# Lines and Circles

Q: write the formula of the line:

① Passes through $(2,-3)$ with slope $\frac{1}{2}$

$$y = y_1 + m(x - x_1)$$

$$\boxed{y = -3 + \frac{1}{2}(x - 2)}$$

② passes through $(3,4)$ and $(-2,5)$

$$y = y_1 + m(x - x_1)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{5-4}{-2-3} = \frac{-1}{5}$$

$$y = y_1 + m(x - x_1)$$

$$\boxed{y = 4 - \frac{1}{5}(x - 3)}$$

③ passes through $(-8,0)$ and $(-1,3)$

$$y = y_1 + m(x - x_1)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{3-0}{-1+8} = \frac{3}{7}$$

$$\boxed{y = 0 + \frac{3}{7}(x + 8)}$$

(4) has slope $\frac{-5}{4}$ and $\underbrace{\text{y-intercept } 6}_{\text{passes through } (0,6)}$

$y = y_1 + m(x - x_1)$

$$\boxed{y = 6 - \frac{5}{4}(x - 0)}$$

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(5) has slope $\frac{1}{2}$ and $\underbrace{\text{y-intercept } -3}_{(0,-3)}$

$y = y_1 + m(x - x_1)$

$$\boxed{y = -3 + \frac{1}{2}(x - 0)}$$

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(6) passes through $(-12, -9)$ and has slope 0

has slope (0) $\Rightarrow \Delta y = 0 \Rightarrow y = \text{constant}$

$\Rightarrow y = y_1 \Rightarrow \boxed{y = -9}$

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(7) passes through $(\frac{1}{3}, 4)$ and has no slope

has no slope $\Rightarrow \Delta x = 0 \Rightarrow x = \text{constant}$

$\Rightarrow x = x_1 \Rightarrow \boxed{x = \frac{1}{3}}$

---

⑧ has $\underbrace{\text{y-intercept } 4}_{\text{passes } (0,4)}$ and $\underbrace{\text{X-intercept } -1}_{\text{passes } (-1,0)}$

$$m = \frac{\Delta y}{\Delta x} = \frac{0-4}{-1-0} = \frac{-4}{-1} = 4$$

$$y = y_1 + m(X - X_1)$$

$$\boxed{y = 4 + 4(X - 0)}$$

⑨ has $\underbrace{\text{y-intercept } -6}_{\text{passes } (0,-6)}$ and $\underbrace{\text{X-intercept } 2}_{\text{passes } (2,0)}$

$$m = \frac{\Delta y}{\Delta x} = \frac{0+6}{2-0} = 3$$

$$y = y_1 + m(X - X_1)$$

$$\boxed{y = -6 + 3(X - 0)}$$

⑩ passes through $(5,-1)$ and is parralled to the lin $2X + 5y = 15$.

Solution we will fix the formula to become like $y = y_1 + m(X - X_1)$

$$5y = -2X + 15$$

$$y = \underbrace{\frac{-2}{5}}_{m_1} X + 15$$

parralled $\Rightarrow m_1 = m_2 \Rightarrow m_2 = \frac{-2}{5}$

$$y = y_1 + m(X - X_1)$$

$$\boxed{y = -1 \; \frac{-2}{5}(X - 5)}$$

(11) passes through $(-\sqrt{2}, 2)$ and parallel to the line
$\sqrt{2}\,X + 5y = \sqrt{3}$

Solution

$$5y = -\sqrt{2}\,X + \sqrt{3}$$

$$y = \frac{-\sqrt{2}}{5} X + \frac{\sqrt{3}}{5}$$

$\rightarrow m_1$

parallel $\Rightarrow$ $m_1 = m_2$ $\Rightarrow$ $\boxed{m_2 = \frac{-\sqrt{2}}{5}}$

$$y = y_1 + m\,(X - X_1)$$

$$\boxed{y = 2 \quad \frac{-\sqrt{2}}{5}\,(X + \sqrt{2})}$$

(12) passes through $(4, 10)$ and is perpendiculer to the line $6X - 3y = 5$

Solution

$$-3y = -6X + 5$$

$$y = 2X - \frac{5}{3}$$

$\rightarrow m_1$

Perpendiculer $\Rightarrow$ $m_1 \cdot m_2 = -1$ $\Rightarrow$ $\boxed{m_2 = \frac{-1}{2}}$

$$y = y_1 + m\,(X - X_1)$$

$$\boxed{y = 10 \quad -\frac{1}{2}\,(X - 4)}$$

(13) passes through $(0,1)$ and is perpendicular to the line $8X - 13y = 13$

$$-13y = -8X + 13$$

$$y = \frac{+8}{13} X - 1$$

$m_1$

perpendicular $\Rightarrow$ $m_1 \cdot m_2 = -1 \Rightarrow m_2 = \frac{-1}{\frac{8}{13}} = \frac{-13}{8}$

$$y = y_1 + m(X - X_1)$$

$$\boxed{y = 1 - \frac{13}{8}(X - 0)}$$

* Question: find the distance between $P(-1,2)$ and $Q(3,4)$

Solution

$$d = \sqrt{(\Delta X)^2 + (\Delta y)^2}$$

$$d = \sqrt{(4-2)^2 + (3+1)^2}$$

$$= \sqrt{4 + 16} = \sqrt{2} = 2\sqrt{5}$$

N

* Question: if you know that the length of a line $\overleftrightarrow{PQ}$ equals to (5) when $P(2,3)$ and $Q(X_Q, 6)$, then find out $X_Q$

$$d = \sqrt{(X_2 - X_1)^2 + (y_2 - y_1)^2} \Rightarrow 5 = \sqrt{(6-3)^2 + (X_Q - 2)^2}$$

we square both sides $\Rightarrow$ $25 = 9 + (X_Q - 2)^2$

$\Rightarrow (X_Q - 2)^2 = 16 \Rightarrow$ either $X_Q - 2 = +4 \Rightarrow \boxed{X_Q = 6}$

or $X_Q - 2 = -4 \Rightarrow \boxed{X_{Q_2} = -2}$

## Question 1 :

If $2 < x < 6$, which of the following statements about x are necessarily true, and which are not necessarily true?

a. $0 < x < 4$ ✗
b. $0 < x - 2 < 4$ ✓
c. $1 < \frac{x}{2} < 3$ ✓
d. $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$ ✓
e. $1 < \frac{6}{x} < 3$ ✓
f. $|x - 4| < 2$ ✓
g. $-6 < -x < 2$ ✗
h. $-6 < -x < -2$ ✓

**Solution:**

(b) $0 < x - 2 < 4$ (+2)
$2 < x < 6$ ✓

(c) $1 < \frac{x}{2} < 3$ (×2)
$2 < x < 6$ ✓

(d) $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$
both sides are positive ⇒
$6 > x > 2$ ✓

(e) $1 < \frac{6}{x} < 3$
both sides are positive
$1 > \frac{x}{6} > \frac{1}{3}$
$6 > x > 2$ ✓

(f) $|x-4| < 2$
$-2 < x-4 < 2$
$2 < x < 6$ ✓

(g) $-6 < -x < 2 \Rightarrow 6 > x > -2$
$-2 < 2$ True

(h) $-6 < -x < -2$
$6 > x > 2$ ✓

## Question 2 :

In Exercises 5–12, solve the inequalities and show the solution sets on the real line.

