Tishk International university

Mechatronics Engineering Department

Grade 1 - Calculus II

Final Questions Bank

of

Calculus II

for

Mechatronics Engineering

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Questions about integration

$\boxed{1}$ (a) $\displaystyle\int \frac{4x^{10} - 2x^4 + 15x^2}{x^3} \, dx$

$= \int 4x^7 - 2x + 15x^{-1} \, dx$

$= 4\frac{x^8}{8} - \frac{2x^2}{2} + 15 \ln|x| + C$ ✓

(b) $\int (w + \sqrt[3]{w})(4 - w^2) \, dw$

$= \int 4w - w^3 + 4w^{\frac{1}{3}} - w^{\frac{7}{3}} \, dw$ ✓

$= \frac{4w^2}{2} - \frac{w^4}{4} + \frac{4w^{\frac{4}{3}}}{\frac{4}{3}} - \frac{w^{\frac{10}{3}}}{\frac{10}{3}} + C$

(c) $\displaystyle\int \frac{7 - 6\sin^2\theta}{\sin^2\theta} \, d\theta$ ✓

$= \int \frac{7}{\sin^2\theta} - \frac{6\sin^2\theta}{\sin^2\theta} \, d\theta$

$= 7\int \frac{1}{\sin^2\theta} - 6\int 1 \, d\theta$

$= 7\int \csc^2\theta - 6\int 1 \, d\theta$

$= -7\cot\theta - 6\theta + C$

2 Evaluate the Integration using substitution.

(a) $\int 18X^2 \cdot \sqrt[4]{6X^3+5} \cdot dX$

$u = 6X^3+5 \Rightarrow du = 18X^2 \cdot dX$

$I = \int u^{\frac{1}{4}} \cdot du$

$= \dfrac{u^{\frac{5}{4}}}{\frac{5}{4}} + c = \frac{4}{5} \cdot u^{\frac{5}{4}} + c = \frac{4}{5} \cdot (6X^3+5)^{\frac{5}{4}} + c$

(b) $\int \sec^2(4t)(3-\tan(4t))^3 \cdot dt$

$u = 3-\tan(4t) \Rightarrow du = -4\sec^2(4t) \cdot dt$

$\Rightarrow \sec^2(4t) \cdot dt = \dfrac{-du}{4}$

$\Rightarrow I = \int u^3 \cdot \left(\frac{-du}{4}\right) = \frac{-1}{4}\int u^3 \cdot du = \frac{-1}{4} \cdot \frac{u^4}{4} + c$

$= -\frac{1}{16}(3-\tan(4t))^4 + c$

(c) $\int \dfrac{3y}{5y^2+4} \cdot dy$

$u = 5y^2+4$

$du = 10y \cdot dy \Rightarrow y \cdot dy = \dfrac{du}{10}$

$I = \dfrac{3 \cdot \frac{du}{10}}{u} = \frac{3}{10}\int \frac{du}{u}$

$= \frac{3}{10} \ln|u| = \frac{3}{10} \ln|5y^2+4| + c$

2

$\boxed{3}$ integer using substitution.

(a) $\int \frac{x}{\sqrt{1-4x^2}} \cdot dx$

$u = 1-4x^2 \qquad du = -8x \cdot dx$

$x \cdot dx = \frac{du}{-8}$

$\Rightarrow I = \int \frac{\frac{-du}{8}}{(u)^{\frac{1}{2}}} = -\frac{1}{8} \int u^{-\frac{1}{2}} \cdot du = -\frac{1}{8} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -\frac{1}{4} \cdot (1-4x^2)^{\frac{1}{2}} + c$

(b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1-\cos x}$.

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{(1+\cos x) \cdot dx}{1-\cos^2 x}$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos x \cdot dx}{\sin^2 x} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 x} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \cdot dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2(x) \cdot dx + \int \left(\underbrace{\frac{\cos x}{\sin x}}_{\cot x} \cdot \underbrace{\frac{1}{\sin x}}_{\csc}\right) \cdot dx$

$= \left[-\cot(x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \left[\csc(x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + C$

3

$\boxed{4}$ integer using " integration techniques"

$$\int \frac{1}{\sqrt{8x - x^2}} \, .dx$$

$\hookrightarrow$ let's complete to a perfect square

$$\int \frac{1}{\sqrt{-\left(x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2\right)}} \, .dx$$

$$= \int \frac{1}{\sqrt{-(x-4)^2 + 16}} \, .dx$$

$$= \int \frac{1}{\sqrt{16\left(-\frac{(x-4)^2}{16} + 1\right)}} \, .dx$$

$$= \frac{1}{4} \int \frac{1}{\left(1 - \left(\frac{x-4}{4}\right)^2\right.} \, .dx$$

$$= \frac{1}{4} \sin^{-1}\left(\frac{x-4}{4}\right) + c$$

5 $\int (\sec t + \tan t)^2 \cdot dt$

↳ we gonna expand the power.

$= \int \sec^2 t + 2\sec t \cdot \tan t + \tan^2 t \; dt$

$= \int \sec^2 t \cdot dt + 2\int \sec t \cdot \tan t \cdot dt + \int \tan^2 t \cdot dt$

$= \tan t + 2\sec t + \underbrace{\int \tan^2 t \cdot dt}_{*}$

$* = \int \tan^2 t \cdot dt = \int \frac{\sin^2 t}{\cos^2 t} \cdot dt$

$= \int \frac{1 - \cos^2 t}{\cos^2 t} \cdot dt = \int \frac{1}{\cos^2 t} dt - \int \frac{\cos^2 t}{\cos^2 t} \cdot dt$

$= \int \sec^2 t \cdot dt - \int 1 \cdot dt = \underline{\tan t - t}$

$\int = \tan t + 2\sec t + \tan t - t + C$

$= 2\tan t + 2\sec t - t + C$

5

$\boxed{6}$ $\int \sqrt{1+\cos 4x}\,.dx$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos^2 2x = \frac{1+\cos 4x}{2}$$

$$\boxed{\cos 4x = 2\cos^2 2x - 1}$$

$$\int = \int \sqrt{1+2\cos^2 2x - 1}\,.dx = \int \sqrt{2\cos^2 2x}\,.dx$$

$$= \int \sqrt{2}\,\cos 2x\,.dx$$

$$= \sqrt{2}\,.\,\frac{1}{2}\,.\sin 2x + c$$

$$= \frac{\sqrt{2}}{2}\,.\,\sin 2x + c$$

$\boxed{7}$ $\int \frac{3x^2 - 7x}{3x+2} \cdot dx$

We realize that the degree of the upper side of the fraction is greater than the degree of the down part

$\Rightarrow$ so we devide

$$\begin{array}{r|l} & x - 3 \\ \hline 3x+2 & 3x^2 - 7x \\ & \mp 3x^2 \mp 2x \\ \hline & 0 \quad -9x \\ & \pm 9x \pm 6 \\ \hline & \quad\quad 6 \end{array}$$

$\int = \int x - 3 + \frac{6}{3x+2} \cdot dx$

$= \frac{x^2}{2} - 3x + 6 \underbrace{\int \frac{1}{3x+2} \cdot dx}_{*}$

$* = \int \frac{1}{3x+2} \cdot dx$

using subtitution $= \frac{1}{3} \ln|3x+2|$

$\int = \frac{x^2}{2} - 3x + 6 \cdot \frac{1}{3} \cdot \ln|3x+2| + c$

$= \frac{x^2}{2} - 3x + 2 \ln|3x+2| + c$

7

$\boxed{8}$ $\displaystyle\int \frac{3x+2}{\sqrt{1-x^2}}$

$$= \underbrace{\int \frac{3x}{\sqrt{1-x^2}}\,dx}_{\text{using substitution}} + \underbrace{\int \frac{2}{\sqrt{1-x^2}}\,dx}_{2\sin^{-1}(x)}$$

$u = 1-x^2 \Rightarrow du = -2x\,dx \Rightarrow x\,dx = \frac{-du}{2}$

$$= -\frac{3}{2} \cdot \int u^{-\frac{1}{2}}\,du$$

$$= -\frac{3}{2}\,\frac{u^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= -3 \cdot (1-x^2)^{\frac{1}{2}}$$

$$\Rightarrow \int \frac{3x+2}{\sqrt{1-x^2}} = -3(1-x^2)^{\frac{1}{2}} + 2\sin^{-1}x + c$$

8

$\boxed{9}$ $\int \sec t \, .dt$

$= \int \sec t \, . 1 . dt$

$= \int \sec t \, . \frac{\sec t + \tan t}{\sec t + \tan t} .dt$

$= \int \frac{\sec t^2 + \sec t . \tan t}{\tan t + \sec t} .dt$

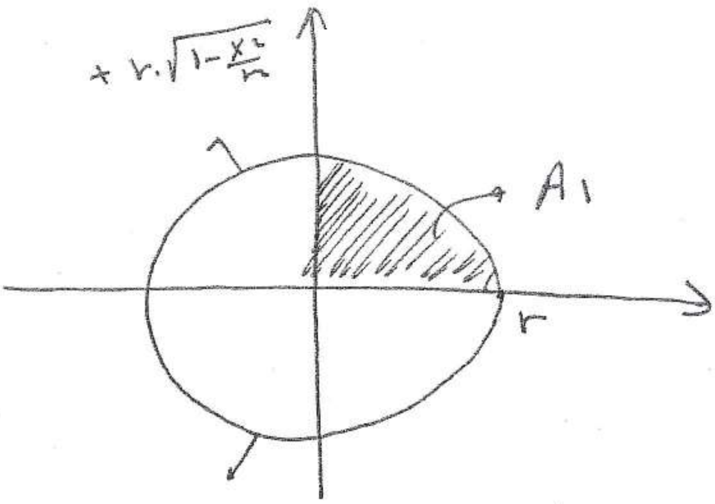
$= \int \frac{u'}{u} . du$

$= \ln |u| + c$

$= \ln | \tan t + \sec t | + c$

$\boxed{10}$ Calculate the area of a circle using integration.

$A = 4 \cdot A_1$



$A_1 = \int_0^r f(x) \cdot dx$

the formula of a circle is

$y^2 + x^2 = r^2$

$\Rightarrow y = \pm\sqrt{r^2 - x^2}$

$= \pm r\sqrt{1 - \frac{x^2}{r^2}}$

$A_1 = \int_0^r r \cdot \sqrt{1 - \frac{x^2}{r^2}} \cdot dx$

if we consider $\frac{x}{r} = \sin\theta \Rightarrow x = r \sin\theta$, $\theta = \sin^{-1}\left(\frac{x}{r}\right)$

$\Rightarrow dx = r \cdot \cos\theta \cdot d\theta$

$x = 0 \Rightarrow \theta = \sin^{-1}\left(\frac{0}{r}\right) = 0$

$x = r \Rightarrow \theta = \sin^{-1}\left(\frac{r}{r}\right) = \frac{\pi}{2}$

$\Rightarrow A_1 = r \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2\theta} \cdot r \cdot \cos\theta \cdot d\theta$

$= r^2 \int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta} \cdot \cos\theta \cdot d\theta = r^2 \int_0^{r} \cos^2\theta \cdot d\theta$

we consider $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$\Rightarrow A_1 = \frac{r^2}{2} \int_0^{\frac{\pi}{2}} 1 + \cos 2\theta \, d\theta = \frac{r^2}{2} \left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= \frac{r^2}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\left(2 \cdot \frac{\pi}{2}\right) - \left(0 + \frac{1}{2}\sin(0)\right)\right] = \frac{r^2}{2} \cdot \frac{\pi}{2} = \frac{\pi r^2}{4}$

