

Mechatronics Engineering
First Grade
Calculus



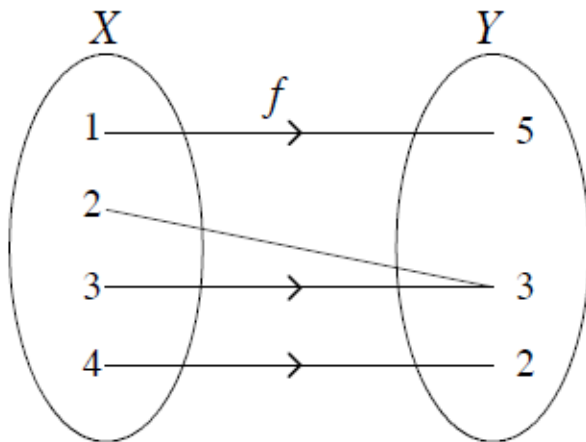
Lesson 3

Functions

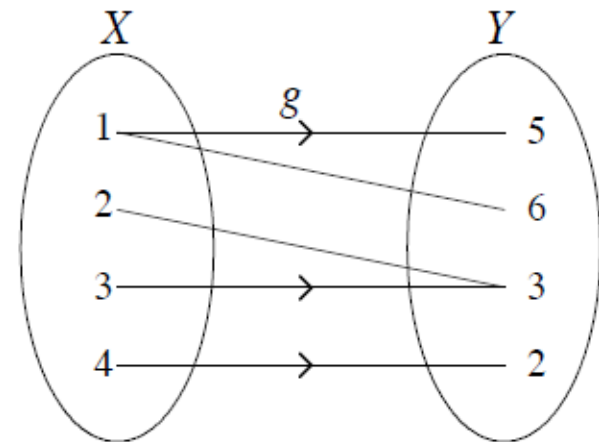
By: Rasha Alkabbanie (MSc)

Definition of a function

A function f from a set of elements X to a set of elements Y is a rule that assigns to each element x in X exactly **one** element y in Y .



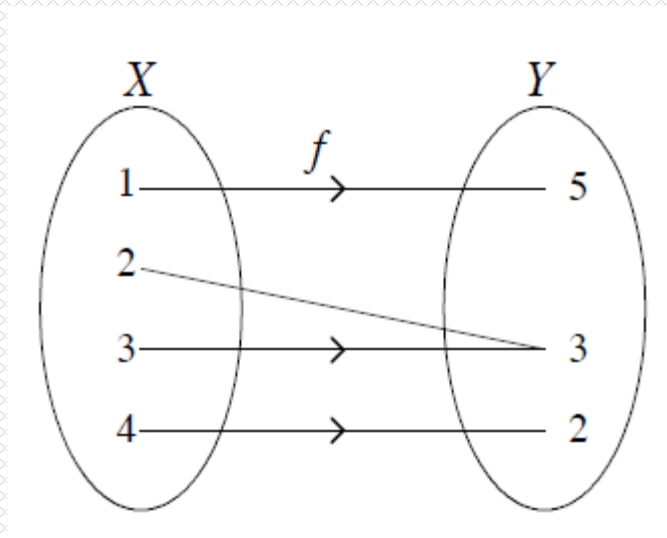
$f : X \rightarrow Y$ is a function. Every element in X has associated with it exactly one element of Y .



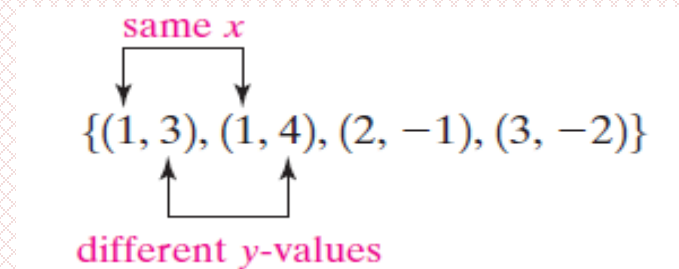
$g : X \rightarrow Y$ is not a function. The element 1 in set X is assigned two elements, 5 and 6 in set Y .

Description of a function

A function can also be described as a set of ordered pairs (x, y)

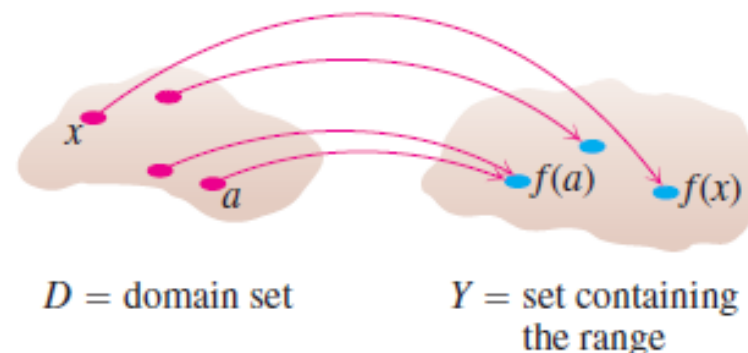


$$F = \{(1, 5), (3, 3), (2, 3), (4, 2)\}$$



we often describe a function using the rule, $y = f(x)$, and create a graph of that function by plotting the ordered pairs $(x, f(x))$ on the Cartesian Plane.

