Mechatronics Engineering
First Grade
Calculus

Lesson 5

Completion - Functions

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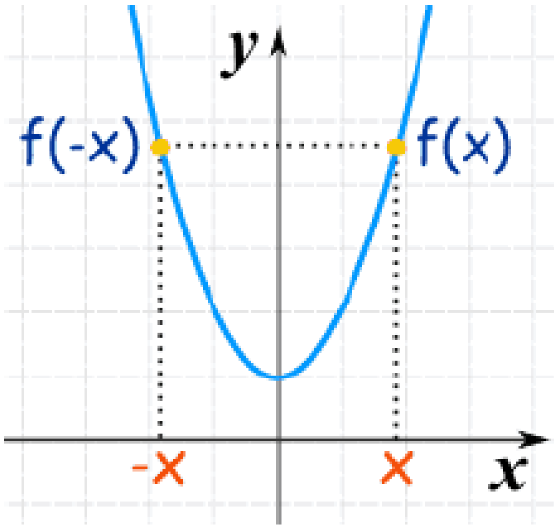
Even and Odd Functions

They are special types of functions:
- *Even Functions*

A function is "even" when:

$f(x) = f(-x)$ for all x

In other words there is symmetry about the y-axis (like a reflection):



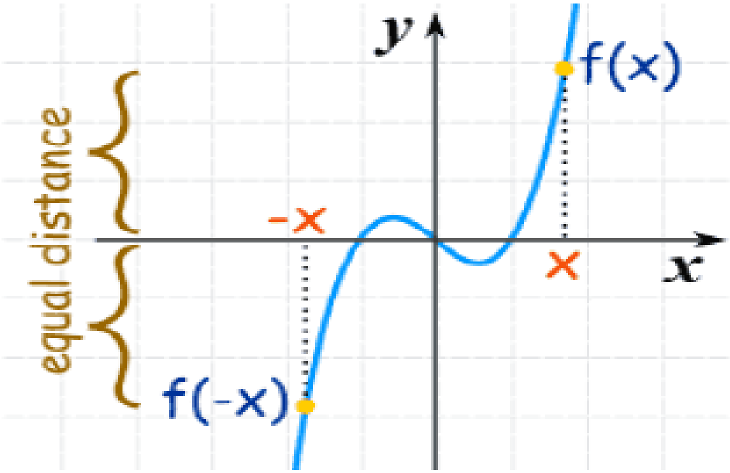
This is the curve $f(x) = x^2+1$

Odd Functions

A function is "odd" when:

-f(x) = f(-x) for all x

Note the minus in front of f: **-f(x)**.
And we get origin symmetry:

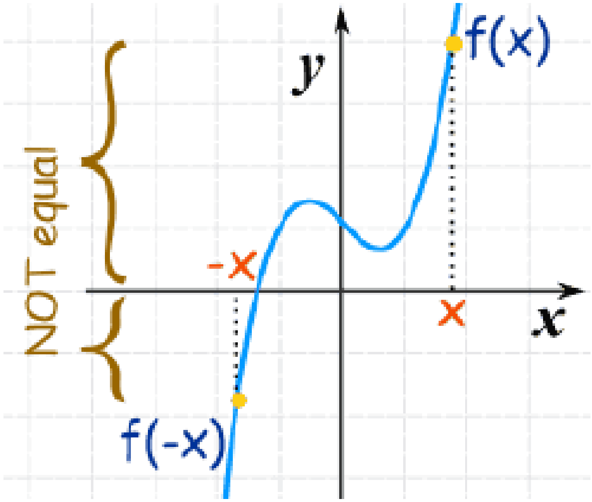


This is the curve $f(x) = x^3-x$

Neither Odd nor Even

Don't be misled by the names "odd" and "even" ... they are just **names** ... and a function does **not have to be** even or odd.

In fact most functions are neither odd nor even. For example, just adding 1 to the curve above gets this:



This is the curve $f(x) = x^3 - x + 1$

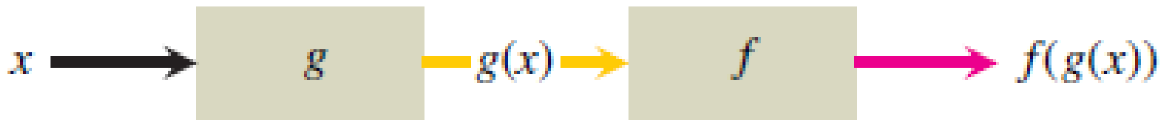
It is **not an odd function**, and it is **not an even function** either.

It is neither odd nor even!

Composite Functions

Composition is another method for combining functions.

> **DEFINITION Composition of Functions**
>
> If f and g are functions, the **composite** function $f \circ g$ ("f composed with g") is defined by
>
> $$(f \circ g)(x) = f(g(x)).$$
>
> The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f.



Notice that if $f(x) = x^2$ and $g(x) = \sqrt{x}$, then $(f \circ g)(x) = \left(\sqrt{x}\right)^2 = x$. However, the domain of $f \circ g$ is $[0, \infty)$, not $(-\infty, \infty)$.

EXAMPLE

If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$.

Solution

	Composite	Domain
(a)	$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$	$[-1, \infty)$
(b)	$(g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
(c)	$(f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$	$[0, \infty)$
(d)	$(g \circ g)(x) = g(g(x)) = g(x) + 1 = (x + 1) + 1 = x + 2$	$(-\infty, \infty)$

Thanks