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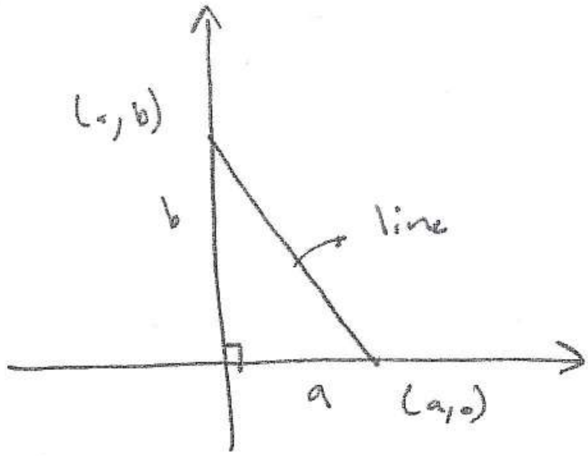
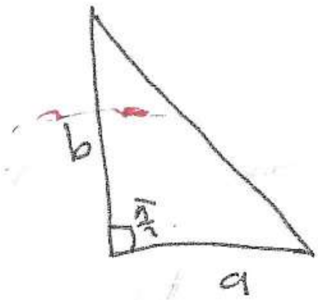
$\Rightarrow A = 4A_1 = \frac{\pi r^2}{4} \cdot 4 = \boxed{\pi r^2}$

11 Calculate the Area of a ~~rectangle~~ Tringle





the formud of the line passing (a,0) , (0,b)

$$y - y_0 = m(x - x_0)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{b-0}{0-a} = -\frac{b}{a} .$$

$$y - 0 = -\frac{b}{a}(x-a)$$

$$y = -\frac{b}{a}(x-a)$$

$$\Rightarrow A = \int f(x)\, dx$$

$$= \int_0^a -\frac{b}{a}(x-a)\cdot dx$$

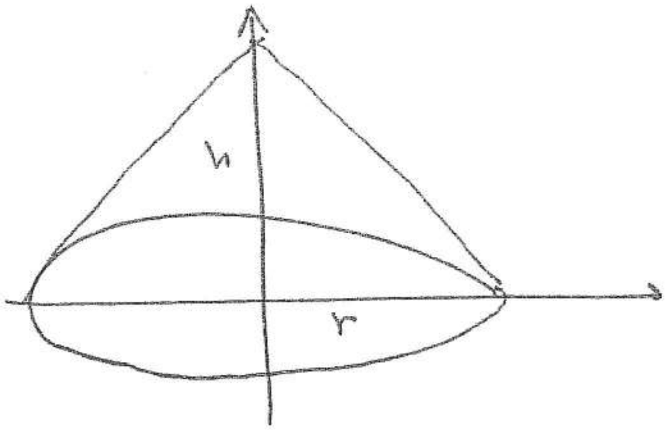
$$= -\frac{b}{a}\left[\frac{x^2}{2} - ax\right]_0^a$$

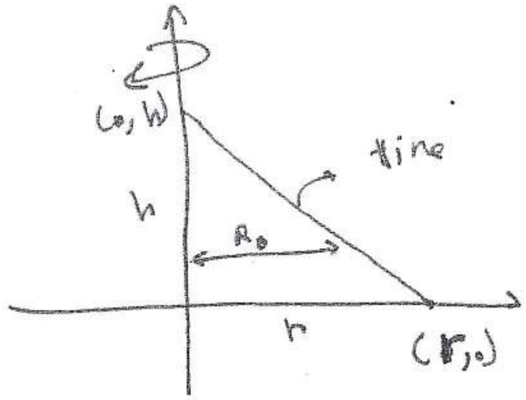
$$= -\frac{b}{a}\left[\frac{a^2}{2} - a^2 - (0-0)\right]$$

$$= -\frac{b}{a}\left(-\frac{a^2}{2}\right) = \boxed{\frac{b \cdot a}{2}}$$

11

$\boxed{12}$ Calculate the volume of the cone





the cone is obtained from rotating the line around y-axis

the formula of the line is (line passing (r, 0), (0, h))

$$y - y_0 = m(x - x_0)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{h-0}{0-r} = \frac{-h}{r}$$

$$\Rightarrow \; y - h = \frac{-h}{r}(x - 0)$$

$$y = h - \frac{h}{r}(x) \Rightarrow x = \frac{-r}{h}(y - h).$$

$$v = \int_{y=0}^{y=h} A(y)\, d(y)$$

$$A(y) = \pi \Big((\underset{\substack{\swarrow \\ x_0 \\ \text{formula}}}{\overset{\text{outer}}{R}})^2 - (\underset{\substack{\searrow \\ 0}}{\overset{\text{inner}}{R}})^2 \Big)$$

$$= \pi \left(\left(\frac{-r}{h}(y-h) \right)^2 - (0)^2 \right) = \frac{\pi r^2}{h^2} \cdot (y-h)^2$$

$$\Rightarrow v = \frac{\pi r^2}{h^2} \int_0^h (y-h)^2 \, dy = \frac{\pi r^2}{h^2} \int_0^h y^2 - 2hy + h^2 \, dy$$

$$= \frac{\pi r^2}{h^2} \left[\frac{y^3}{3} - \frac{2hy^2}{2} + h^2 y \right]_0^h$$

$$= \frac{\pi r^2}{h^2} \cdot \left[\frac{h^3}{3} - \frac{2h^3}{2} + h^3 \right] = \boxed{\frac{\pi r^2 h}{3}}$$

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③

(P1)

13 Determine the Volume of the solid obtained by Rotating the Region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x-axis about the x-axis

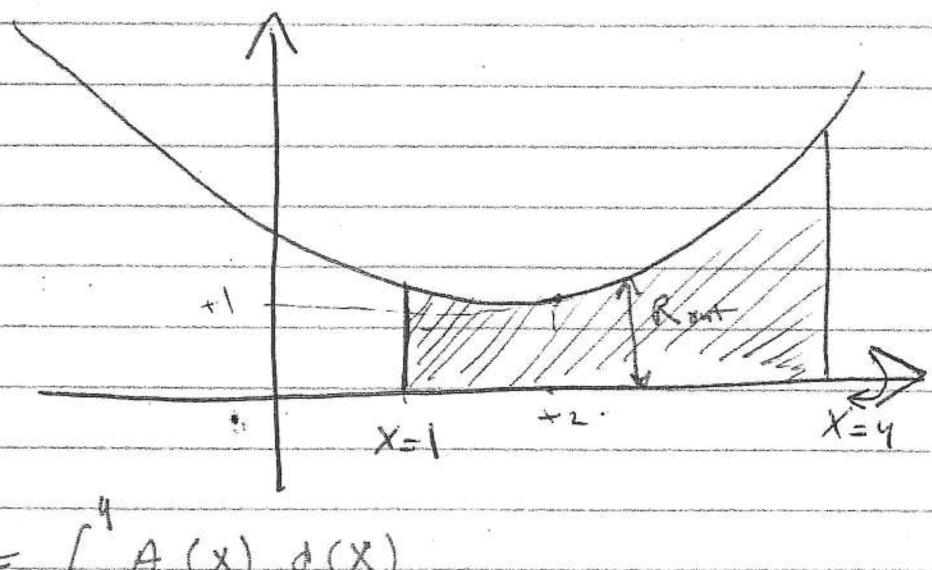
(1) using Discs Method:

* finding intersection Point and graphing

$y = x^2 - 4x + 5$ ⟶ completing to a perfect square

$y = x^2 - 4x + 4 - 4 + 5$

$y = (x-2)^2 + 1$



$v = \int_1^4 A(x)\, d(x)$

$A(x) = \pi\left((\text{outer } R)^2 - (\text{inner } R)^2\right)$

y_{formula} 0

$= \pi\left((x^2 - 4x + 5)^2 - (0)^2\right)$

$= \pi(x^2 - 4x + 5)(x^2 - 4x + 5)$

$= \pi(x^4 - 8x^3 + 26x - 40x + 25)$

$\Rightarrow v = \pi\left[\frac{x^5}{5} - \frac{8x^4}{4} + \frac{21x^3}{2} - \frac{40x^2}{2} + 25x\right]_1^4 = \frac{78\pi}{5}$

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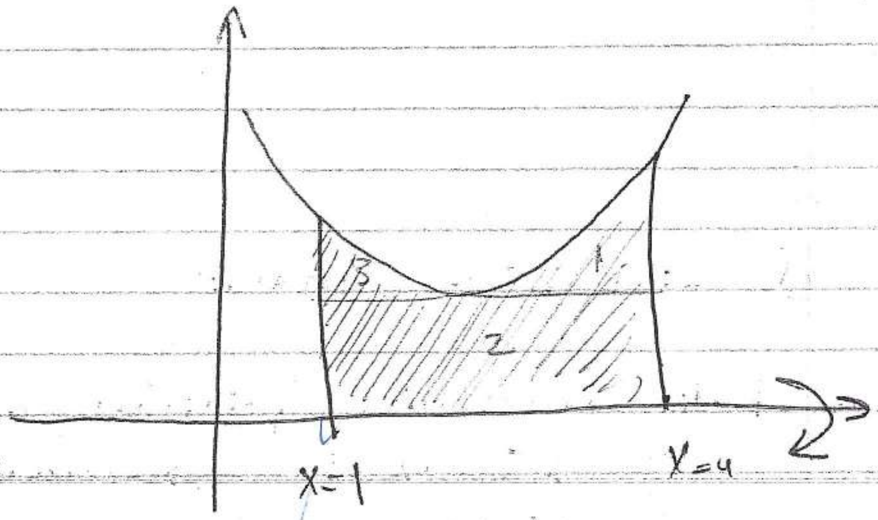
(2) Using Cylindrical Shells Method



it is Complicated to Solve ~~by~~ this exercise by this Way because it will be 3 Volumes.

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Class

(1)

(P3)

$\boxed{14}$

$\boxed{14}$ Determine the volume of the solid obtained by Rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about y axis

$x = y^3$ $\quad$ $x = 4y$

the intersection points

$y_1 = y_2 \Rightarrow$

$\sqrt[3]{x} = \frac{x}{4}$

$\frac{x}{4} - x^{\frac{1}{3}} = 0$

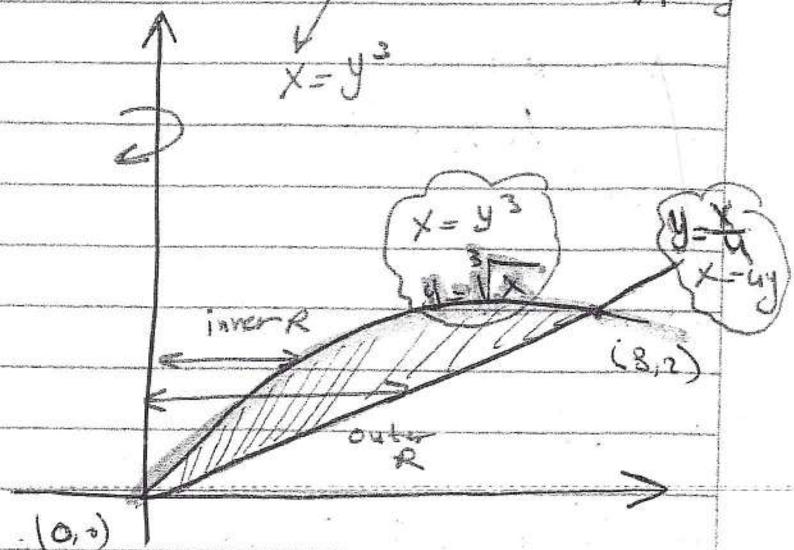
$x^{\frac{1}{3}}\left(\frac{x^{\frac{2}{3}}}{4} - 1\right) = 0$

$x^{\frac{1}{3}} = 0 \Rightarrow \boxed{x = 0} \Rightarrow \boxed{y = 0}$

or $\frac{1}{4}x^{\frac{2}{3}} = 1 \Rightarrow \sqrt[3]{x^2} = 4$

$\Rightarrow x^2 = 4^3 = 4.4.4 \Rightarrow x = \sqrt{4.4.4} = 2.2.2 = 8$

$\boxed{x = 8} \Rightarrow \boxed{y = 2}$



$\boxed{1}$ Using discs Method:

$V = \int_{y=0}^{y=2} A(y)\, d(y)$

$A(y) = \pi\left(\left(\text{outer } R\right)^2 - \left(\text{inner } R\right)^2\right)$

x_o formula $\quad$ x_i formula

when $y = \frac{x}{4}$

$\Rightarrow x = 4y$

When $y = \sqrt[3]{x}$

$x = y^3$

$A(y) = \pi\left((4y)^2 - (y^3)^2\right).dy$

$V = \pi \int_0^2 16y^2 - y^6 \,.dy$

$= \pi \left[\frac{16y^3}{3} - \frac{y^7}{7}\right]_0^2 = \ldots$

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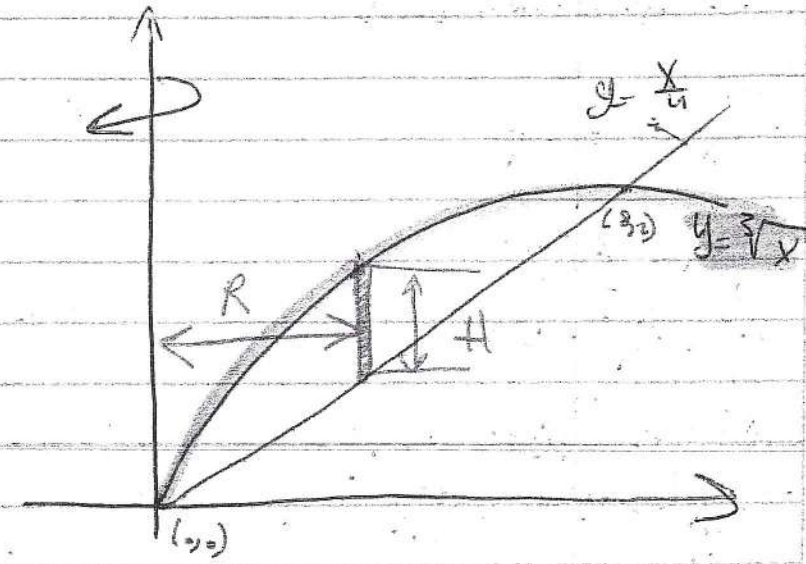
Class

2 Using Cylindrical shells:

$$V = \int_{x=0}^{x=8} A(x)\cdot dx$$



$$A(x) = 2\pi R H$$

$$R = X$$

$$H = y_{up} - y_{down} = \sqrt[3]{x} - \frac{x}{4}$$

formula formula

$$\Rightarrow A(x) = 2\pi (x)\left(\sqrt[3]{x} - \frac{x}{4}\right)$$

$$V = \int_0^8 2\pi (x)\left(\sqrt[3]{x} - \frac{x}{4}\right)\cdot dx$$

(2)

$\boxed{15}$

Determine the volume of the solid obtained by Rotating the Region bonded by $y = X^2 - 2X$ and $y = X$ about the line $y = 4$ (P5)

* intersection points and graphing

$y_1 = y_2$

$X^2 - 2X = X \Rightarrow X^2 - 3X = 0$

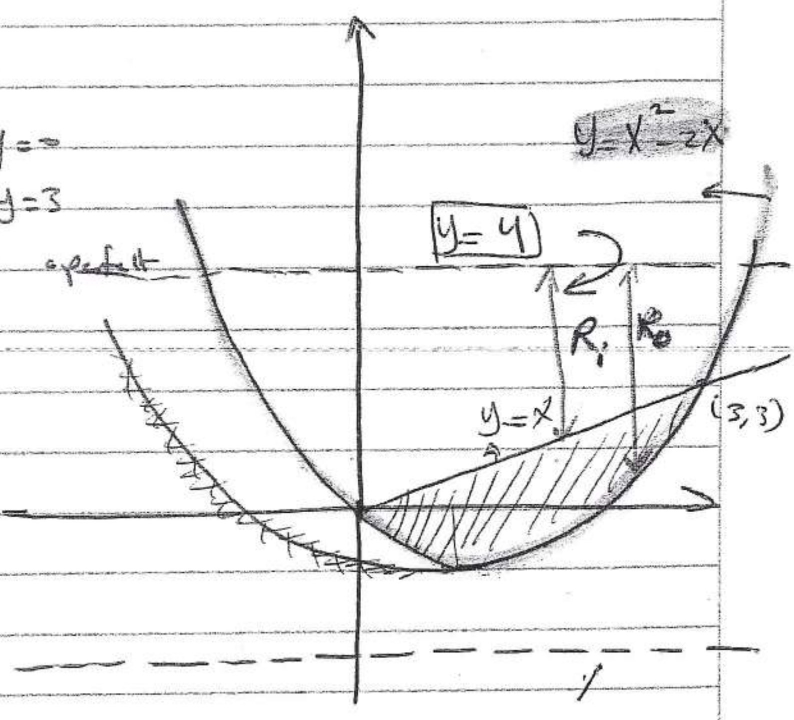
$X(X-3) = 0 \rightarrow X = 0 \Rightarrow y = 0$

$X = 3 \Rightarrow y = 3$

$y = X^2 - 2X$ .. complete to a perfect square

$y = X^2 - 2X + 1 - 1$

$y = (X-1)^2 - 1$



$\boxed{1}$ Using Discs Method

$$V = \int_{x=0}^{x=3} A(x)\, d(x)$$

$$A(x) = \pi \left(\left(\underset{R}{\text{outer}}\right)^2 - \left(\underset{R}{\text{inner}}\right)^2 \right)$$

$4 - y_{\text{outer}}$ formula?　　$4 - y_{\text{inner}}$ formula

$$A(x) = \pi \left((4 - (x^2 - 2x))^2 - (4 - x)^2 \right)$$

$$V = \int_0^3 \pi \left((4 - x^2 + 2x)^2 - (4 - x)^2 \right)$$

⋮

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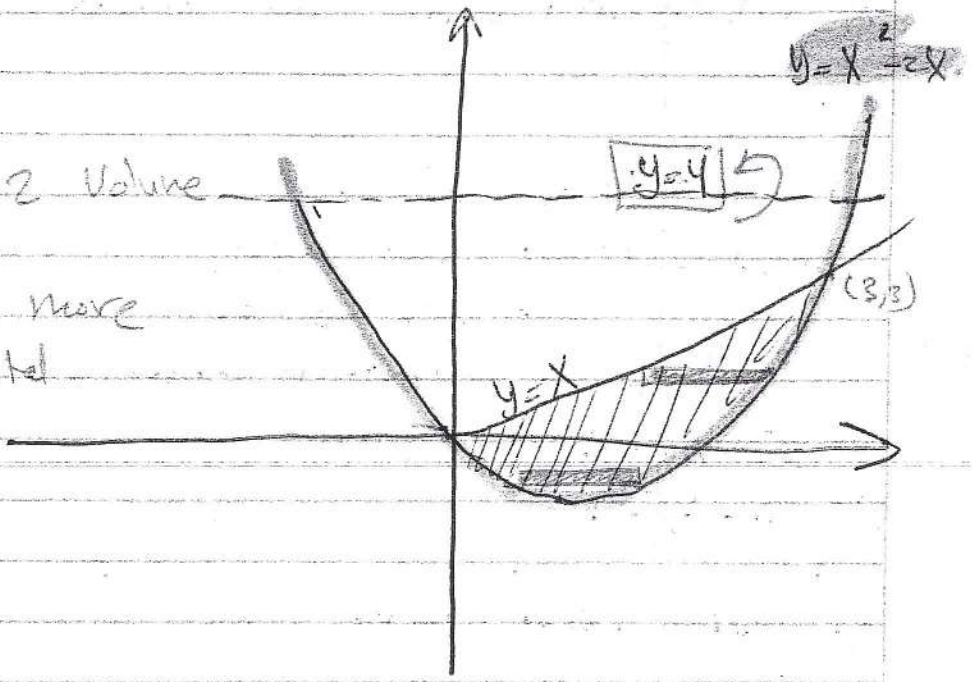
[2] Using Cylindrical shells:

there will be 2 Volume

$\Rightarrow$ it will be more complicated

$\Downarrow$

Crude



(4)

(P7)

$\boxed{16}$

Determine the volume of the solid obtained by Rotating the Region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$

$\boxed{x = y + 1}$

$\sqrt{x-1} = \frac{y}{2}$

$x - 1 = \frac{y^2}{4} \Rightarrow \boxed{x = \frac{y^2}{4} + 1}$

* intersection points :

$y_1 = y_2 \Rightarrow 2\sqrt{x-1} = x - 1$

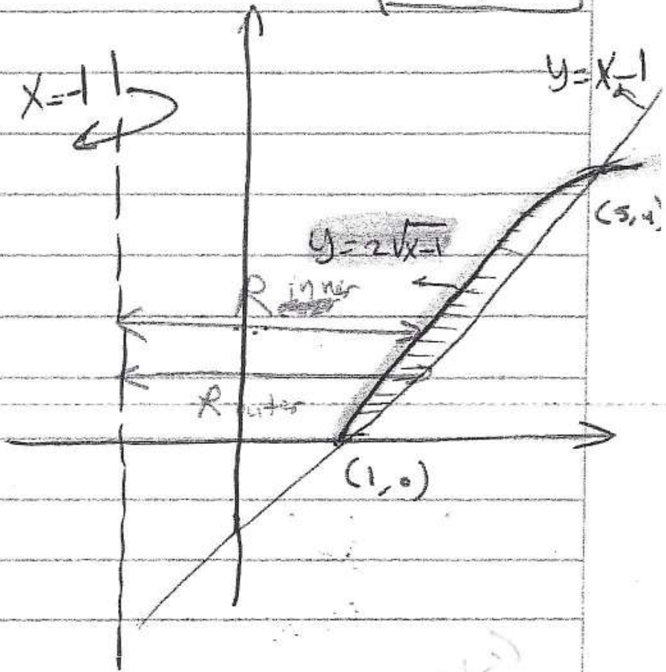
$\Rightarrow (x-1) - 2(x-1)^{\frac{1}{2}} = 0$

$(x-1)^{\frac{1}{2}}\left((x-1)^{\frac{1}{2}} - 2\right) = 0$

$(x-1)^{\frac{1}{2}} = 0 \Rightarrow \boxed{x = 1} \Rightarrow \boxed{y = 0}$

or $(x-1)^{\frac{1}{2}} = 2 \Rightarrow x - 1 = 4$

$\boxed{x = 5} \Rightarrow \boxed{y = 4}$



$\boxed{1}$ using disks Method

$V = \int_{y=0}^{y=4} A(y)\, d(y)$

$A(y) = \pi\left(\left(\underset{R}{\text{outer}}\right)^2 - \left(\underset{R}{\text{inner}}\right)^2\right)$

$x_o + 1$ ↳ formula $\qquad x_i + 1$ ↳ formula

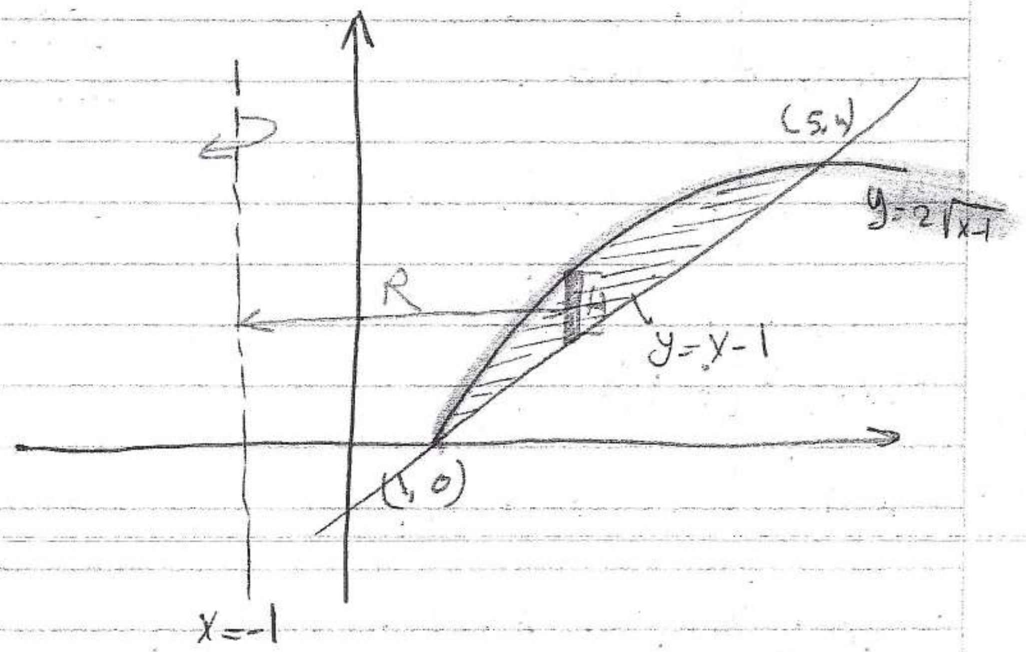
$A(y) = \pi\left(((y+1)+1)^2 - \left(\frac{y^2}{4} + 1 + 1\right)^2\right)$