Functions; Domain and Range

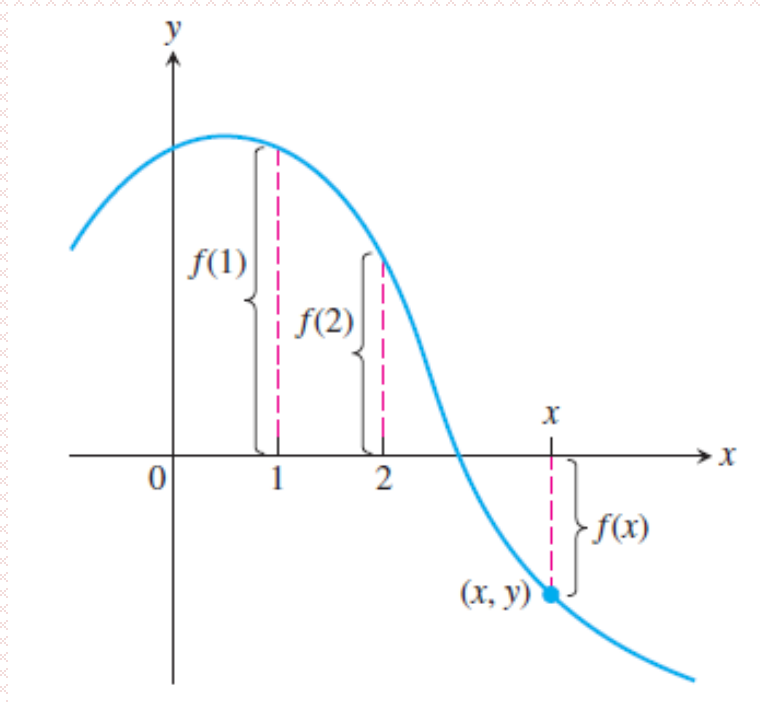


Verify the domains and ranges of these functions.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$

Graphs of Functions

Another way to visualize a function is its graph. If f is a function with domain D , its **graph** consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f



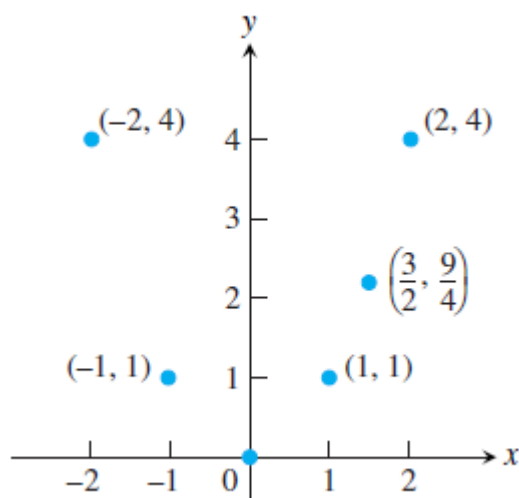
Sketching a Graph

Graph the function $y = x^2$ over the interval $[-2, 2]$.

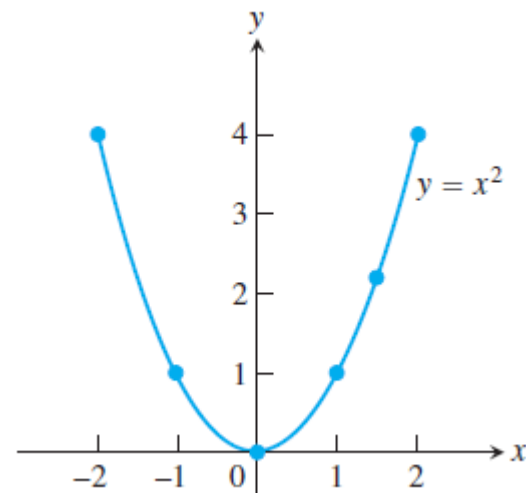
1-Make a table of xy -pairs

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

2-Plot the points (x, y)



3-Draw a smooth curve through the plotted points.



The Vertical Line Test

Not every curve you draw is the graph of a function. A function f can have only one value $f(x)$ for each x in its domain, so no *vertical line* can intersect the graph of a function more than once.