5. $-2x > 4$
6. $8 - 3x \geq 5$
7. $5x - 3 \leq 7 - 3x$
8. $3(2 - x) > 2(3 + x)$
9. $2x - \frac{1}{2} \geq 7x + \frac{7}{6}$
10. $\frac{6 - x}{4} < \frac{3x - 4}{2}$
11. $\frac{4}{5}(x - 2) < \frac{1}{3}(x - 6)$
12. $-\frac{x + 5}{2} \leq \frac{12 + 3x}{4}$

**Solution:**

(5) $-2x > 4$
$x < -2$

$-2$

(6) $8 - 3x \geq 5$
$-3x \geq -3$
$x \leq 1$

$1$

(7) $5x - 3 \leq 7 - 3x$
$5x + 3x \leq 7 + 3$
$8x \leq 10$
$x \leq \frac{5}{4}$

$\frac{5}{4}$

(8) $3(2-x) > 2(3+x)$
$6 - 3x > 6 + 2x$
$0 > 5x$
$x < 0$

$0$

(9) $2x - \frac{1}{2} \geq 7x + \frac{7}{6}$
$-\frac{7}{6} - \frac{1}{2} \geq 5x$
$\frac{-10}{6} \geq 5x$
$-\frac{2}{6} \geq x$
$x \leq -\frac{1}{3}$

$-\frac{1}{3}$

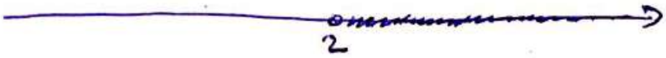
(10) $\frac{6-x}{4} < \frac{3x-4}{2}$

$*(4)$

$6 - x < 6x - 8$

$14 < 7x$

$2 < x$




(11) $\frac{4}{5}(x-2) < \frac{1}{3}(x-6)$

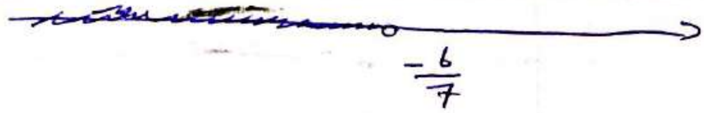
multibly by (15)

$12(x-2) < 5(x-6)$

$12x - 24 < 5x - 30$

$7x < -6$

$x < \frac{-6}{7}$




(12) $-\left(\frac{x+5}{2}\right) \leq \frac{12+3x}{4}$

$(* 4)$

$-2x - 10 \leq 12 + 3x$

$-22 \leq 5x$

$\frac{-22}{5} \leq x$

## Question 3 :

(8)

Solve the inequalities in Exercises 19–34, expressing the solution sets as intervals or unions of intervals. Also, show each solution set on the real line.

| | | |
|---|---|---|
| 19. $\lvert x\rvert < 2$ | 20. $\lvert x\rvert \le 2$ | 21. $\lvert t-1\rvert \le 3$ |
| 22. $\lvert t+2\rvert < 1$ | 23. $\lvert 3y-7\rvert < 4$ | 24. $\lvert 2y+5\rvert < 1$ |
| 25. $\left\lvert \frac{z}{5}-1\right\rvert \le 1$ | 26. $\left\lvert \frac{3}{2}z-1\right\rvert \le 2$ | 27. $\left\lvert 3-\frac{1}{x}\right\rvert < \frac{1}{2}$ |
| 28. $\left\lvert \frac{2}{x}-4\right\rvert < 3$ | 29. $\lvert 2s\rvert \ge 4$ | 30. $\lvert s+3\rvert \ge \frac{1}{2}$ |
| 31. $\lvert 1-x\rvert > 1$ | 32. $\lvert 2-3x\rvert > 5$ | 33. $\left\lvert \frac{r+1}{2}\right\rvert \ge 1$ |
| 34. $\left\lvert \frac{3r}{5}-1\right\rvert > \frac{2}{5}$ | | |

28/27/34

*Solution:*

(19) $|x| < 2$

$-2 < x < 2$


$x \in (-2, 2)$

(20) $|x| \le 2$

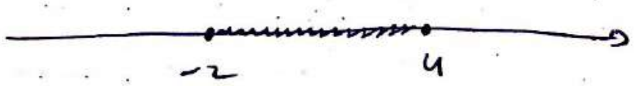
$-2 \le x \le 2$


$x \in [-2, 2]$

(21) $|t-1| \le 3$

$-3 \le t-1 \le 3$

$-2 \le t \le 4$

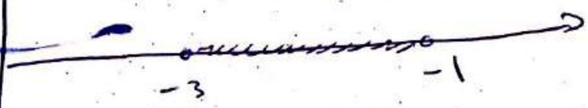



$x \in [-2, 4]$

(22) $|t+2| < 1$

$-1 < t+2 < 1$

$-3 < t < -1$



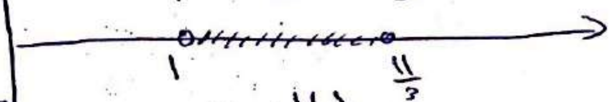

$x \in (-3, -1)$

(23) $|3y-7| < 4$

$-4 < 3y-7 < +4$

$3 < 3y < 11$

$1 < y < \frac{11}{3}$



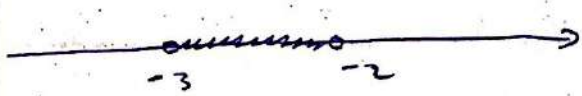

$x \in (1, \frac{11}{3})$

(24) $|2y+5| < 1$

$-1 < 2y+5 < +1$

$-6 < 2y < -4$

$-3 < y < -2$




$x \in (-3, -2)$

(25) $\left|\frac{Z}{5}-1\right| \leq 1$

$-1 \leq \frac{Z}{5}-1 \leq +1$

$0 \leq \frac{Z}{5} \leq 2$

$0 \leq Z \leq 10$

0 10

$Z \in [0, 10]$

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(26) $\left|\frac{3}{2}Z-1\right| \leq 2$

$-2 \leq \frac{3}{2}Z-1 \leq 2$

$-1 \leq \frac{3Z}{2} \leq 3$ $\times\left(\frac{2}{3}\right)$

$-\frac{2}{3} \leq Z \leq 2$

$-\frac{2}{3}$ 2

$X \in \left[-\frac{2}{3}, 2\right]$

---

(27) $\left|3-\frac{1}{X}\right| < \frac{1}{2}$

$-\frac{1}{2} < 3-\frac{1}{X} < \frac{1}{2}$ add $(-3) = -\frac{6}{2}$

$-\frac{7}{2} < -\frac{1}{X} < \frac{-5}{2}$

here $\left(-\frac{1}{X}\right)$ is a number between two negative numbers $\left(-\frac{7}{2}, -\frac{5}{2}\right)$ so both sides are negative