$V = \int_0^4 \pi\left((y+2)^2 - \left(\frac{y^2}{4} + 2\right)^2\right)$

(19)

Class

[2] using Cylindrical shells Method



$$v = \int_{x=1}^{x=5} A(x)\, d(x)$$

$A(x) = 2\pi R H$

$R = x + 1$

~~$H = y_{up} - y_d$~~

$H = y_{up} - y_{down} = 2\sqrt{x-1} - (x-1)$

formula formula

$\Rightarrow A(x) = 2\pi\,(x+1)\left(2\sqrt{x-1} - (x-1)\right)$

$$v = \int_1^5 2\pi\,(x+1)\left(2\sqrt{x-1} - (x-1)\right) dx$$

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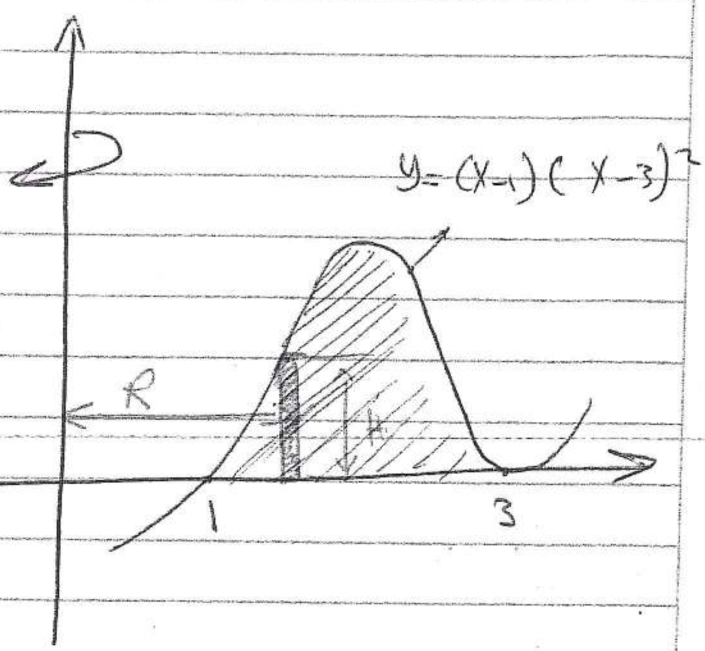
Class

17

Determine the volume of the solid obtained by Rotating the Region bounded by $y=(x-1)(x-3)^2$ and x-axis about y-axis

intersection point



x-axis $\Rightarrow y=0$

$y_1 = y_2$

$(x-1)(x-3)^2 = 0$

$x-1=0 \Rightarrow \boxed{x=1}$

$(x-3)^2=0 \Rightarrow \boxed{x=3}$

the solution is by using cylindrical shells.

$$v = \int_{x=1}^{x=3} A(x)\,dx$$

$A(x) = 2\pi R H$

$R = x$

$H = \underbrace{y_{up}}_{\text{formula}} - \underbrace{y_{down}}_{\text{formula}} = (x-1)(x-3)^2 - 0$

$\Rightarrow A(x) = 2\pi (x)(x-1)(x-3)^2$

$$v = \int_1^3 2\pi (x)(x-1)(x-3)^2 \, dx$$

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P10

22

18

(P11)

6) Determine the volume of the solid obtained by Rotating the Region bounded by $y = \sqrt[3]{x}$, $x = 8$ and the x-axis about the x-axis

$\boxed{x = y^3}$

1) using discs Methods

$$v = \int_{x=0}^{x=8} A(x)\, d(x)$$

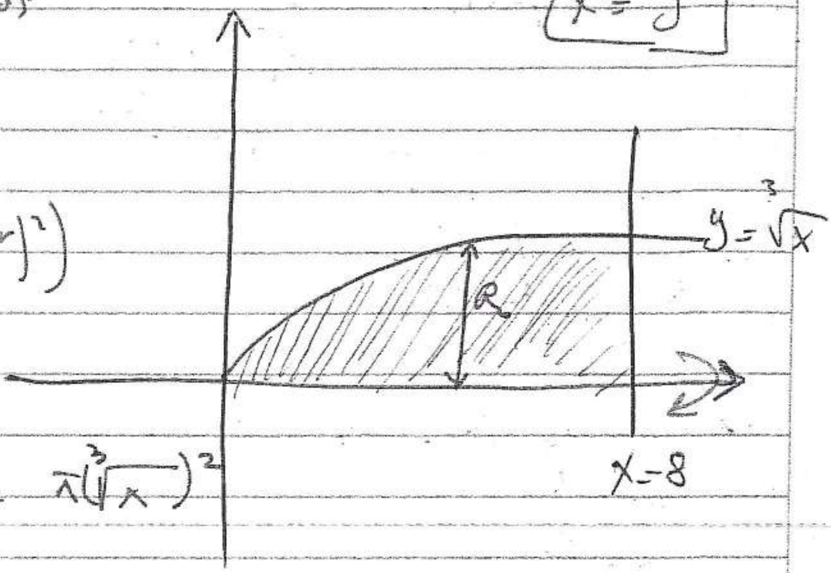
$$A(x) = \pi\left(\left(\underset{R}{\text{outer}}\right)^2 - \left(\underset{R}{\text{inner}}\right)^2\right)$$

y.   0

$$= \pi\left(\left(\sqrt[3]{x}\right)^2 - 0\right) = \pi\left(\sqrt[3]{x}\right)^2$$

$$v = \int \pi\left(\sqrt[3]{x}\right)^2 . dx$$

$$= \pi \int_0^8 (x)^{\frac{2}{3}} . dx = \pi\left[\frac{x^{\frac{5}{3}}}{\frac{5}{3}}\right] = \frac{3\pi}{5}\left[x^{\frac{5}{3}}\right]_0^8 = \frac{96\pi}{5}$$



2) Using cylindrical shells Method

$x = 8 \Rightarrow y = \sqrt[3]{8} = 2$

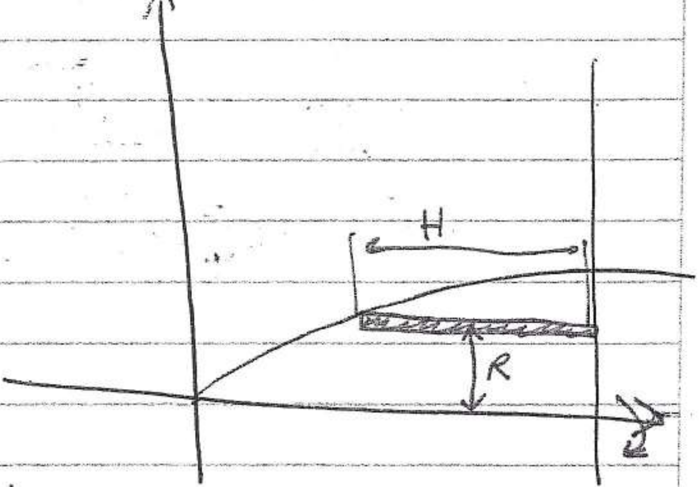
look at this

$$v = \int_{y=0}^{y=2} A(y)\, d(y)$$

$$A(y) = 2\pi R H$$

$$R = y$$

$$H = x_{Right} - x_{left} = 8 - y^3$$



$$\Rightarrow v = \int_0^2 2\pi . y(8 - y^3) . dy$$

$$= 2\pi \int_0^2 (8y - y^4) . dy = 2\pi\left[\frac{8y^2}{2} - \frac{y^5}{5}\right]_0^2 = \frac{96\pi}{5}$$

23

$\boxed{19}$ (P12)

3) Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x-1$ about the line $x = 6$

$\frac{y}{2} = \sqrt{x-1}$

$\frac{y^2}{4} = x-1$

$\boxed{x = \frac{y^2}{4} + 1}$

$\boxed{x = y+1}$

intersection points:

$y_1 = y_2$

$\Rightarrow x-1 = 2\sqrt{x-1}$

$(x-1) - 2(x-1)^{\frac{1}{2}} = 0$

$(x-1)^{\frac{1}{2}}\left((x-1)^{\frac{1}{2}} - 2\right) = 0$

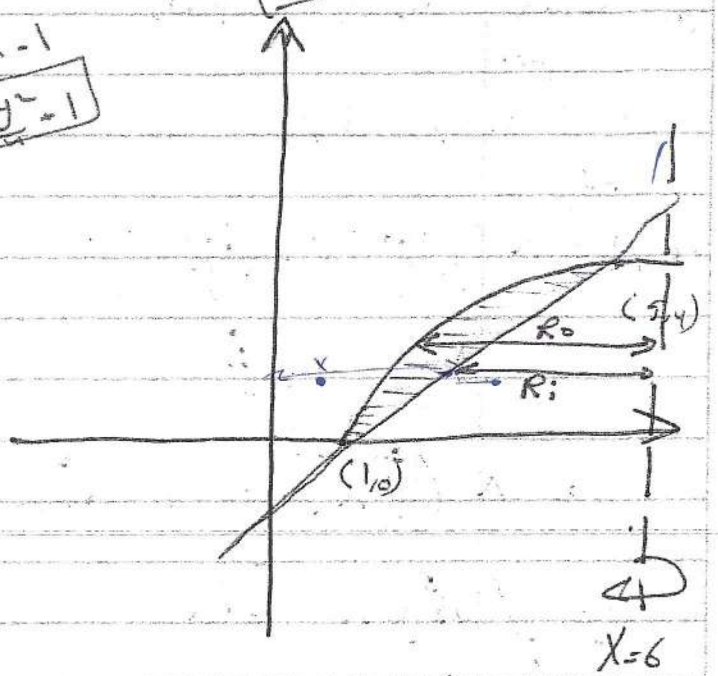
$x-1 = 0 \Rightarrow x = 1 \rightarrow y = 0$

or $(x-1)^{\frac{1}{2}} - 2 = 0$

$(x-1)^{\frac{1}{2}} = 2$

$\Rightarrow x-1 = 4$

$x = 5 \rightarrow y = 4$



$\boxed{1}$ using Discs Method

$$V = \int_{y=0}^{y=4} A(y)\, d(y)$$

$$A(y) = \pi\left((\text{outer } R)^2 - (\text{inner } R)^2\right)$$

$$= \pi\left((6 - x_o)^2 - (6 - x_i)^2\right)$$

for x_o ... for x_i

$$= \pi\left(\left(6 - \left(\frac{y^2}{4} + 1\right)\right)^2 - \left(6 - (y+1)\right)^2\right)$$

24

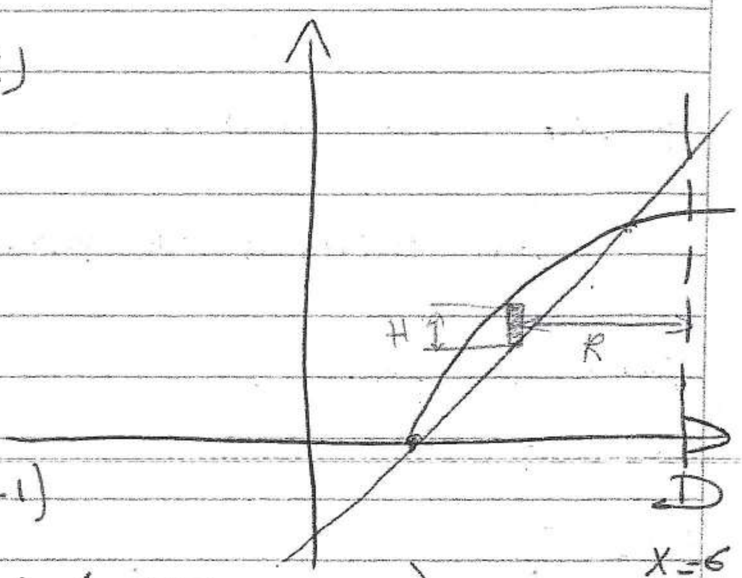
Class

(P3)

