Mechatronics Engineering
First Grade
Calculus

Lesson 7

Completion – Continuity

By: Rasha Alkabbanie (MSc)

Continuity

Introduction:

Definition

A function $f(x)$ is said to be **continuous** at $x = a$ if

$$\lim_{x \to a} f(x) = f(a)$$

A function is said to be continuous on the interval $[a, b]$ if it is continuous at each point in the interval.

Fact 1

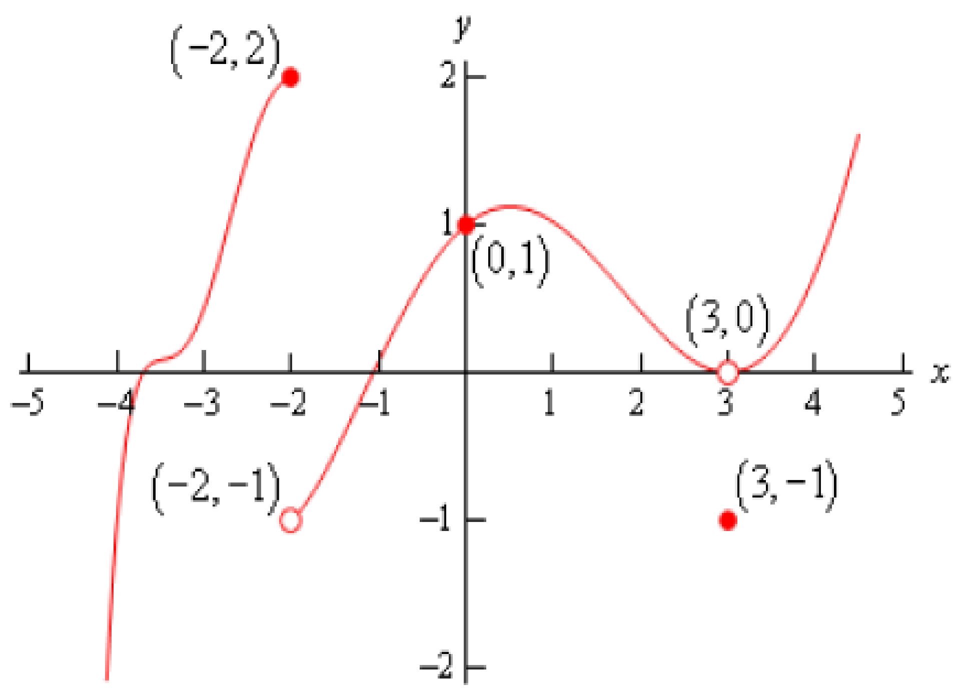
If $f(x)$ is continuous at $x = a$ then,

$$\lim_{x \to a} f(x) = f(a) \qquad \lim_{x \to a^-} f(x) = f(a) \qquad \lim_{x \to a^+} f(x) = f(a)$$

<u>Continuity at a point:</u>

Example:

Given the graph of *f(x)*, shown below, determine if *f(x)* is continuous at $x = -2, x = 0, and\ x = 3.$



For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x?

For what value of b is

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

continuous at every x?

Rates of Change and Tangent Lines

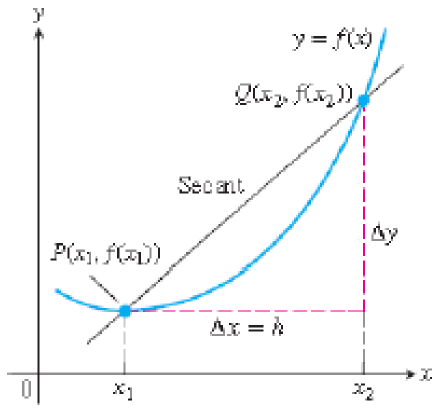
Rates of Change :

Given an arbitrary function $y = f(x)$, we calculate the average rate of change of y with respect to x over the interval $[x_1, x_2]$ by dividing the change in the value of y, $\Delta y = f(x_2) - f(x_1)$, by the length $\Delta x = x_2 - x_1 = h$ of the interval over which the change occurs.

DEFINITION Average Rate of Change over an Interval

The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

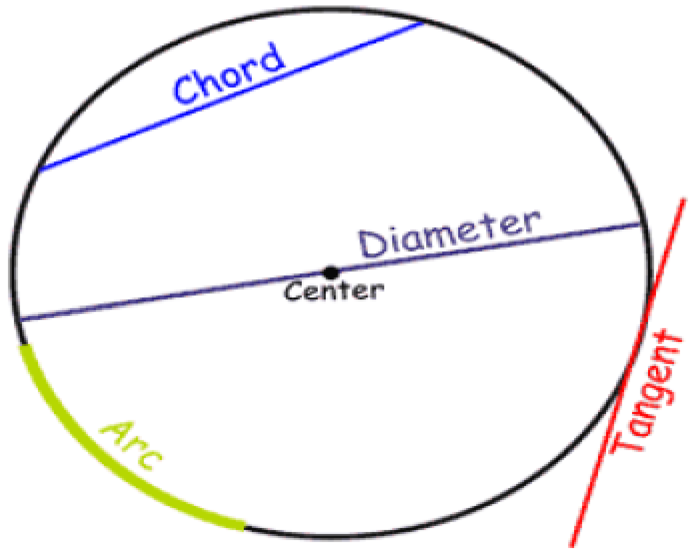
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}, \qquad h \neq 0.$$



Geometrically, the rate of change of f over is the slope of the line through the point s *P and Q*. In geometry, a line joining two points of a curve is a secant to the curve. Thus, the average **rate of change of f from x_1 to x_2** is **identical** with the **slope of secant *PQ***.

Tangent Lines:

A line that just **touches** a curve at **one point**, without cutting across it. To find the equation of a tangent line of a curve, we first find the slope of the curve at the tangent point. The tangent line is the line with this slope and passes through the tangent point (see the following example).



Example: Find the slope of the parabola $y = x^2$ at the point $P(2, 4)$. Write an equation for the tangent to the parabola at this point.

$$\text{Secant slope} = \frac{\Delta y}{\Delta x} = \frac{(2+h)^2 - 2^2}{h} = \frac{h^2 + 4h + 4 - 4}{h}$$

$$= \frac{h^2 + 4h}{h} = h + 4.$$

$$\lim_{h \to 0} (h + 4) = 4.$$

We take 4 to be the parabola's slope at P.

The tangent to the parabola at P is the line through P with slope 4:

$$y = 4 + 4(x - 2) \qquad \text{Point-slope equation}$$

$$y = 4x - 4.$$