=> we can make procepnettion

$-\frac{2}{7} > -X > \frac{-2}{5}$ multiply by $(-1)$

$\frac{2}{7} < X < \frac{2}{5}$

$2/7$ $2/5$ $X \in \left(\frac{2}{7}, \frac{2}{5}\right)$

---

(28) $\left|\frac{2}{X}-4\right| < 3$

$-3 < \frac{2}{X}-4 < 3$

$1 < \frac{2}{X} < 7$

$\left(\frac{2}{X}\right)$ is a number between two positive numbers $(1, 7)$

=> $\frac{2}{X}$ is positive

$1 > \frac{X}{2} > \frac{1}{7}$

$2 > X > \frac{2}{7}$

$2/7$ 2

$X \in \left(\frac{2}{7}, 2\right)$

---

(29) $|2S| \geq 4$

$2S \geq +4 \Rightarrow S \geq 2$

or $2S \leq -4 \rightarrow S \leq -2$

-2 2

$X \in (-\infty, -2] \cup [2, +\infty)$

---

(30) $|S+3| \geq \frac{1}{2}$

$S+3 \geq +\frac{1}{2} \Rightarrow S \geq \frac{-5}{2}$

or $S+3 \leq -\frac{1}{2} \Rightarrow S \leq -\frac{7}{2}$

$-\frac{7}{2}$ $-\frac{5}{2}$

$X \in \left(-\infty, -\frac{7}{2}\right] \cup \left[\frac{-5}{2}, \infty\right)$

---

(31) $|1-X| > 1 \Rightarrow$ $1-X > +1 \Rightarrow X < 0$

or $1-X < -1 \Rightarrow X > 2$

0 2

$X \in (-\infty, 0) \cup (2, +\infty)$

(5)

## Question 4:

Solve the inequalities in Exercises 35–42. Express the solution sets as intervals or unions of intervals and show them on the real line. Use the result $\sqrt{a^2} = |a|$ as appropriate.

| | | |
|---|---|---|
| 35. $x^2 < 2$ | 36. $4 \le x^2$ | 37. $4 < x^2 < 9$ |
| 38. $\frac{1}{9} < x^2 < \frac{1}{4}$ | 39. $(x-1)^2 < 4$ | 40. $(x+3)^2 < 2$ |
| 41. $x^2 - x < 0$ | 42. $x^2 - x - 2 \ge 0$ | |

***Solution:***

35) $x^2 < 2 \Rightarrow |x| < \sqrt{2}$

$-\sqrt{2} < x < \sqrt{2}$

$-\sqrt{2} \quad 0 \quad \sqrt{2}$

$x \in (-\sqrt{2}, +\sqrt{2})$

36) $4 \le x^2 \Rightarrow 2 \le |x|$

$x \ge +2$

or

$x \le -2$

$-2 \quad 0 \quad 2$

$x \in (-\infty, -2] \cup [2, +\infty)$

37) $4 < x^2 < 9$

$2 < |x| < 3$

and

$2 < |x|$

$\Rightarrow x > +2$

or $x < -2$

$-2 \quad 2$

$|x| < 3$

$\Rightarrow -3 < x < +3$

$-3 \quad +3$

intersection

$-3 \quad -2 \quad 2 \quad 3$

$x \in (-3, -2) \cup (2, 3)$

38) $\frac{1}{9} < x^2 < \frac{1}{4}$

$\frac{1}{3} < |x| < \frac{1}{2}$

and

$\frac{1}{3} < |x|$

$x > +\frac{1}{3}$

or $x < -\frac{1}{3}$

$-\frac{1}{3} \quad +\frac{1}{3}$

$|x| < \frac{1}{2}$

$-\frac{1}{2} < x < +\frac{1}{2}$

$-\frac{1}{2} \quad \frac{1}{2}$

intersection

$-\frac{1}{2} \quad -\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{2}$

$x \in (-\frac{1}{2}, -\frac{1}{3}) \cup (\frac{1}{3}, \frac{1}{2})$

39) $(x-1)^2 < 4$

$|x-1| < 2$

$-2 < x-1 < 2$

$-1 < x < 3$

$-1 \quad 3$

$x \in (-1, 3)$

(40) $(x+3)^2 < 2$

$|x+3| < \sqrt{2}$

$-\sqrt{2} < x+3 < +\sqrt{2}$

$-3-\sqrt{2} < x < \sqrt{2}-3$


$x \in (-3-\sqrt{2}, \sqrt{2}-3)$

---

(41) $x^2 - x < 0$

$\underbrace{x^2 - x + \left(\tfrac{1}{2}\right)^2} - \left(\tfrac{1}{2}\right)^2 < 0$

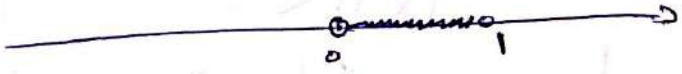
Completing to a perfect square

$\left(x - \tfrac{1}{2}\right)^2 < \tfrac{1}{4}$

$\left|x - \tfrac{1}{2}\right| < \tfrac{1}{2}$

$-\tfrac{1}{2} < x - \tfrac{1}{2} < +\tfrac{1}{2}$

$0 < x < 1$




$x \in (0, 1)$

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(6) Completetion of question: 3

(32) $|2-3x| > 5$

$2-3x > +5$

$-3 > 3x$

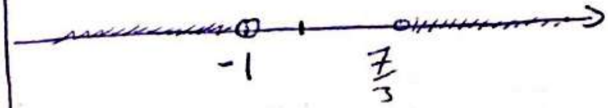
$-1 > x$

or

$2-3x < -5$

$7 < 3x$

$\tfrac{7}{3} < x$

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(33) $\left|\dfrac{r+1}{2}\right| \geq 1$

$\dfrac{r+1}{2} \geq +1$

$r+1 \geq 2$

$r \geq 1$

or

$\dfrac{r+1}{2} \leq -1$

$r+1 \leq -2$

$r \leq -3$


---

(34) $\left|\dfrac{3r}{5} - 1\right| > \dfrac{2}{5}$

$\dfrac{3r}{5} - 1 > +\dfrac{2}{5}$

$\dfrac{3r}{5} > \dfrac{7}{5} \Rightarrow r > \dfrac{7}{3}$

or $\dfrac{3r}{5} - 1 < -\dfrac{2}{5}$

$\dfrac{3r}{5} < \dfrac{3}{5} \Rightarrow r < 1$

# Exercises of Lesson 3+4

* Find the Domain and the range of the following functions:

$$\boxed{y = x^2}$$

Domain = ؟

the function has no problem when $x = 0$ / with negative values

$\Rightarrow$ Domain $(-\infty, +\infty)$

Range = ؟

we realize that what ever was $X$ $\rightarrow$ the result is always

positive $\Rightarrow$ Range $[0, +\infty)$

$$\boxed{y = \frac{1}{x}}$$

Domain = ؟

the function has no problem with negative values but has a problem with 0

$\Rightarrow$ Domain $= (-\infty, 0) \cup (0, +\infty)$

Range = ؟

we realize that the output of the function can be any real number but 0.

$\Rightarrow$ range $= (-\infty, 0) \cup (0, +\infty)$

$$\boxed{y = \sqrt{X}}$$

Domain = ?

$X \geq 0 \Rightarrow$ Doman $= [0, \infty)$

Range

$\sqrt{X}$ is always a positive number

Range $= [0, \infty)$

---

$y = \sqrt{4 - X}$

$4 - X \geq 0 \Rightarrow X \leq 4 \Rightarrow$ Domain $= (-\infty, 4]$

| $X$ | $-\infty$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| | | 1,7 | 1,4 | 1 | 0 |

$\Rightarrow$ Range $= [0, +\infty)$

---

$$y = \left| \frac{-3}{2X} \right|$$

Daman $(-\infty, 0) \cup (0, +\infty)$

Range = ?

$y$ → always positive

$y$ → cant be 0

$\Rightarrow$ Range $= (0, +\infty)$

---

$y = 1 + X^2$

Damin $= (-\infty, +\infty)$

Range = ?