[2] using cylindrical shells:

$$V = \int_{x=1}^{x=5} A(x)\, d(x)$$

$$A(x) = 2\pi R H$$



$$R = 6 - X$$

$$H = y_{up} - y_{down}$$

$$= 2\sqrt{X-1} - (X-1)$$

$$V = \int_{1}^{5} 2\pi (6 - X)\left(2\sqrt{X-1} - (X-1)\right) dX$$

25

Class

$\boxed{20}$ P14

Determine the volume of the solid obtained by rotating the region bounded by:

$x = (y-2)^2$ and $y = x$ about the line $y = -1$

* using Discs method it will be difficult

⇒ <u>Using cylindrical shells method</u>

intersection points

$x_1 = x_2$

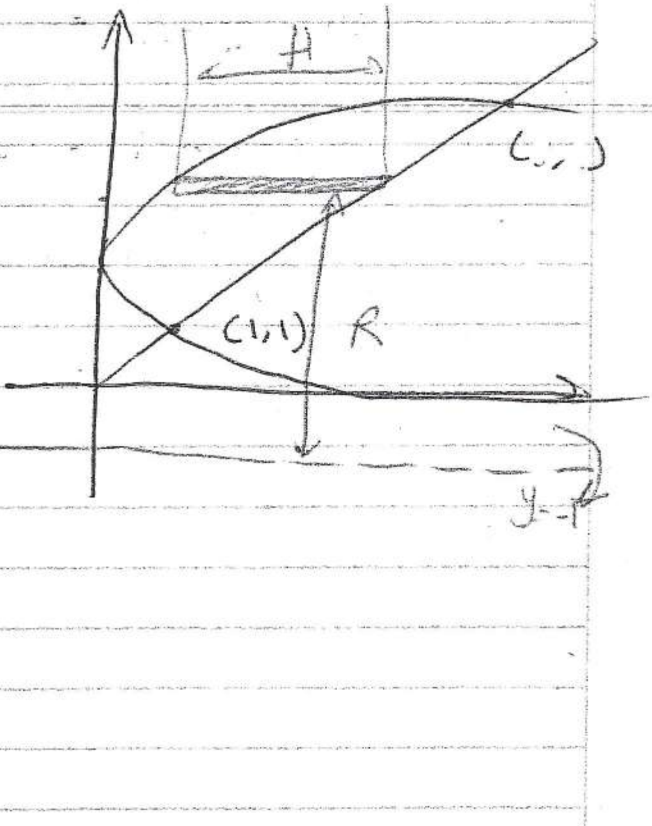
$(y-2)^2 = y$

$y^2 - 4y + 4 = y$

~~[illegible]~~

$y^2 - 5y + 4 = 0$

$(y-4)(y-1) = 0 \quad y = 4 \rightarrow x = 4 \quad y = 1 \rightarrow x = 1$



$$V = \int_{y=1}^{y=4} A(y)\, d(y)$$

$A(y) = 2\pi R H$

$R = y + 1$

$H = \underset{\text{Right}}{x} - \underset{\text{left}}{x} = y - (y-2)^2$

$$V = \int_1^4 2\pi (y+1)\left(y - (y-2)^2\right) . dy$$

26

Class

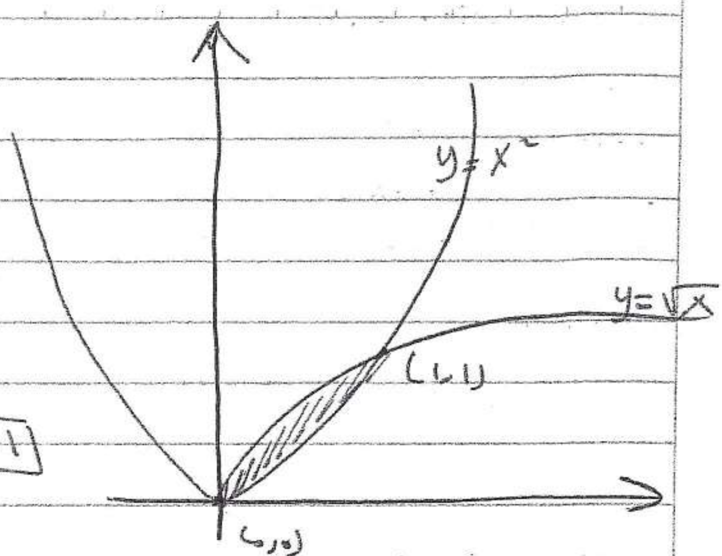
21

P15

Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$

$x = \sqrt{y}$

$x = y^2$



intersection points

$y_1 = y_2 \Rightarrow x^2 = \sqrt{x}$

$x^2 - \sqrt{x} = 0$

$\sqrt{x}\,(x^{\frac{3}{2}} - 1) = 0$

$\sqrt{x} = 0 \Rightarrow \boxed{x=0} \to \boxed{y=0}$

$x^{\frac{3}{2}} - 1 = 0 \Rightarrow x^{\frac{3}{2}} = 1 \Rightarrow \boxed{x=1} \to \boxed{y=1}$

[1] up - down

$$A = \int_{x=0}^{x=1} f(x) - g(x) \, dx$$

$$= \int_0^1 \sqrt{x} - x^2 \, dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3} \right]_0^1$$

$$= \boxed{\frac{1}{3}}$$

[2] Right - left

$$A = \int_{y=0}^{y=1} f(y) - g(y) \, dy$$

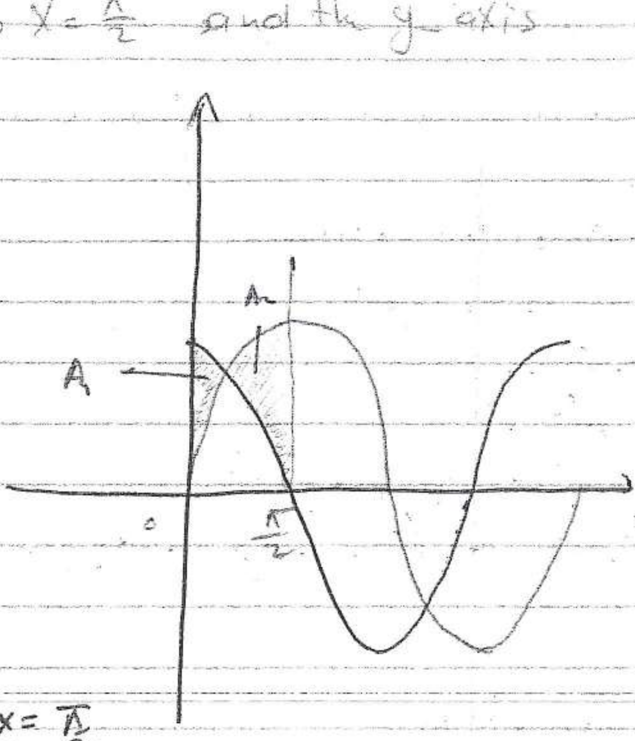
$$= \int_0^1 \sqrt{y} - y^2 \, dy = \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^3}{3} \right]_0^1 = \boxed{\frac{1}{3}}$$

27

22

Determine the area of the region enclosed by $y = \sin x$, $y = \cos x$, $x = \frac{\pi}{2}$ and the y-axis

intersection points



$y_1 = y_2$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

$\Rightarrow \boxed{x = \frac{\pi}{4}}$

* up-down

$$A = \int_{x=0}^{x=\frac{\pi}{4}} f(x) - g(x)\, dx + \int_{x=\frac{\pi}{4}}^{x=\frac{\pi}{2}} g(x) - f(x) \cdot dx$$

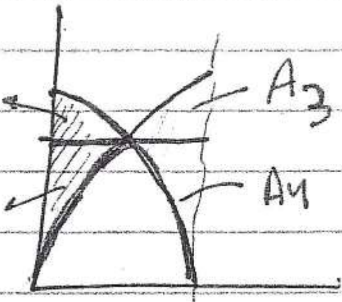
$$= \int_0^{\frac{\pi}{4}} \cos x - \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x \, dx$$

$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1)\right] + \left[0 - 1 - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right]$$

$$= \boxed{2\sqrt{2}}$$

Right-left



$A = A_1 + A_2 + A_3 + A_4$

↓

long way

→ not Required for exam.

28

23

(P17)

Determine the area of the region enclosed by $X = \frac{1}{2}y^2 - 3$ and $y = X - 1$

intersection point

$X_1 = X_2$

$\frac{1}{2}y^2 - 3 = y + 1$

$y^2 - 6 = 2y + 2$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

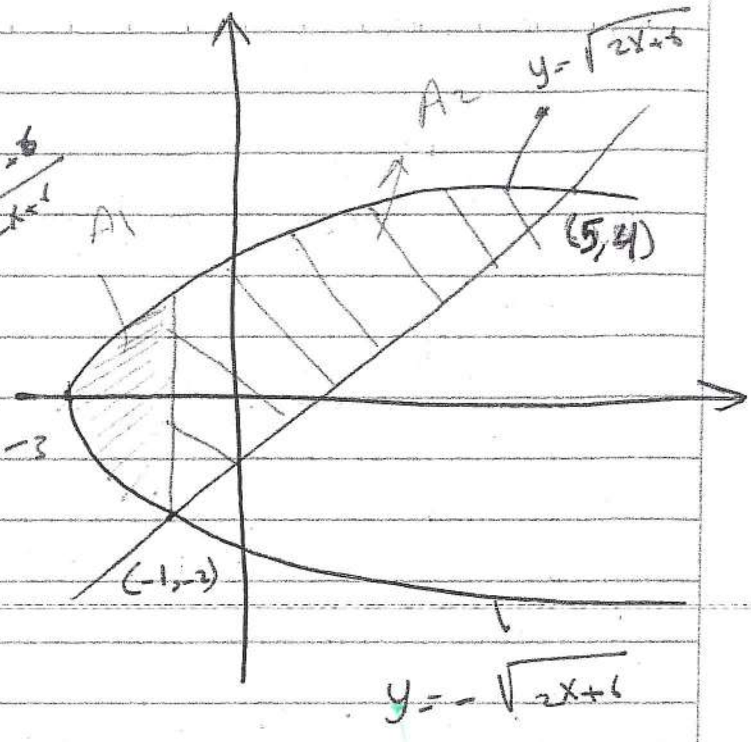
$y = 4 \rightarrow x = 5$

$y = -2 \rightarrow x = -1$

$y^2 - 6 = 2x$

$y^2 = 2x + 6$

$y = \pm\sqrt{2x+6}$



Up-down

$A = A_1 + A_2$

$$= \int_{-3}^{-1} \sqrt{2x+6} - \left(-\sqrt{2x+6}\right) + \int_{-1}^{5} \sqrt{2x+6} - (x-1)$$

$$= \int_{-3}^{-1} 2 \cdot \sqrt{2x+6} + \int_{-1}^{5} \sqrt{2x+6} - x + 1$$

$\boxed{A = 18}$

Class

P18

Right - left

$$A = \int_{-2}^{4} (y+1) - \left(\tfrac{1}{2}y^2 - 3\right) \cdot dy$$

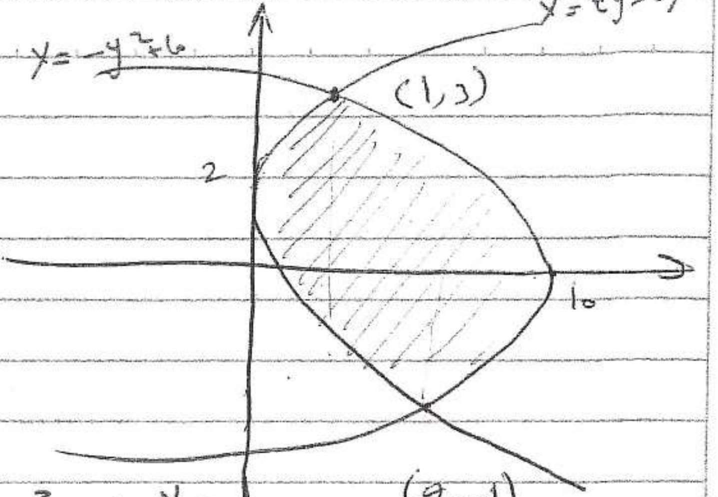
$$= \int_{-2}^{4} \left(y + 1 - \tfrac{1}{2}y^2 + 3\right) \cdot dy$$

$$= \int_{-2}^{4} \left(y - \tfrac{1}{2}y^2 + 4\right) \cdot dy$$

$$= \left[\frac{y^2}{2} - \frac{1}{2}\frac{y^3}{3} + 4y\right]_{-2}^{4} = \boxed{18}$$

