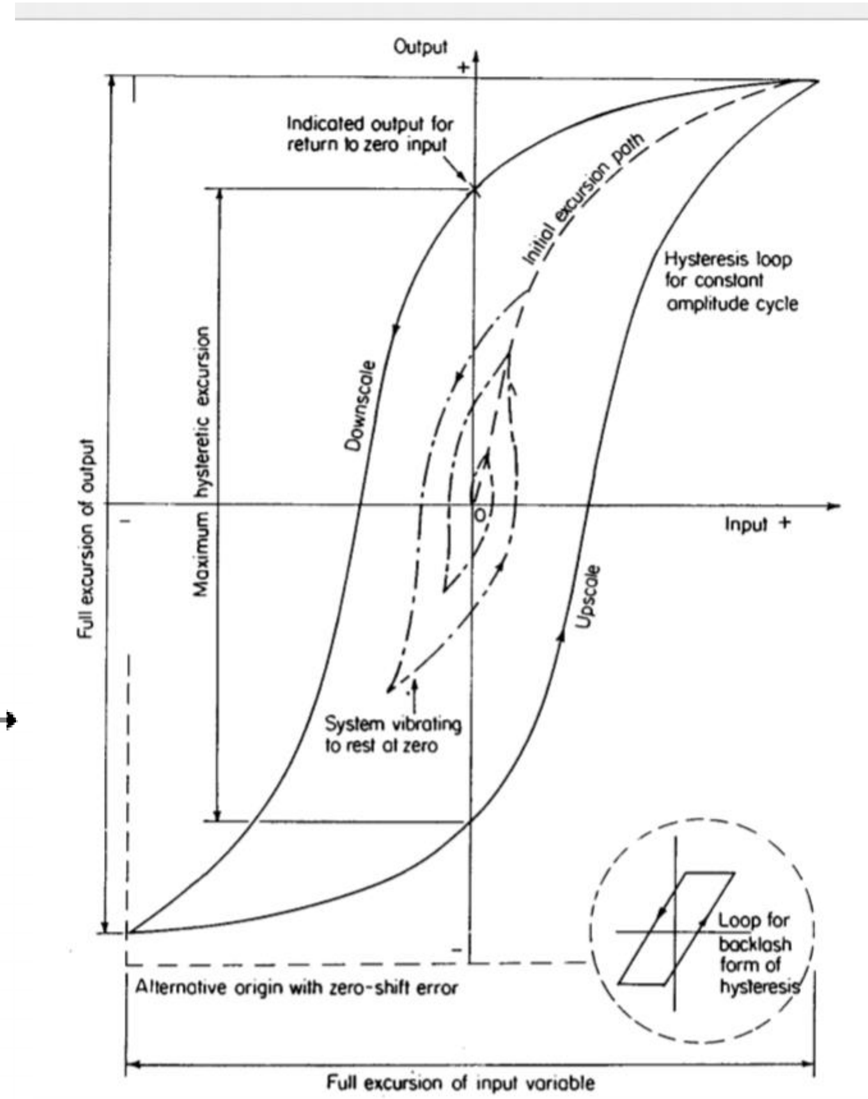
Hysteresis and Backlash





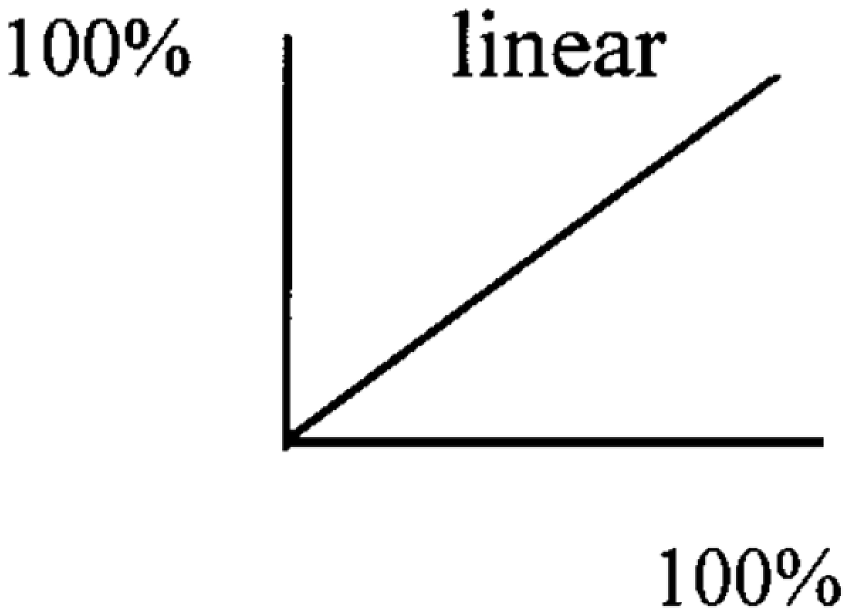
a)