$X^2 \geq 0$

$X^2 + 1 \geq 1 \Rightarrow$ Range $= [1, +\infty)$

---

$$\boxed{y = \frac{-3\sqrt{x}+1}{2}}$$

Domain?

$x \geq 0 \Rightarrow$ Domain $[0, +\infty)$

Range=?

we realize that $\sqrt{x} \geq 0$

$$-3\sqrt{x} \leq 0$$

$$-3\sqrt{x}+1 \leq 1$$

$$\frac{-3\sqrt{x}+1}{2} \leq \frac{1}{2}$$

$$y \leq \frac{1}{2} \Rightarrow \text{Range} = (-\infty, \tfrac{1}{2}]$$

---

$\boxed{y = 3x - x^2}$ , Domain = R

by completing to a perfect square

$$y = -(x^2 - 3x)$$

$$= -(x^2 - 3x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2)$$

$$= -((x - \tfrac{3}{2})^2 - \tfrac{9}{4})$$

$$y = -(x - \tfrac{3}{2})^2 + \tfrac{9}{4}$$

we have

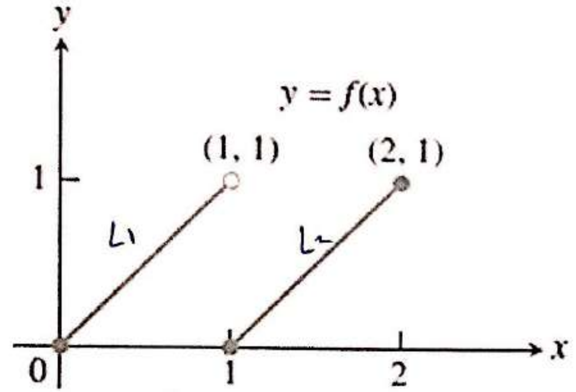
$$(x - \tfrac{3}{2})^2 \geq 0$$

$$-(x - \tfrac{3}{2})^2 \leq 0$$

$$-(x - \tfrac{3}{2})^2 + \tfrac{9}{4} \leq \tfrac{9}{4}$$

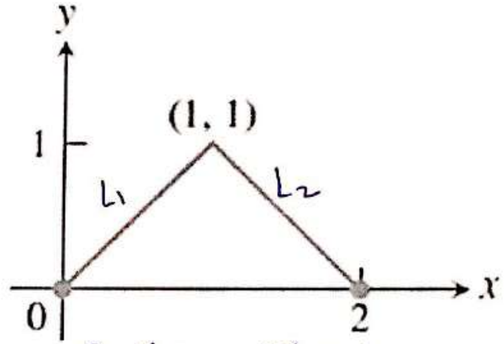
$$y \leq \tfrac{9}{4}$$

So the range is $(-\infty, \frac{9}{4}]$

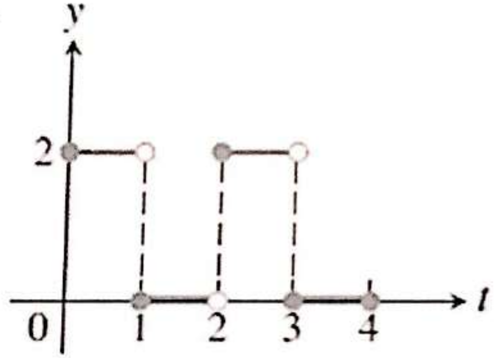



Find a formula for each function graphed.

**a.**




**b.**



formula $y = \begin{cases} X : 0 \leq X < 1 \\ X-1 : 1 \leq X \leq 2 \end{cases}$

(L2) line passes two points (1,0) and (2,1)

$\Rightarrow m = \frac{1-0}{2-1} = 1$

$y - 0 = 1(X-1) \Rightarrow \boxed{y = X-1}$

$y = \begin{cases} X : 0 \leq X \leq 1 \\ -X+2 : 1 < X \leq 2 \end{cases}$

$L_2$ is a line passes two points (1,1) and (2,0) $\Rightarrow m = \frac{0-1}{2-1} = -1$

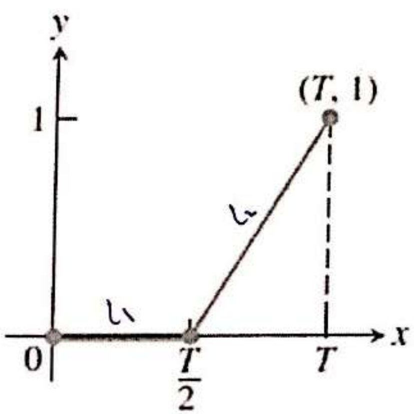
$\Rightarrow y - 0 = -1(X-2)$

$\boxed{y = -X+2}$

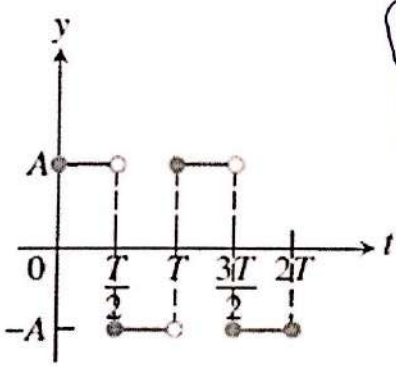
$y = \begin{cases} 2 : 0 \leq X < 1 \\ 0 : 1 \leq X < 2 \\ 2 : 2 \leq X < 3 \\ 0 : 3 \leq X \leq 4 \end{cases}$

(4)

**a'.**




**b'.**



(a'): $y = \begin{cases} 0 : 0 < X < \frac{T}{2} \\ \frac{2}{T}X - 1 : \frac{T}{2} < X < T \end{cases}$

$L_1$: is a line that passes two points $(\frac{T}{2}, 0)$, $(T, 1)$

$m = \frac{1-0}{T-\frac{T}{2}} = \frac{2}{T}$

$\Rightarrow y - 0 = \frac{2}{T}(X - \frac{T}{2}) \Rightarrow \boxed{y = \frac{2}{T}X - 1}$

(b') $y = \begin{cases} A : 0 \leq X < \frac{T}{2} \\ -A : \frac{T}{2} \leq X < T \\ A : T \leq X < \frac{3T}{2} \\ -A : \frac{3T}{2} \leq X \leq 2T \end{cases}$

Exercises

Lesson 5 + Lesson 6

Calculus

find the limits of the following functions

$$\lim_{t \to -\infty} \frac{t^2 - 5t - 9}{2t^4 + 3t^3} = \frac{(-\infty)^2 - 5(-\infty) - 9}{2(-\infty)^4 + 3(-\infty)^3} = \frac{\infty + \infty - 9}{\underbrace{\infty - \infty}_{\text{undefined}}}$$

$\Rightarrow$ we will make some changes in the function.