24 (P19)

Determine the area of the region bounded by
$x = -y^2 + 10$ and $x = (y-2)^2$



$x_1 = x_2$

$-y^2 + 10 = (y-2)^2$

$-y^2 + 10 = y^2 - 4y + 4$

$2y^2 - 4y - 6 = 0$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0 \Rightarrow$ $y = 3 \rightarrow x = 1$; $y = -1 \rightarrow x = 9$

Right − left

$$A = \int_{y=-1}^{y=3} f(y) - g(y) \cdot dy$$

$$= \int_{-1}^{3} (-y^2 + 10) - (y-2)^2 \cdot dy$$

$$= \int_{-1}^{3} (-y^2 + 10 - y^2 + 4y - 4) \cdot dy$$

$$= \int_{-1}^{3} (-2y^2 + 4y + 14) \cdot dy = \_\_\_\_\_\_$$

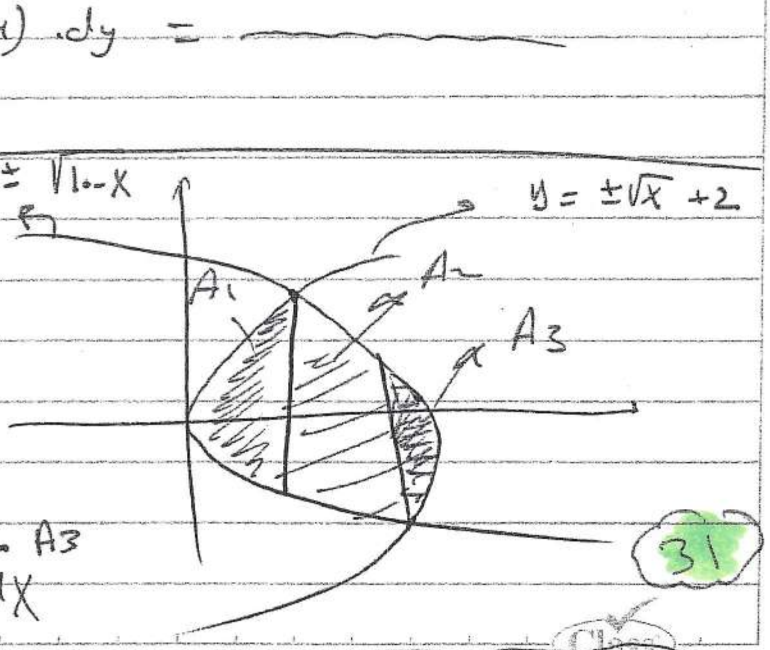
Up − down

$A = A_1 + A_2 + A_3$

$$= \int_{0}^{1} (\sqrt{x} + 2) - (-\sqrt{x} + 2) \cdot dx$$

$$+ \int_{1}^{9} (\sqrt{10 - x}) - (-\sqrt{x} + 2)$$

$$+ \int_{9}^{10} (\sqrt{10 - x}) - (-\sqrt{10 - x}) \cdot dx$$



31

Questions about "Integration by Partiation"

25 $\int \frac{3x+11}{x^2-x-6} \cdot dx$

$$\frac{3x+11}{x^2-x-6} = \frac{3x+11}{(x-3)(x+2)} = \frac{A}{(x-3)} + \frac{B}{(x+2)}$$

$$\Rightarrow \quad 3x+11 = A(x+2) + B(x-3)$$

$$x = -2 \Rightarrow 3(-2)+11 = 0 + 5B$$
$$\Rightarrow \boxed{B = -1}$$

$$x = 3 \Rightarrow 3(3)+11 = 5A + 0$$
$$\Rightarrow \boxed{A = 4}$$

$$\Rightarrow \int \frac{3x+11}{x^2-x-6} \cdot dx = \int \frac{4 \cdot dx}{x-3} + \int \frac{-1}{x+2} \cdot dx$$

$$= 4 \cdot \ln|x-3| - \ln|x+2| + c$$

26 $\displaystyle\int \frac{x^2+4}{x(x-1)(x-3)}\,dx$

$$\frac{x^2+4}{x(x-1)(x-3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-3}$$

$$\Rightarrow x^2+4 = A(x-1)(x-3) + B(x)(x-3) + C(x)(x-1)$$

$x=1 \Rightarrow 5 = -2B \Rightarrow \boxed{B = \frac{-5}{2}}$

$x=3 \Rightarrow 13 = 6C \Rightarrow \boxed{C = \frac{13}{6}}$

$x=0 \Rightarrow 4 = 3A \Rightarrow \boxed{A = \frac{4}{3}}$

$$\Rightarrow \int \frac{x^2+4}{x(x-1)(x-3)}\,dx = \int \frac{\frac{4}{3}}{x}\,dx + \int \frac{\frac{-5}{2}}{x-1}\,dx + \int \frac{\frac{13}{6}}{x-3}\,dx$$

$$= \frac{4}{3}\ln|x| - \frac{5}{2}\ln|x-1| + \frac{13}{6}\ln|x-3| + c$$

$\boxed{27}$ $\displaystyle\int \frac{X^2 - 29X + 5}{(X-4)^2 (X^2+3)}$

$$\frac{X^2 - 29X + 5}{(X-4)^2(X^2+3)} = \frac{A}{(X-4)^1} + \frac{B}{(X-4)^2} + \frac{CX+D}{X^2+3}$$

$$X^2 - 29X + 5 = A(X-4)(X^2+3) + B(X^2+3) + (CX+D)(X-4)^2$$

Because I have A, B, C, D : more than 3 variables

$\Rightarrow$ I can put any value for A, B, C, D.

$$\left.\begin{array}{l} X = 4 \\ X = 0 \\ X = 1 \\ X = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \boxed{A = 1} \\ \boxed{B = -5} \\ \boxed{C = -1} \\ \boxed{D = 2} \end{array}\right\}$$ 4 formulas.

$$\int \frac{X^2 - 29X + 5}{(X-4)^2(X^2+3)} \cdot dX = \underbrace{\int \frac{1}{X-4} \cdot dX}_{intg_1} + \underbrace{\int \frac{-5}{(X-4)^2} \cdot dX}_{intg_2} + \underbrace{\int \frac{-X+2}{X^2+3}}_{intg_3}$$

$$intg_1 = \int \frac{1}{X-4} \cdot dX = \ln|X-4|$$

Substitution $u = X-4$, $du = dX$

$$intg_2 = \int \frac{-5}{(X-4)^2} \cdot dX = -5 \int \frac{dX}{(X-4)^2}$$
$$= -5 \int \frac{du}{u^2} = -5 \int u^{-2} \cdot du = -5 \frac{u^{-1}}{-1} = 5(X-4)^{-1}$$

$$intg_3 = \int \frac{-X+2}{X^2+3} = \int \frac{-X}{X^2+3} \cdot dX + \int \frac{2 \cdot dX}{X^2+3}$$

$u = X^2+3$
$du = 2X \cdot dX$
$\Rightarrow X \cdot dX = \frac{du}{2}$

$$\Rightarrow \int \frac{-\frac{du}{2}}{u} = -\frac{1}{2} \ln|u| = -\frac{1}{2} \ln|X^2+3|$$

35

$$= 2 \cdot \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{X}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}} \tan^{-1} \frac{X}{\sqrt{3}}$$

28 $\int \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2} \, dx$

$$\frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+4)} + \frac{Dx+E}{(x^2+4)^2}$$

$$\Rightarrow x^3+10x^2+3x+36 = A(x^2+4)^2 + (Bx+C)(x-1)(x^2+4)^2 + (Dx+E)(x-1)$$

We need (5) formulas because we have (5) variables A, B, C, D, E

$$\left.\begin{array}{l} x=1 \\ x=0 \\ x=-1 \\ x=2 \\ x=3 \end{array}\right\} \Rightarrow \begin{array}{l} A=2 \\ B=-2 \\ C=-1 \\ D=1 \\ E=0 \end{array}$$

Substitut Rule

$\frac{-2x}{x^2+4}$ $\frac{-1}{x^2+4}$

Rule Separate Substitution

$$\int \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2} = \int \frac{2}{x-1} + \frac{-2x-1}{(x^2+4)} + \frac{x}{(x^2+4)^2}$$

$$= 2\ln|(x-1)| - \ln|x^2+4| - \frac{1}{\sqrt{4}}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\cdot\frac{1}{x^2+4} + C$$

$\boxed{29}$ $\displaystyle\int \frac{x^4 - 5x^3 + 6x^2 - 18}{x^3 - 3x^2} \, dx$

We realize that the max grade of the upper side (x^4) is greater than the max grade of the Down Side (x^3)

$\Rightarrow$ we devide then we integer.

$$\int = \int x - 2 - \underbrace{\frac{18}{x^3 - 3x^2}}_{*} \, dx \Leftarrow$$

$$\begin{array}{r|l} & x - 2 \\ \hline x^3 - 3x^2 & x^4 - 5x^3 + 6x^2 - 18 \\ & \mp x^4 \pm 3x^3 \\ \hline & 0 \; -2x^3 + 6x^2 - 18 \\ & \pm 2x^3 \mp 6x^2 \\ \hline & 0 \quad 0 \quad -18 \end{array}$$

$$* = \int \frac{18}{x^3 - 3x^2} \, dx$$

$$\frac{18}{x^3 - 3x^2} = \frac{18}{x^2(x-3)} = \frac{A}{x} + \frac{B}{(x)^2} + \frac{C}{(x-3)}$$

$18 = A \cdot x \cdot (x-3) + B(x-3) + c \cdot x^2$

$= 0 \Rightarrow 18 = -3B \Rightarrow \boxed{B = -6}$

$= 3 \Rightarrow 18 = 6 \cdot 9 \Rightarrow \boxed{C = 2}$

$= 1 \Rightarrow 18 = -2A - 2B + c$

$\Rightarrow \boxed{A = -2}$

$$\Rightarrow \int \frac{18}{x^3 - 3x^2} = \int \frac{-2}{x} + \int \frac{-6}{x^2} + \int \frac{2}{x-3}$$

$$= -2\ln|x| + 6x^{-1} + 2\ln|x-3|$$

we put $*$ in its place

$$\Rightarrow \int \frac{x^4 - 5x^3 + 6x^2 - 18}{x^3 - 3x^2} \, dx$$

$$= \frac{x^2}{2} - 2x - \left(-2\ln|x| + \frac{6}{x} + 2\ln|x-3|\right) + C$$

new

30

$$\int \frac{X^2}{X^2-1} \cdot dX = ?$$

$$\int \frac{X^2}{X^2-1} \cdot dX = \int \frac{X^2-1+1}{X^2-1} \cdot dX = \int \frac{X^2-1}{X^2-1} + \frac{1}{X^2-1} \cdot dX$$

$$= \int 1 \cdot dX + \underbrace{\int \frac{1}{X^2-1} \cdot dX}_{*}$$

$$* = \int \frac{1}{X^2-1} \cdot dX = \int \frac{1}{(X-1)(X+1)} \cdot dX$$

$$\frac{1}{(X-1)(X+1)} = \frac{A}{(X-1)} + \frac{B}{(X+1)}$$

$\Rightarrow 1 = A.(X+1) + B(X-1) \rightarrow$ we need (2) formulas

$X = -1 \Rightarrow \boxed{B = \frac{-1}{2}}$

$X = 1 \Rightarrow \boxed{A = \frac{1}{2}}$

$$\Rightarrow * = \int \frac{1}{X^2-1} \cdot dX = \int \frac{\frac{1}{2}}{X-1} \cdot dX + \int \frac{-\frac{1}{2}}{X-1} \cdot dX$$

$$* = \frac{1}{2} \ln |X-1| - \frac{1}{2} \ln |X-1| *$$

lets put $*$ in its place

$$\int \frac{X^2}{X^2-1} \cdot dt = X + \frac{1}{2} \ln |X-1| - \frac{1}{2} \ln |X-1| + C$$