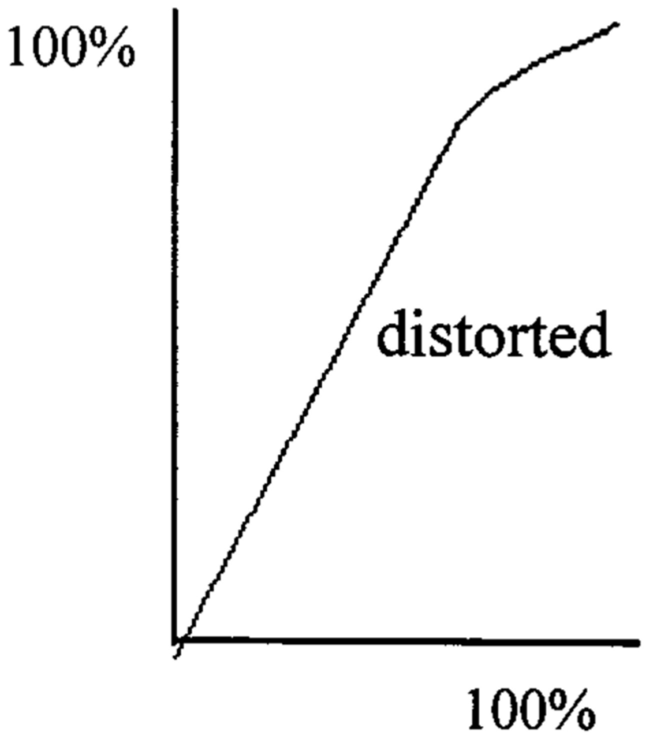
b)



Saturation

Measurements are linear over certain part of the characteristic curve. (compression at top end)