$$\lim_{t \to -\infty} \frac{t^4\left(\frac{t^2}{t^4} - 5\frac{t}{t^4} - \frac{9}{t^4}\right)}{t^4\left(2\frac{t^4}{t^4} + 3\frac{t^3}{t^4}\right)} = \lim_{t \to -\infty} \frac{\left(\frac{1}{t^2} - 5\frac{1}{t^3} - \frac{9}{t^4}\right)}{\left(2 + \frac{3}{t}\right)} =$$

$$= \frac{\frac{1}{(-\infty)^2} - \frac{5}{(-\infty)^3} - \frac{9}{(-\infty)^4}}{2 + \frac{3}{-\infty}}$$

$$= \frac{0 - 0 - 0}{2 + 0} = \frac{0}{2} = \boxed{0}$$

---

$$\lim_{x \to \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7} = \frac{2(\infty)^4 - (\infty)^2 + 8(\infty)}{-5(\infty)^4 + 7} = \frac{\infty - \infty + \infty}{-\infty}$$

undefined

$$= \lim_{x \to \infty} \frac{x^4\left(\frac{2x^4}{x^4} - \frac{x^2}{x^4} + \frac{8x}{x^4}\right)}{x^4\left(\frac{-5x^4}{x^4} + \frac{7}{x^4}\right)} = \lim_{x \to \infty} \frac{\left(\frac{2}{\cancel{1}} - \frac{1}{x^2} + \frac{8}{x^3}\right)}{\left(-5 + \frac{7}{x^4}\right)}$$

$$= \frac{2 - \frac{1}{(\infty)^2} + \frac{8}{(\infty)^3}}{-5 + \frac{7}{(\infty)^4}} = \frac{2 - 0 + 0}{-5 + 0} = \boxed{\frac{-2}{5}}$$

---

$$\lim_{x\to\infty} \frac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+e^{-x}} = \frac{6e^{4(\infty)}-e^{-2(\infty)}}{8e^{4(\infty)}-e^{2(\infty)}+e^{-(\infty)}} = \frac{\infty-0}{\infty-\infty+0}$$

undefined

$$\lim_{x\to\infty} \frac{e^{4x}\left(6\frac{e^{4x}}{e^{4x}}-\frac{e^{-2x}}{e^{4x}}\right)}{e^{4x}\left(8\frac{e^{4x}}{e^{4x}}-\frac{e^{2x}}{e^{4x}}+\frac{e^{-x}}{e^{4x}}\right)} = \lim_{x\to\infty}\frac{\left(6-\frac{1}{e^{6x}}\right)}{\left(8-\frac{1}{e^{2x}}+\frac{1}{e^{5x}}\right)} = \frac{6-0}{8-0+0}$$

$$= \boxed{\frac{6}{8}}$$

---

$$\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}} = \frac{16-16}{2-2} = \frac{0}{0} \text{ undefined}$$

$$\lim_{x\to 4} \frac{x^2\left(\frac{4x}{x^2}-\frac{x^2}{x^2}\right)}{x^2\left(\frac{2}{x^2}-\frac{x^{\frac{1}{2}}}{x^2}\right)} = \lim_{x\to 4}\frac{\frac{4}{x}-1}{\frac{2}{x^2}-\frac{1}{x^{\frac{3}{2}}}} = \frac{\frac{4}{4}-1}{\frac{2}{16}-\frac{1}{8}} = \frac{0}{0}$$

again not Defined

we will try another way.

$$\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}} = \lim_{x\to 4}\frac{x(4-x)}{(2-\sqrt{x})} = \lim_{x\to 4}\frac{x\cancel{(2-\sqrt{x})}(2+\sqrt{x})}{\cancel{(2-\sqrt{x})}}$$

$$= \lim_{x\to 4} x(2+\sqrt{x}) = 4(2+\sqrt{4}) = \boxed{16}$$

---

$$\lim_{t\to 4} \frac{t-\sqrt{3t+4}}{4-t} = \frac{4-\sqrt{3.4+4}}{4-4} = \frac{0}{0}\Big) \to \text{undefined}$$

$$\lim_{t\to 4} \frac{\left(t-\sqrt{3t+4}\right)\left(t+\sqrt{3t+4}\right)}{(4-t)\cdot\left(t+\sqrt{3t+4}\right)} = \lim_{t\to 4} \frac{t^2-(3t+4)}{(4-t)\left(t+\sqrt{3t+4}\right)}$$

$$= \lim_{t\to 4} \frac{(t^2-3t-4)}{(4-t)\left(t+\sqrt{3t+4}\right)} = \lim_{t\to 4} \frac{(t-4)(t+1)}{-(t-4)\left(t+\sqrt{3t+4}\right)}$$

$$= \lim_{t\to 4} \frac{t+1}{-\left(t+\sqrt{3t+4}\right)} = \frac{4+1}{-\left(4+\sqrt{3.4+4}\right)}$$

$$= \boxed{\frac{5}{-8}}$$



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$$\lim_{X \to -2} \frac{X^2 - 3X + 2}{X^3 - 4X} = \frac{(-2)^2 - 3(-2) + 2}{(-2)^3 - 4(-2)} = \frac{4+6+2}{-8+8}$$

$$= \frac{12}{0}$$

$+\infty \quad -\infty$

we will do the limit from both sides. $\Leftarrow$

* the limit from the right side

$$\lim_{X \to -2^+} \frac{X^2 - 3X + 2}{X^3 - 4X} = \lim_{X \to -2^+} \frac{(X-2)(X-1)}{X(X^2-4)} = \lim_{X \to -2^+} \frac{(X-2)(X-1)}{X(X-2)(X+2)}$$

$$= \lim_{X \to -2^+} \frac{X-1}{X(X+2)} = \frac{-2^+ - 1}{-2^+(-2^+ + 2)} = \frac{-3}{-2(0^+)}$$

$$= \frac{-3}{0^-} = \underline{\underline{+\infty}}$$

* the limit from the left side:

$$\lim_{X \to -2^-} \frac{X-1}{X(X+2)} = \frac{-2^- - 1}{-2^-(-2^- + 2)} = \frac{-3}{-2(0^-)} = \frac{-3}{0^+} = \underline{\underline{-\infty}}$$

$\Rightarrow$ So the limit of the function doesn't exist when $X \to -2$

there are only

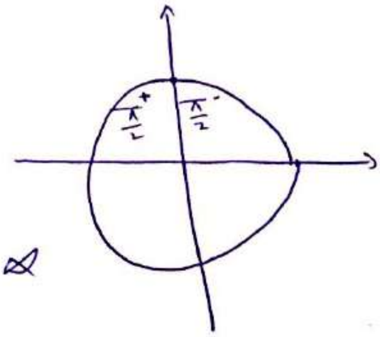
* limit from the left side $(-\infty)$

* " " " right " $(+\infty)$

$$\lim_{x \to \frac{\pi}{2}} \tan x = \lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cos x} = \frac{\sin(\frac{\pi}{2})}{\cos(\frac{\pi}{2})} = \frac{1}{0} \begin{cases} +\infty \\ -\infty \end{cases}$$

* the limit from the right side

$$\lim_{x \to \frac{\pi}{2}^+} \frac{\sin x}{\cos x} = \frac{\sin(\frac{\pi}{2}^+)}{\cos(\frac{\pi}{2}^+)} = \frac{1}{0^-} = -\infty$$




* the limit from the left side

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\sin x}{\cos x} = \frac{\sin(\frac{\pi}{2}^-)}{\cos(\frac{\pi}{2}^-)} = \frac{1}{0^+} = +\infty$$