Exercises about "integration by parts"

31 $\int x \cdot e^{6x} \cdot dx$

$u = x$ $\qquad dv = e^{6x} \cdot dx$

$du = dx$ $\qquad v = \frac{1}{6} \cdot e^{6x}$

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

$$\int x \cdot e^{6x} \cdot dx = \frac{x}{6} \cdot e^{6x} - \int \frac{1}{6} \cdot e^{6x} \cdot dx$$

$$= \frac{x}{6} \cdot e^{6x} - \frac{1}{36} \cdot e^{6x} + C$$

---

32 $\int (3t+5) \cdot \cos\left(\frac{t}{4}\right) \cdot dt$

$u = 3t + 5$ $\qquad dv = \cos \frac{t}{4}$

$du = 3 \cdot dt$ $\qquad v = 4 \sin \frac{t}{4}$

$$\int = (3t+5)\left(4 \sin \frac{t}{4}\right) - \int 3 \cdot 4 \cdot \sin \frac{t}{4} \cdot du$$

$$= (3t+5)\left(4 \cdot \sin \frac{t}{4}\right) + 48 \cos \frac{t}{4} + C$$

33 $\int w^2 . \sin(10w) . dw$

$u = w^2$ $\qquad dv = \sin(10w)$

$du = 2w . dw$ $\qquad v = -\frac{1}{10} \cos 10w$

$\int = -\frac{w^2}{10} . \cos 10w - \int -\frac{2w}{10} . \cos 10w . dw$

$= \frac{-w^2}{10} . \cos 10w + \frac{1}{5} \underbrace{\int w . \cos 10w . dw}_{*}$

$* = \int w . \cos 10w . dw$

$u = w$ $\qquad dv = \cos 10w$

$du = dw$ $\qquad v = +\frac{1}{10} \sin 10w$

$* = \frac{w}{10} . \sin 10w - \int \frac{1}{10} \sin 10w . dw$

$* = \frac{w}{10} . \sin 10w + \frac{1}{100} \cos 10w$

lets put * in its plac

$\int w^2 . \sin 10w \, dw = \frac{-w^2}{10} . \cos 10w + \frac{1}{5}\left(\frac{w}{10} . \sin 10w + \frac{1}{100} . \cos 10w\right) + c$

$\boxed{34}$ $\int \ln X \cdot dX$

$u = \ln X \qquad dv = dX$

$du = \frac{1}{X} \cdot dX \qquad v = X$

$\int \ln X \cdot dX = X \cdot \ln X - \int X \cdot \frac{1}{X} \cdot dX$

$= X \cdot \ln X - \int 1 \cdot dX$

$= X \cdot \ln X - X + C$

$\boxed{35}$ $\int X^5 \cdot \sqrt{X^3 + 1} \cdot dX$

$u = X^5 \qquad dv = \sqrt{X^3+1} \cdot dX$ → in this way it will be more diffecult

~~du~~

⇒ We gonna use a trick.

$\int = \int X^3 \cdot \left(X^2 \cdot \sqrt{X^3+1} \right) \cdot dX$

$u = X^3 \qquad dv = X^2 \cdot \sqrt{X^3+1} \cdot dX$

$du = 3X^2 \cdot dX \qquad v = \frac{2}{9} \cdot (X^3+1)^{\frac{3}{2}}$

$\int = \frac{2X^3}{9} \cdot (X^3+1)^{\frac{3}{2}} - \int \underbrace{\frac{2}{9} \cdot X^2 \cdot (X^3+1)^{\frac{3}{2}}}_{\text{Substitution}} \cdot dX$

$= \frac{2X^3}{9} (X^3+1)^{\frac{3}{2}} - \frac{2}{9} \cdot \frac{2}{5} (X^3+1)^{\frac{5}{2}} + C$

the integration is done using Substitution

$dv = X^2 \cdot \sqrt{X^3+1} \cdot dX$

$u = X^3+1 \Rightarrow du = 3X^2 \cdot dX$

$\Rightarrow X^2 \cdot dX = \frac{du}{3}$

$v = \int (u)^{\frac{1}{2}} \cdot \frac{du}{3}$

$= \frac{1}{3} \cdot \frac{2}{3} \cdot u^{\frac{3}{2}}$

$= \frac{2}{9} (X^3+1)^{\frac{3}{2}}$

$\boxed{36}$ $\int e^{\theta} . \cos\theta . d\theta$

$u = e^{\theta}$. $dv = \cos\theta . d\theta$

$du = e^{\theta} . d\theta$ $v = \sin\theta$

$\int e^{\theta} . \cos\theta . d\theta = e^{\theta} . \sin\theta - \underbrace{\int e^{\theta} . \sin\theta . d\theta}_{*}$

$* = \int e^{\theta} . \sin\theta . d\theta$

$u = e^{\theta}$. $dv = \sin\theta . d\theta$

$du = e^{\theta} . d\theta$ $v = -\cos\theta$

$* = -e^{\theta} . \cos\theta + \int e^{\theta} . \cos\theta . d\theta$

$\int e^{\theta} . \cos\theta . d\theta = e^{\theta} . \sin\theta - \left(-e^{\theta} . \cos\theta + \int e^{\theta} . \cos\theta . d\theta\right)$

$\int e^{\theta} . \cos\theta . d\theta = e^{\theta} . \sin\theta + e^{\theta} . \cos\theta - \int e^{\theta} . \cos\theta . d\theta$

$\int e^{\theta} . \cos\theta . d\theta + \int e^{\theta} . \cos\theta . d\theta = e^{\theta} \sin\theta + e^{\theta} . \cos\theta$

$2 \int e^{\theta} . \cos\theta . d\theta = e^{\theta} . \sin\theta + e^{\theta} . \cos\theta$

$\int e^{\theta} . \cos\theta . d\theta = \frac{1}{2}\left(e^{\theta} . \sin\theta + e^{\theta} . \cos\theta\right).$

The Solution of the First homework / Civil Engineering / First grade / Calculus II

(37) $\int \frac{\sin X + \sec X}{\tan X} \cdot dX = \int \frac{\sin X}{\tan X} \cdot dX + \frac{\sec X}{\tan X} \cdot dX$

$= \int \frac{\sin X}{\frac{\sin X}{\cos X}} \cdot dX + \int \frac{\frac{1}{\cos X}}{\frac{\sin X}{\cos X}} \cdot dX$

$= \int \cos X \cdot dX + \int \frac{1}{\sin X} \cdot dX$

$= \int \cos X \cdot dX + \int \csc X \cdot dX$

$= -\sin X - \ln|\csc X + \cot X| + c$

this rule was concluded in the class and it has to be memorised.

$\int \sec X \cdot dX = \ln|\sec X + \tan X| + c$

$\int \csc X \cdot dX = -\ln|\csc X + \cot X| + c$

(38) $\int \frac{X-1}{X^2-4X+5} = \int \frac{X-1}{X^2-4X+4-4+5} = \int \frac{X-1}{(X-2)^2+1}$

$= \int \frac{(X-2)+1}{(X-2)^2+1} \cdot dX$

$\left.\begin{array}{l} u = X-2 \\ du = dx \end{array}\right\} \Rightarrow = \int \frac{u+1}{u^2+1} \cdot du$

$$= \int \frac{u}{u^2+1}\,dx + \int \frac{1}{u^2+1}$$

$$= \frac{1}{2}\ln(u^2+1) + \tan^{-1} u + C$$

---

(39) $\int \sin^3\theta \cdot \cos^5\theta \, d\theta$

$$= \int \cos^5\theta \cdot \sin^2\theta \cdot \sin\theta \cdot d\theta$$

$$= \int \cos^5\theta \, (1 - \cos^2\theta) \cdot \sin\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta \Rightarrow \sin\theta \, d\theta = -du \Big\} \Rightarrow$

$$= -\int u^5 (1-u^2) \cdot du = -\int u^5 - u^7 \cdot du$$

$$= -\frac{u^6}{6} + \frac{u^8}{8} + C$$

$$= -\frac{\cos^6\theta}{6} + \frac{\cos^8\theta}{8} + C$$

Class

(40) $\int e^{x+e^x} \cdot dx = \int e^x \cdot e^{e^x} \, dx = \int e^{e^x} \cdot e^x \, dx$

$u = e^x \Rightarrow du = e^x \cdot dx \} \Rightarrow$

$= \int e^u \cdot du = e^u + c = e^{e^x} + c$

---

(41) $\int \frac{3w-1}{w+2} = \int \frac{3(w+2-2)-1}{w+2}$

$= \int \frac{3(w+2)-6-1}{(w+2)}$

$= \int \frac{3(w+2)}{(w+2)} + \int \frac{-7}{(w+2)}$

$= \int 3 \, dw - 7 \int \frac{1}{w+2}$

$= 3w - 7 \ln|w+2| + c$

* Write the a list of Points must be mentioned in the AD Board meeting

for evaluating / Eva..

Evaluation of the results

Success statistics

H.W (1)

Calculus (2)
Mechatronics Engineering
Grade I

Find the following Integrations:

1. $\int X^3 . \ln X \; dX$

B. $\int \sin X . \ln(\cos X) . dX$

C. $\int \sin^{-1}(X) \; dX$

D. $\int X^3 . \sqrt{4 - X^2} \; . dX$

E. $\int \frac{X^2}{X^2 - 1} \; . dX$

F. $\int \frac{2X^3 - 4X^2 - X - 3}{X^2 - 2X - 3} \; . dX$

G. $\int \frac{-2X + 4}{(X^2 + 1) . (X - 1)^2} \; . dX$

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Rasha Alkabbani

Solution of HW (1)

(A): $\int X^3 . \ln X . dX$

$U = \ln X$ | $dv = X^3 . dX$

$du = \frac{1}{X} . dX$ | $v = \frac{X^4}{4}$

$I = U.v - \int v\,du$

$= \ln X \left(\frac{X^4}{4}\right) - \int \frac{X^4}{4} . \frac{1}{X} . dX$

$= \frac{X^4 . \ln X}{4} - \frac{1}{4}\int X^3 . dX$

$= \frac{X^4 . \ln X}{4} - \frac{1}{4} . \frac{X^4}{4}$

$$\boxed{I = \frac{X^4 . \ln X}{4} - \frac{X^4}{16}}$$

(B): $\int \sin X . \ln(\cos X) . dX$

$U = \ln(\cos X)$ | $dv = \sin X . dX$

$du = \frac{-\sin X}{\cos X} . dX$ | $v = -\cos X$

$I = U.v - \int v . du$

$= -\cos X . \ln(\cos X) - \int (-\cos X) . \left(\frac{-\sin X}{\cos X}\right) . dX$

$= -\cos X . \ln(\cos X) - \int \sin X . dX$

$$\boxed{I = \quad '' \quad + \cos X}$$

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(c) $\int \sin^{-1}(x) \cdot dx$

$U = \sin^{-1}(x)$ | $dv = 1 \cdot dx$

$du = \frac{1}{\sqrt{1-x^2}} \cdot dx$ | $v = x$

$I = U \cdot v - \int v \cdot du$

$= x \cdot \sin^{-1}(x) - \underbrace{\int \frac{x}{\sqrt{1-x^2}} \cdot dx}$

* will be solved by substitution rule

$* = \int \frac{x}{\sqrt{1-x^2}} \cdot dx$

$U = 1 - x^2 \Rightarrow du = -2x \cdot dx \Rightarrow \boxed{dx = \frac{du}{-2x}}$