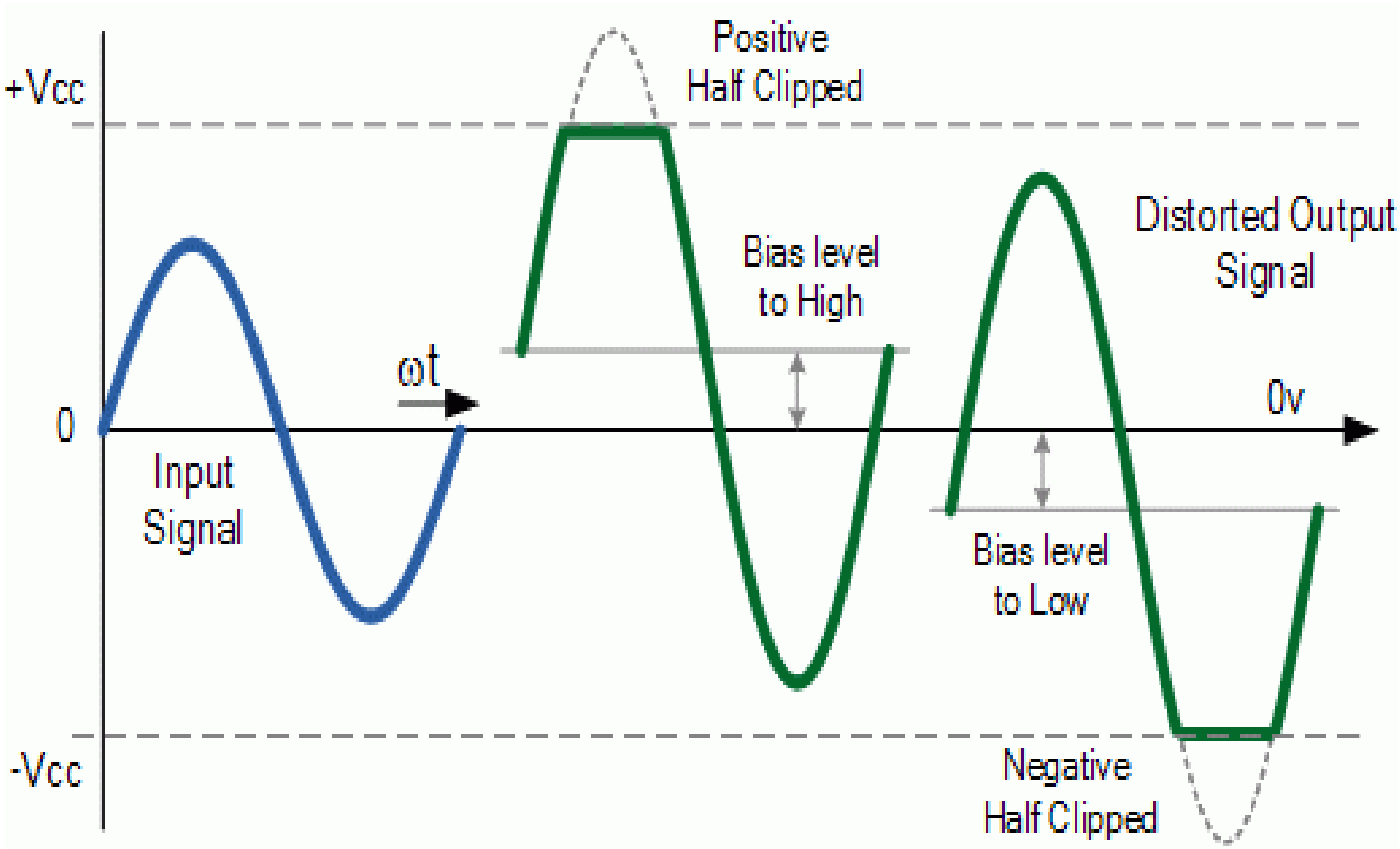




Bias

- **Sometimes, the electronic signal processing situation calls for the input signal to be processed at a higher average voltage or current than arises normally. Here a dc value is added to the input signal to raise the level to a higher state as shown in Figure 3.10. A need for this is met where only one polarity of signal can be amplified by a single semiconductor element. This is usually done to overcome noise.**

Error due to nonlinearity

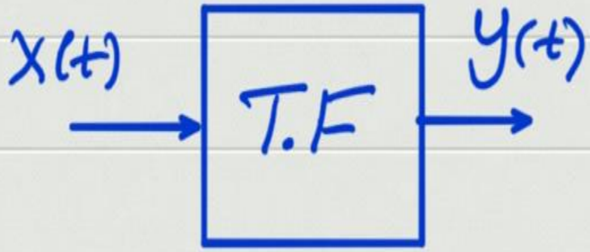


Dynamic Characteristics of Instrumentation Systems

- Due to high nonlinearity in characteristic of most instrumentation systems, finding a precise mathematical formula is not always an easy task. However generic formulas can be developed for limited spans of input variables. This way the characteristic curve can then be divided into zones each having its own formula (Transfer Function), this is called linearization.
- Lookup Tables in computers and other digital systems can provide good solution, but these remains: non-mathematical; experimental; and/or numerically obtained (usually iterative) models.

Dynamic Characteristics of Instruments

In this system, we have the measurand $\underline{x(t)}$ and the output $\underline{y(t)}$



$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

In the complex frequency domain "s", this can be written:

$$a_n s^n Y(s) + a_{n-1} s^{n-1} Y(s) + \cdots a_1 s Y(s) + a_0 Y(s) = b_0 X(s)$$

- $\boldsymbol{a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t)}$
- Laplace Transform Form

$$a_2 Y(s) + a_1 Y(s) + a_0 Y(s) = X(s)$$

$$T.F. = \frac{Y(s)}{X(s)} = \frac{1}{a_2 s^2 + a_1 s + a_0}$$

Laplace Transform

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n + 1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

$$\boxed{\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}}$$

$$R(s) \longrightarrow \boxed{\frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}} \longrightarrow C(s)$$

- Hence the T.F. of this instrument can be written as:

$$\frac{Y(s)}{X(s)} = T.F. = \frac{b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots a_1 s + a_0}$$

- The order of the system is the highest power of "s" in the T.F.
- The order could be zero, one, two, ---

<u>Zero order:</u>

$$\frac{Y(s)}{X(s)} = \frac{b_0}{a_0} \ \underline{\underline{or}} = k$$

$$Y(s) = kX(s)$$

Exercise:

- Find the **transfer function** of the measuring instrument that have the following differential equation for its input/output characteristics