$\Rightarrow$ The limit of the function doesn't exist when $x \to \frac{\pi}{2}$

we have only the limit from right side $(-\infty)$

" " left " $(+\infty)$

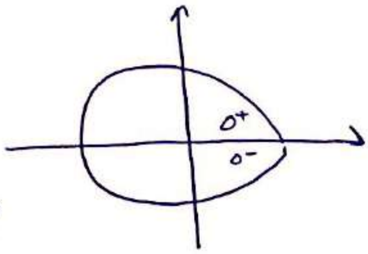
---

$$\lim_{x \to 0} \left(1 + \frac{1}{\sin x}\right) = 1 + \frac{1}{\sin(0)} = 1 + \frac{1}{0} = \begin{cases} +\infty \\ -\infty \end{cases}$$

from the right side:

$$\lim_{x \to 0^+} \left(1 + \frac{1}{\sin x}\right) = 1 + \frac{1}{\sin(0^+)} = 1 + \frac{1}{0^+} = +\infty$$

$$\lim_{x \to 0^-} \left(1 + \frac{1}{\sin x}\right) = 1 + \frac{1}{\sin(0^-)} = 1 + \frac{1}{0^-} = -\infty$$



What is the type of the following functions

$f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x) \Rightarrow$ Even function

$f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = -f(x) \Rightarrow$ odd function

$f(x) = \dfrac{1}{x-1}$

$f(-x) = \dfrac{1}{-x-1} = \dfrac{1}{-(x+1)} \left. \begin{matrix} \neq f(x) \\ \neq -f(x) \end{matrix} \right\} \Rightarrow$ the function is neither odd nor even

$f(x) = \dfrac{1}{x^2-1}$

$f(-x) = \dfrac{1}{(-x)^2-1} = \dfrac{1}{x^2-1} = f(x) \Rightarrow$ the function is Even

$f(x) = x^{-5}$

$f(-x) = (-x)^{-5} = \dfrac{1}{(-x)^5} = \dfrac{1}{-x^5} = -x^{-5} = -f(x) \Rightarrow$ the function is odd.

$f(x) = x^2 + x$

$f(-x) = (-x)^2 - x = x^2 - x < \begin{matrix} \neq f(x) \\ \neq -f(x) \end{matrix} \Big\}$ the function is neither odd nor even.

$f(x) = 2x+1$

$f(-x) = -2x+1 \left\langle \begin{array}{l} \neq f(x) \\ \neq -f(x) \end{array} \right\}$ the function is neither odd nor even.

---

$f(x) = x^4 + 3x^2 - 1$

$f(-x) = (-x)^4 + 3(-x)^2 - 1$

$= x^4 + 3x^2 - 1 = f(x) \Rightarrow$ even function

---

$f(x) = \dfrac{x}{x^2-1}$

$f(-x) = \dfrac{-x}{(-x)^2-1} = \dfrac{-x}{x^2-1} = -\left(\dfrac{x}{(x^2-1)}\right) = -f(x) \Rightarrow$ odd function

---

$f(x) = |x^3|$

$f(-x) = |(-x)^3| = |x^3| = f(x) \Rightarrow$ even function

---

$f(x) = 2|x|+1$

$f(-x) = 2|-x|+1 = 2|x|+1 \Rightarrow$ even function

---

Exercises about
the composite functions

Find the domain of fog and gof of each of thi

* $\boxed{F(x) = x^2+2 \quad , \quad g(x) = \sqrt{7-x}}$

Step 1: the domain of the inside function
Step 2: " " " Resulted function
step 3: the intersection between the two ~~functions~~ domains

First: fog

Step 1: the domain of the inside function $(g(x))$ is

$$7-x \geq 0$$
$$x \leq 7$$

Step 2: the domain of the Resulted function

$$f\circ g(x) = f(g(x)) = (\sqrt{7-x})^2+2 = 7-x+2 = 9-x$$
$$f\circ g(x) = 9-x \Rightarrow \text{Domain} = R$$

Step 3 the intersection of both domains:

$$\boxed{x \in (-\infty, 7]} \Leftarrow$$

7

Second gof:

Step 1: domain of $f(x)$ is $R$

Step 2: domain of gof

$$g\circ f(x) = g(f(x)) = \sqrt{7-x^2-2} = \sqrt{5-x^2}$$

$$5-x^2 \geq 0 \Rightarrow x^2 \leq 5$$
$$\Rightarrow |x| \leq \sqrt{5}$$
$$-\sqrt{5} \leq x \leq \sqrt{5}$$

$-\sqrt{5}$ $+\sqrt{5}$

Step 3: the intersection

$-\sqrt{5}$ $+\sqrt{5}$

$$\boxed{x \in [-\sqrt{5}, +\sqrt{5}]}$$

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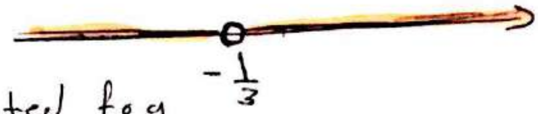
* $f(x)=\frac{1-x}{3x}$ $\qquad$ $g(x)=\frac{1}{1+3x}$

First: $f\circ g$

step 1: doman of $g(x)$ is $1+3x\neq 0 \Rightarrow x\neq -\frac{1}{3}$



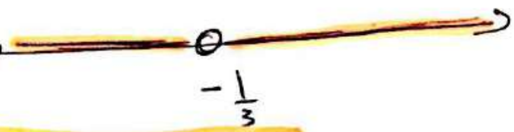

step 2: doman of the Resulted $f\circ g$

$$f\circ g\,(x)= f(g(x)) = \frac{1-\frac{1}{1+3x}}{\frac{3}{1+3x}} = \frac{\frac{1+3x-1}{1+3x}}{\frac{3}{1+3x}}$$

$$= \frac{3x}{1+3x}\cdot\frac{1+3x}{3} = \boxed{x}$$

$\Rightarrow$ Doman $= R$

step 3: intersection $= R/\{-\frac{1}{3}\}$




$$\boxed{x\in R/\{-\tfrac{1}{3}\}}$$

Second: $g\circ f$

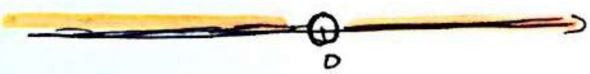
step 1: doman of $f(x)$: $x\neq 0$


step 2: " " Resulted

$$g\circ f\,(x)= g(f(x)) = \frac{1}{1+3\left(\frac{1-x}{3x}\right)} = \frac{1}{\frac{x+1-x}{x}} = \boxed{x}$$

Doman $R$

step 3 the intersection is $R/\{0\}$




$$\Rightarrow \boxed{x\in R/\{0\}}$$

$$f(x) = \sqrt{x-2} \qquad g(x) = \sqrt{x^2-1}$$

* first: $f \circ g$

step 1: doman of $g(x)$ is

$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow |x| \geq 1$

$\Rightarrow x \geq 1$ or $x \leq -1$

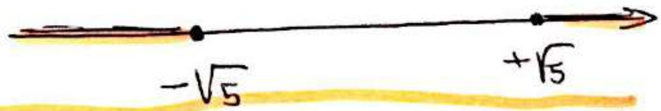

step 2: doman of the Resulted

$f \circ g(x) = f(g(x)) = \sqrt{\sqrt{x^2-1} - 2}$

$\sqrt{x^2-1} - 2 \geq 0 \Rightarrow \sqrt{x^2-1} \geq 2 \Rightarrow x^2 - 1 \geq 4$