$* = \int \frac{x}{U^{\frac{1}{2}}} \cdot \frac{du}{-2x} = \frac{-1}{2} \int U^{-\frac{1}{2}} \cdot dU$

$= -\frac{1}{2} \cdot \frac{U^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$

$= -\frac{1}{2} \cdot \frac{2}{1} \cdot U^{\frac{1}{2}}$

$= -U^{\frac{1}{2}} = \underbrace{-(1-x^2)^{\frac{1}{2}}}_{*}$

$$\boxed{I = x \cdot \sin^{-1}(x) + (1-x^2)^{\frac{1}{2}} + c}$$

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D

$$\int x^3 \cdot \sqrt{4-x^2}\, dx$$

$$I = \int x^2 \cdot x(4-x^2)^{\frac{1}{2}} \cdot dx$$

$u = x^2$

$du = 2x \cdot dx$

$$\begin{cases} dv = x(4-x^2)^{\frac{1}{2}} \cdot dx \\ \underbrace{v = -\frac{1}{3} \cdot (4-x^2)^{\frac{3}{2}}}_{\text{How } \ast} \end{cases}$$

$$\ast = \int x(4-x^2)^{\frac{1}{2}} \cdot dx$$

$$u = 4-x^2 \Rightarrow du = -2x\, dx \Rightarrow dx = \frac{du}{-2x}$$

$$\ast = \int x\,(u)^{\frac{1}{2}} \cdot \frac{du}{-2x} = -\frac{1}{2}\int u^{\frac{1}{2}} \cdot du$$

$$= -\frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = -\frac{1}{3} \cdot (4-x^2)^{\frac{3}{2}}$$

$$I = u \cdot v - \int v\, du$$

$$= x^2 \cdot \left(-\frac{1}{3}(4-x^2)^{\frac{3}{2}}\right) - \int 2x \cdot \left(-\frac{1}{3}(4-x^2)^{\frac{3}{2}}\right) \cdot dx$$

$$I = -\frac{1}{3} x^2 \cdot (4-x^2)^{\frac{3}{2}} + \underbrace{\int x\,(4-x^2)^{\frac{3}{2}} \cdot dx}_{☼}$$

$$☼ = \int x\,(4-x^2)^{\frac{3}{2}} \cdot dx$$

$$u = 4-x^2 \Rightarrow du = -2x \cdot dx \Rightarrow dx = \frac{du}{-2x}$$

$$☼ = \int x \cdot u^{\frac{3}{2}} \cdot \frac{du}{-2x} = -\frac{1}{2}\int u^{\frac{3}{2}} \cdot du$$

$$= -\frac{1}{2} \cdot \frac{u^{\frac{5}{2}}}{\frac{5}{2}} = \underbrace{-\frac{1}{5} \cdot (4-x^2)^{\frac{5}{2}}}_{☼}$$

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$$\Rightarrow \boxed{I = -\frac{1}{3} x^2 (4-x^2)^{\frac{3}{2}} - \frac{1}{5}(4-x^2)^{\frac{5}{2}}}$$

(E) $\int \frac{x^2}{x^2-1} \cdot dx$

$I = \int \frac{x^2}{(x-1)(x+1)} = \int \frac{A}{x-1} + \int \frac{B}{x+1}$

$\Rightarrow \quad x^2 = A(x+1) + B(x-1)$

$x = 1 \Rightarrow 1 = 2A + 0 \Rightarrow \boxed{A = \frac{1}{2}}$

$x = -1 \Rightarrow 1 = 0 - 2B \Rightarrow \boxed{B = \frac{-1}{2}}$

$I = \int \frac{\frac{1}{2}}{x-1} dx + \int \frac{\frac{-1}{2}}{x+1} dx$

$= \frac{1}{2} \cdot \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1}$

$\boxed{I = \frac{1}{2} \cdot \ln|x+1| - \frac{1}{2} \ln|x+1|}$

(F): $I = \int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3} \cdot dx$

$I = \int 2x + \frac{5x-3}{x^2-2x-3}$

$= \frac{2x^2}{2} + \underbrace{\int \frac{5x-3}{(x-3)(x+1)}}_{*}$

$$\begin{array}{r|l} & 2x \\ \hline x^2-2x-3 & 2x^3 - 4x^2 - x - 3 \\ & \mp 2x^3 \pm 4x^2 \pm 6x \\ \hline & 0 \quad 0 \quad 5x - 3 \end{array}$$

$* = \int \frac{5x-3}{(x-3)(x-1)} = \int \frac{A}{x-3} + \frac{B}{x+1}$

$\Rightarrow 5x - 3 = A(x+1) + B(x-3)$

5.

$x = -1 \Rightarrow -8 = -4B \Rightarrow \boxed{B = 2}$

$x = 3 \Rightarrow 12 = 4A \Rightarrow \boxed{A = 3}$

$$* = \int \frac{3}{X-3} + \int \frac{2}{X+1}$$

$$* = 3 \ln|X-3| + 2 \ln|X+1|$$

$$\Rightarrow \boxed{I = X^2 + 3\ln|X-3| + 2\ln|X+1|}$$

G) $$I = \int \frac{-2X+4}{(X^2+1)(X-1)^2} \cdot dX$$

$$I = \int \frac{-2X+4}{(X^2+1)(X-1)^2} = \frac{AX+B}{(X^2+1)} + \frac{C}{(X-1)} + \frac{D}{(X-1)^2}$$

$$\Rightarrow -2X+4 = (AX+B)(X-1)^2 + C(X^2+1)(X-1) + D(X^2+1)$$

because we have four variables A, B, C, D (more than three)

$\Rightarrow$ we can submit any values to the A, B, C, D

A = 2 , B = 3 , C = 4 , D = 5

$$I = \underbrace{\int \frac{2X+3}{(X^2+1)}}_{I_1} + \underbrace{\int \frac{4}{(X-1)}}_{I_2} + \underbrace{\int \frac{5}{(X-1)^2}}_{I_3}$$

$$I_1 = \int \frac{2X}{X^2+1} + \int \frac{3}{X^2+1} \rightarrow 3 \cdot \tan^{-1} X$$

$U = X^2+1 \Rightarrow dU = 2X \cdot dX$

$dX = \frac{du}{2X}$

$$= \int \frac{2X}{U} \cdot \frac{du}{2X} = \ln|U| = \ln|X^2+1|$$

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$I_2 = \int \frac{4}{X-1} = 4 . \ln |X-1|$

$I_3 = 5 \int \frac{dX}{(X-1)^2}$

$U = X-1 \Rightarrow du = dX$

$I_3 = 5 \int \frac{du}{U^2} = \int U^{-2} . du = \frac{U^{-1}}{-1} = \frac{-1}{U} = \frac{-1}{(X-1)}$

$\Rightarrow \boxed{I = \ln |X^2+1| + 3 \tan^{-1} X + 4 \ln |X-1| \frac{-1}{X-1}}$

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H.W ②

Calculus ②

Calculate the Volume of an object resulted by rotating the area bounded by $y = x^2$ and $y = 5x - 6$ about the followings:

(a) $x = 5$

(b) $x = -2$

(c) $\overleftrightarrow{y}$-axis

(d) $y = 2$

(e) $x = 1$

(f) $y = -2$

(g) $y = 11$

(h) $\overrightarrow{X}$-axis

Using both Methods

- Discs Method
- Cylinder Methods

Dead line for the H.W 2 is Monday 27th / May

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quiz for calculus 2
will be on Monday 27th / May
lessons included: → Calculating Areas
→ calculating Volumes.

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intersection points

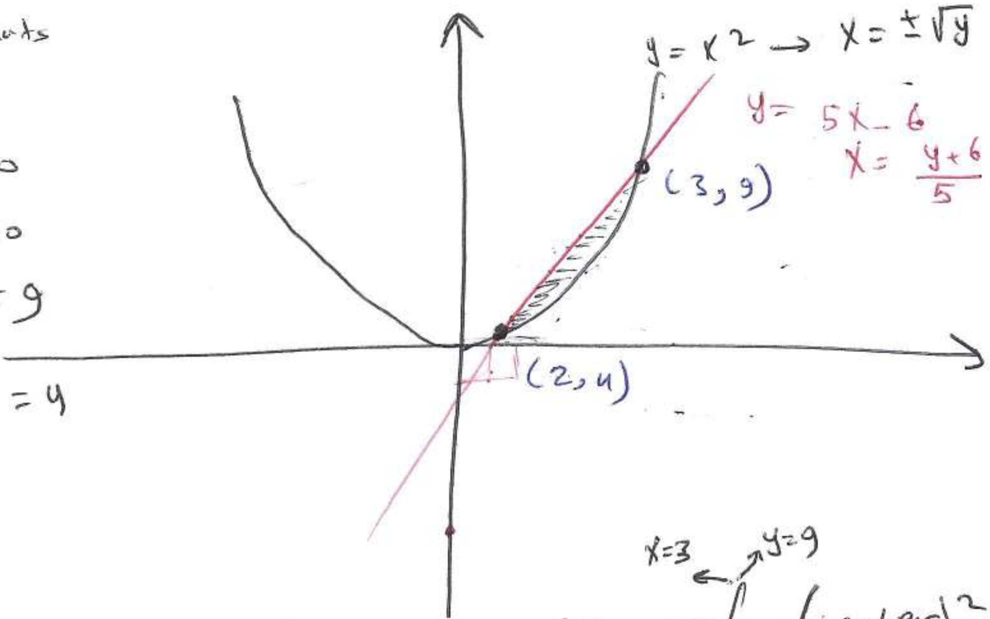
$x^2 = 5x - 6$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\to x = 3 \to y = 9$

$\to x = 2 \to y = 4$



Using Discs Methods

$$V = \pi \int_{x=2,\ y=4}^{x=3,\ y=9} \left(\left(\text{outer Rad}\right)^2 - \left(\text{inner Rad}\right)^2 \right)$$

(a) about $\boxed{x = 5}$ outer Rad $= 5 -$ inner Rad $= 5 - \sqrt{y}$	(b) about $\boxed{x = -2}$ outer Rad $= \sqrt{y} + 2$ inner Rad $= \frac{y+6}{5} + 2$	(c) about $\boxed{y\text{-axis}}$ outer Rad $= \sqrt{y}$ inner Rad $= \frac{y+6}{5}$
(d) about $\boxed{y = 2}$ outer Rad $= (5x-6) - 2$ inner Rad $= x^2 - 2$	(e) about $\boxed{x = 1}$ outer Rad $= \sqrt{y} - 1$ inner $= \frac{y+6}{5} - 1$	(f) about $\boxed{y = -2}$ outer Rad $= (5x-6) + 2$ inner Rad $= x^2 + 2$
(g) about $\boxed{y = 11}$ outer Rad $= 11 - (x^2)$ inner Rad $= 11 - (5x-6)$	(h) about $\boxed{x\text{-axis}}$ outer Rad $= 5x - 6$ inner Rad $= x^2$	

H.W (2)

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Using Cylinders Method:

$$v = \int_{x=2}^{x=3} {}_{y=4}^{y=9}\; 2\pi R H$$

(a) about $\boxed{x = 5}$ $R = 5 - x$ $H = (5x - 6) - x^2$	(b) about $\boxed{x = -2}$ $R = x + 2$ $H = (5x - 6) - x^2$	(c) about $\boxed{y\text{-axis}}$ $R = x$ $H = (5x - 6) - x^2$
(d) about $\boxed{y = 2}$ $R = y - 2$ $H = \sqrt{y} - \left(\frac{y+6}{5}\right)$	(e) about $\boxed{x = 1}$ $R = x - 1$ $H = (5x - 6) - x^2$	(f) about $\boxed{y = -2}$ $R = y + 2$ $H = \sqrt{y} - \left(\frac{y+6}{5}\right)$
(g) about $\boxed{y = 11}$ $R = 11 - y$ $H = \sqrt{y} - \left(\frac{y+6}{5}\right)$	(h) about $\boxed{x\text{-axis}}$ $R = y$ $H = \sqrt{y} - \left(\frac{y+6}{5}\right)$	

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