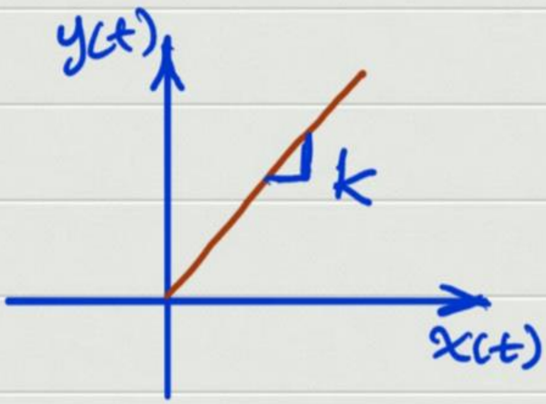
$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Zero, First and Second Order Systems

☆ In zero order system, the output is instantaneous as compared to the input; no lag, no delay

$$Y(s) = k\,X(s) \quad \underline{\text{or}} \quad y(t) = k\,x(t)$$

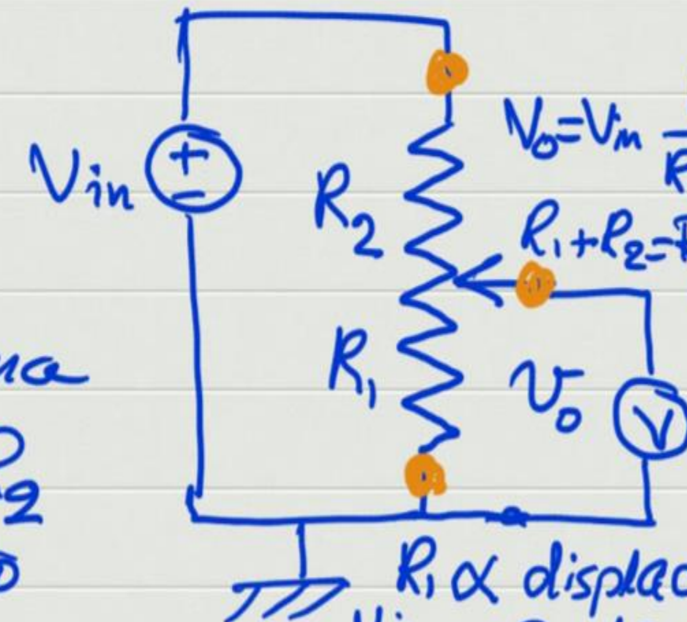
"K" is called the static sensitivity representing the slope of the curve



Example of a zero order instrument:

is the potentiometer, it converts linear displacement into electrical signal and it convert rotational angle "θ" into electrical signal

This is a three terminal device the centre point (slide) divides the resistance into two sections R_1 & R_2. R_1 is directly propotional to the displacement.



$$V_o = V_{in} \frac{R_1}{R_1 + R_2}$$

$$R_1 + R_2 = R_{pot.}$$

$R_1 \propto$ displacement

V_{in} = Constant

hence, $v_o \propto$ displacement

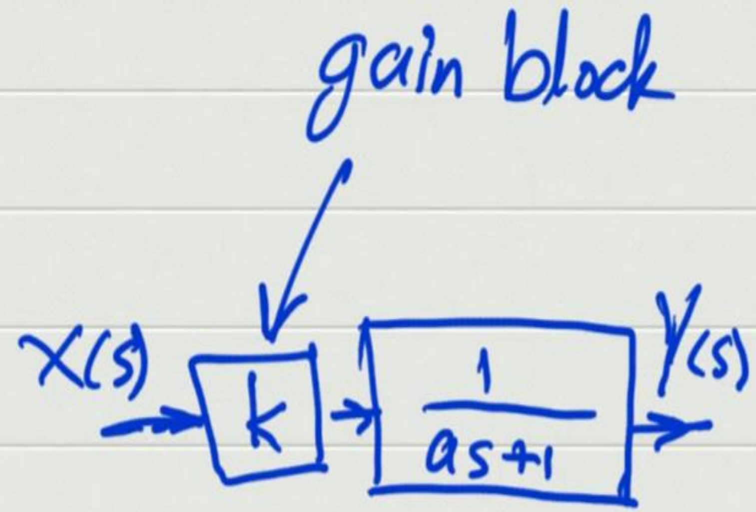
☆ First Order Instrument

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

$$a_1 \dot{y} + a_0 y = b_0 x$$

$$a_1 s Y(s) + a_0 Y(s) = b_0 X(s)$$

$$Y(s)\left(a_1 s + a_0\right) = b_0 X(s)$$



$$\frac{Y(s)}{X(s)} = \frac{b_0}{a_1 s + a_0} = \frac{k}{as+1}$$