$\Rightarrow x^2 \geq 5 \Rightarrow |x| \geq \sqrt{5} \Rightarrow x \geq \sqrt{5}$ or $x \leq -\sqrt{5}$


step 3: the intersectio

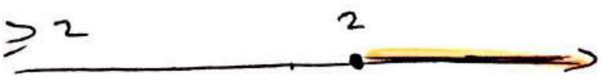



$$\boxed{x \in (-\infty, -\sqrt{5}] \cup [\sqrt{5}, +\infty)}$$

* Second: $g \circ f$

step 1: doman of $f(x)$

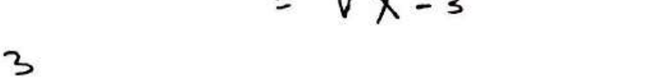
$x - 2 \geq 0 \Rightarrow x \geq 2$




step 2: doman of resulted

$g \circ f(x) = g(f(x)) = \sqrt{(\sqrt{x-2})^2 - 1} = \sqrt{x-2-1}$
$= \sqrt{x-3}$

$x - 3 \geq 0 \Rightarrow x \geq 3$




step 3: the intersection


$$\boxed{x \in [3, +\infty)}$$

question Calculate the following derivation

- $g(x) = \frac{x}{x-1}$

$g'(x) = \frac{1.(x-1) - 1.x}{(x-1)^2} = \frac{x-1-x}{(x+1)^2} = \boxed{\frac{-1}{(x-1)^2}}$

---

- $g(x) = \frac{(x-1)(x^2-2x)}{x^4}$

$g(x) = \frac{x^3 - 2x^2 + 2x - x^2}{x^4} = \frac{x^3 - 3x^2 + 2x}{x^4}$

$g(x) = \frac{x^3}{x^4} - \frac{3x^2}{x^4} + \frac{2x}{x^4}$

$g(x) = \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3}$

$g(x) = x^{-1} - 3x^{-2} + 2x^{-3}$

now we make derivation

$\boxed{g'(x) = -x^{-2} + 6x^{-3} - 6x^{-4}}$

---

question find the derivation of the following function

$$f(x) = \frac{x^2+3}{(x-1)^3+(x+1)^3}$$

$$f(x) = \frac{x^2+3}{x^3+3x^2+3x+1+x^3-3x^2+3x-1}$$

$$f(x) = \frac{x^2+3}{2x^3+6x}$$

$$f(x) = \frac{(x^2+3)}{2x(x^2+3)} = \boxed{\frac{1}{2x}}$$

now we make derivation

$$f'(x) = \frac{0-2}{(2x)^2} = \boxed{\frac{-1}{x^2}}$$

---

question find the derivation of

$$f(x) = \tan(5-\sin(2x))$$

$$f'(x) = \tan'(5-\sin(2x)).(5-\sin(2x))'$$

$$f'(x) = \sec^2(5-\sin 2x)(0-2\cos(2x))$$

---

question find the derivation of $f(x) = \cot\left(\frac{\sin(t)}{t}\right)$

$$f(x) = \cot\left(\frac{\sin(t)}{t}\right)$$

$$\Rightarrow f'(x) = \cot'\left(\frac{\sin(t)}{t}\right).\left(\frac{\sin(t)}{t}\right)'$$

$$= -\csc^2\left(\frac{\sin(t)}{t}\right).\left(\frac{\cos(t).t-\sin(t)}{t^2}\right)$$

---

Question find the derivative of the following function

$$f(x) = \sin\left(\frac{x}{\sqrt{x+1}}\right)$$

$$= \sin'\left(\frac{x}{\sqrt{x+1}}\right) \cdot \left(\frac{x}{\sqrt{x+1}}\right)'$$

$$= \cos\left(\frac{x}{\sqrt{x+1}}\right) \cdot \left(\frac{1 \cdot \sqrt{x+1} - \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1 \cdot x}{(x+1)}\right)$$

Question find the derivation of the following function

$$f(x) = \sin(\cos(x^2))$$

$$f'(x) = \sin'(\cos(x^2)) \cdot (\cos(x)^2)'$$

$$= \cos(\cos(x^2)) \cdot \cos'(x)^2 \cdot (x^2)'$$

$$= \cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot (2x)$$

Question find the derivation of the following function

$$f(x) = \cot(\sin(\csc(3x^3)))$$

$$f'(x) = \cot'(\sin(\csc(3x^3))) \cdot (\sin(\csc(3x^3)))'$$

$$f'(x) = -\csc^2(\sin(\csc(3x^3))) \cdot \sin'(\csc(3x^3)) \cdot (\csc(3x^3))'$$

$$= -\csc^2(\sin(\csc(3x^3))) \cdot \cos(\csc(3x^3)) \cdot \csc'(3x^3) \cdot (3x^3)'$$

$$= -\csc^2(\sin(\csc(3x^3))) \cdot \cos(\csc(3x^3)) \cdot (-\csc(3x^3) \cdot \cot(3x^3) \cdot (9x^2))$$

Question find the $y'$

$$x^2 + y^2 = 25$$

we make derivation for both sides

$$2x + 2yy' = 0$$

$$\Rightarrow \boxed{y' = \frac{-2x}{2y}}$$

---

Question find $y'$

$$y^2 \cdot \cos\left(\frac{1}{y}\right) = 2x + 2y$$

$$(y^2)' \cdot \cos\left(\frac{1}{y}\right) + \left(\cos\left(\frac{1}{y}\right)\right)' \cdot y^2 = 2 + 2y'$$

$$2yy' \cdot \cos\left(\frac{1}{y}\right) - \sin\left(\frac{1}{y}\right)\left(\frac{-y'}{\cancel{y^2}}\right) \cdot \cancel{y^2} = 2 + 2y'$$

$$2yy' \cdot \cos\left(\frac{1}{y}\right) + y' \cdot \sin\left(\frac{1}{y}\right) = 2 + 2y'$$

$$2yy' \cdot \cos\left(\frac{1}{y}\right) + y' \cdot \sin\left(\frac{1}{y}\right) - 2y' = 2$$

$$y'\left(2y\cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) - 2\right) = 2$$

$$y' = \frac{2}{2y\cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) - 2}$$

---

Question find $y'$

Question find the second derivation $y''$

$$3x^3 - 3y^2 = 8$$

$$9x^2 - 6yy' = 0 \Rightarrow y' = \frac{9x^2}{6y}$$

we make derivation again

$$18x - 6(y' \cdot y' + y'' \cdot y) = 0$$

$$18x - 6(y'^2 + y'' \cdot y) = 0$$

$$18x - 6y'^2 - 6y''y = 0$$

$$y'' = \frac{18x - 6y'^2}{6y} \Rightarrow \boxed{y'' = \frac{3x - \left(\frac{9x^2}{6y}\right)^2}{y}}$$

---

Question $y = x^x \rightarrow$ find $y'$

$$\ln y = \ln x^x$$

$$\ln y = x \ln x$$

we make derivation for both sides

$$\frac{y'}{y} = 1 \cdot \ln x + \frac{1}{x} \cdot x$$

$$\boxed{y' = y\left(\ln(x) + 1\right)}$$

---

Question. find $y'$

$$y = \frac{x^5}{(1-10x)\sqrt{x^2+2}}$$ ⇒ we make ln for both sides

$$\ln y = \ln \frac{x^5}{(1-10x)\sqrt{x^2+2}}$$

$$= \ln(x^5) - \ln\left((1-10x)(x^2+2)^{\frac{1}{2}}\right)$$

$$\ln y = 5\ln x - \ln(1-10x) - \frac{1}{2}\ln(x^2+2)$$

we make derivation for both sides

$$\frac{y'}{y} = 5 \cdot \frac{1}{x} + \frac{10}{(1-10x)} - \frac{1}{2} \cdot \frac{2x}{x^2+2}$$

$$\Rightarrow \boxed{y' = y \cdot \left(\frac{5}{x} + \frac{10}{1-10x} - \frac{x}{x^2+2}\right)}$$

Question : find the derivative of the following function

$$g(x) = \frac{3\cos(x)}{4x^2+7}$$

$$g'(x) = \frac{-3 \cdot \sin(x)(4x^2+7) - (8x)(3\cos(x))}{(4x^2+7)^2}$$

# application of derivatives

## L' Hospital's Rule

this rule Helps when the result of the Limit is one of the following situations:

$\frac{0}{0}$ / $\frac{\infty}{\pm\infty}$ / $(0)(\pm\infty)$ / $\infty - \infty$ / $1^{\infty}$ / $0^{0}$ / $\infty^{0}$

indetermined forms.

So ⇓

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

---

* $\lim_{x \to 0} \frac{\sin x}{x} = \frac{\sin(0)}{0} = \frac{0}{0}$ } indetermined form

⇒ we apply L'Hospital's Rule:

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = \frac{\cos(0)}{1} = \frac{1}{1} = \boxed{1}$$

---

$\lim_{x \to 0} \frac{3x - \sin x}{x} = \frac{0 - 0}{0} = \frac{0}{0}$ } in determined form

⇒ we apply L'Hospital's Rule:

$$\lim_{x \to 0} \frac{3x - \sin x}{x} = \lim_{x \to 0} \frac{3 - \cos x}{1} = \frac{3 - 1}{1} = \boxed{2}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} = \frac{\sqrt{1} - 1}{0} = \frac{0}{0} \Big\}$$ indetermined form

→ we apply L'Hospital:

$$\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} =$$

$$= \lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - 0}{1} = \frac{1}{2}(1+0)^{-\frac{1}{2}} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \frac{0 - 0}{0} = \frac{0}{0} \Big\}$$ indetermined form

→ we apply L'Hospital:

$$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{1 - \cos x}{3x^2} = \frac{1 - 1}{0} = \frac{0}{0} \Big\}$$ indetermined form

→ we apply L'Hospital Again

$$\lim_{x\to 0} \frac{1 - \cos x}{3x^2} = \lim_{x\to 0} \frac{+\sin x}{6x} = \frac{0}{0} \Big\}$$ indetermined form

→ we apply L'Hospital Again

$$\lim_{x\to 0} \frac{\sin x}{6x} = \lim_{x\to 0} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2}$$

$$= \frac{\sqrt{1} - 1 - 0}{0} = \frac{0}{0} \Big\}$$ indefined form → We apply L'Hospital

$$\Rightarrow \; = \lim_{x \to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - 0 - \frac{1}{2}}{2x} = \frac{\frac{1}{2}(1+0)^{-\frac{1}{2}} - \frac{1}{2}}{2(0)} = \frac{0}{0} \Big\}$$ indefined form

→ we apply L'Hospital again

$$= \lim_{x \to 0} \frac{\frac{1}{2}(-\frac{1}{2})(1+x)^{-\frac{3}{2}} - 0}{2} = \frac{-\frac{1}{4}(1+0)^{-\frac{3}{2}}}{2} = \boxed{-\frac{1}{8}}$$

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$$\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{e^\infty}{(\infty)^2} = \frac{\infty}{\infty} \Big\}$$ indefined form

We apply L' Hospital.

$$= \lim_{x \to \infty} \frac{e^x}{2x} = \frac{\infty}{\infty} \Big\}$$ indefined form

We apply L' Hospital again.

$$= \lim_{x \to \infty} \frac{e^x}{2} = \frac{\infty}{2} = \boxed{\infty}$$

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$$\lim_{x \to 0} x \cdot \ln x = 0 \cdot (-\infty) \Big\}$$ undefined form

important

$$\lim_{x \to 0} x \cdot \ln x = \lim_{x \to 0} \frac{\ln x}{\frac{1}{x}}$$

now I apply L' Hospital :

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x \to 0} -\frac{x^2}{x} = \lim_{x \to 0} (-x) = (-0) = \boxed{0}$$

$$\lim_{x \to \infty} \left(x \cdot \sin \frac{1}{x}\right) = \infty \cdot \sin \frac{1}{\infty} = \infty \cdot \sin(0) = \infty(0)$$

undefined form

$$\lim_{x \to \infty} \left(x \cdot \sin \frac{1}{x}\right) = \lim_{x \to \infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$$

now we apply L' Hospital:

$$= \lim_{x \to \infty} \frac{\cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}}{\frac{-1}{x^2}} = \lim_{x \to \infty} \cos\left(\frac{1}{x}\right) = \cos\left(\frac{1}{\infty}\right)$$

$$= \cos(0) = \boxed{1}$$

$$\lim_{x\to 1} \frac{1}{\ln(x)} - \frac{1}{(x-1)} = \frac{1}{\ln(1)} - \frac{1}{(1-1)}$$

$$= \frac{1}{0} - \frac{1}{0}$$

$$= \infty - \infty \Big\} \to \text{indefined form}$$

$$= \lim_{x\to 1} \frac{(x-1) - \ln(x)}{\ln(x).(x-1)}$$

now I will apply L'Hospital:

$$= \lim_{x\to 1} \frac{1 - 0 - \frac{1}{x}}{(\frac{1}{x})\cdot(x-1) + (1)(\ln x)} = \lim \frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \ln(x)}$$

$$= \frac{1 - \frac{1}{1}}{\frac{1-1}{1} + \ln(1)} = \frac{0}{0+0} = \frac{0}{0} \Big\} \text{indefined form}$$

we apply L'Hospital again:

$$\lim_{x\to 1} \left(\frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \ln(x)}\right) = \lim_{x\to 0} \frac{0 - \frac{-1}{x^2}}{\frac{x - x + 1}{x^2} + \frac{1}{x}} =$$

$$= \lim_{x\to 1} \frac{\frac{1}{x^2}}{\frac{1}{x^2} + \frac{1}{x}} = \frac{\frac{1}{1}}{\frac{1}{1} + \frac{1}{1}} = \boxed{\frac{1}{2}}$$

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$$\lim_{x \to \alpha} x^{\frac{1}{x}} = \alpha^{\frac{1}{\alpha}} = (\alpha)^0$$ ) indefined form

if we suppose that $y = x^{\frac{1}{x}}$

$$\Rightarrow \ln y = \ln x^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln x = \frac{\ln x}{x}$$

$$\lim_{x \to \alpha} \ln y = \lim_{x \to \alpha} \frac{\ln(x)}{x} = \frac{\alpha}{\alpha}$$ ) indefined form

$\Rightarrow$ we apply l'hospital

$$\lim_{x \to \alpha} \ln(y) = \lim_{x \to \alpha} \frac{\frac{1}{x}}{1} = \frac{\frac{1}{\alpha}}{1} = \frac{0}{1} = \boxed{0}$$

but $\ln y = \ln y$

$$\Rightarrow y = e^{\ln y}$$

$$\Rightarrow \lim_{x \to \alpha} y = e^{\lim_{x \to \alpha} \ln y}$$

$$= e^0 = \boxed{1}$$

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