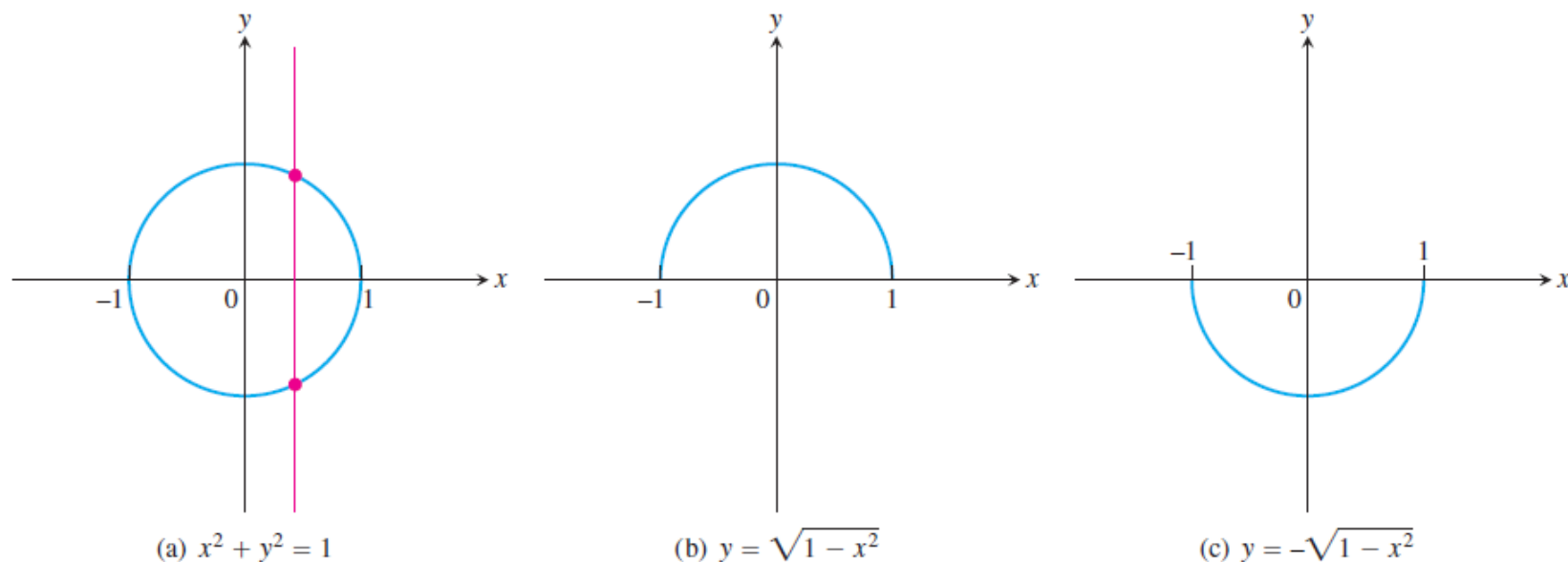
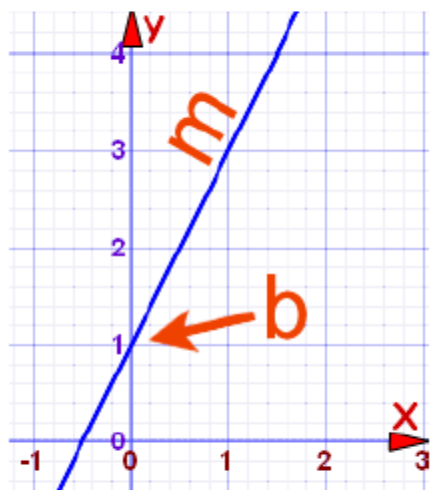


FIGURE 1.28 (a) The circle is not the graph of a function; it fails the vertical line test. (b) The upper semicircle is the graph of a function $f(x) = \sqrt{1 - x^2}$. (c) The lower semicircle is the graph of a function $g(x) = -\sqrt{1 - x^2}$.

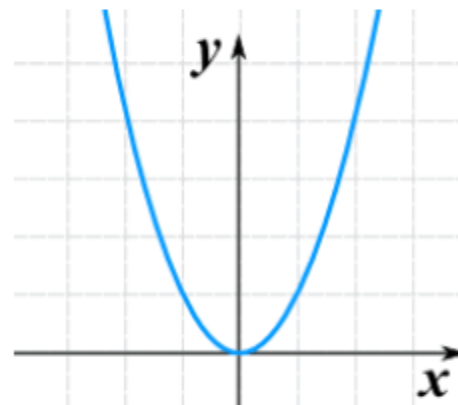
Some types of common functions

Linear Function:



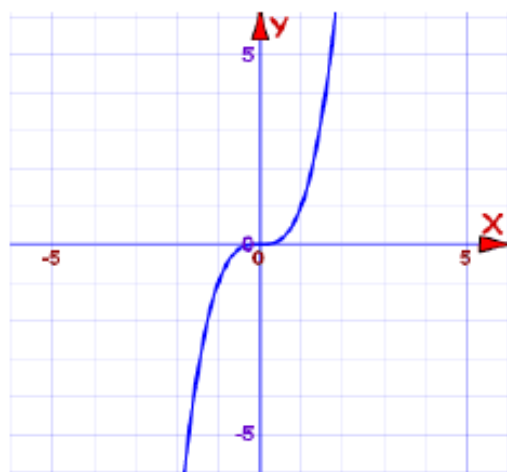
$$f(x) = mx + b$$

Square Function:



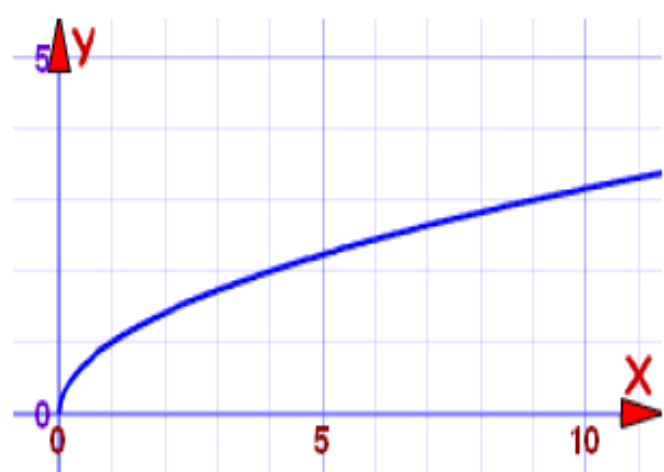
$$f(x) = x^2$$

Cube Function:



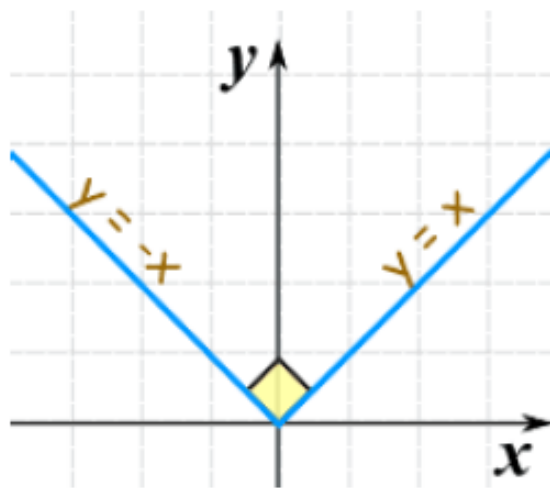
$$f(x) = x^3$$

Square Root Function:



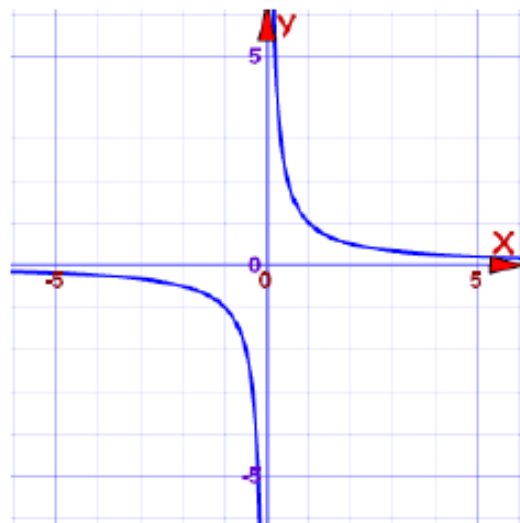
$$f(x) = \sqrt{x}$$

Absolute Value Function:



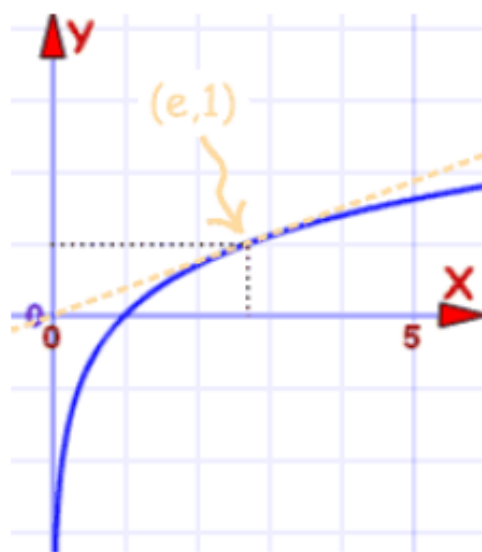
$$f(x) = |x|$$

Reciprocal Function



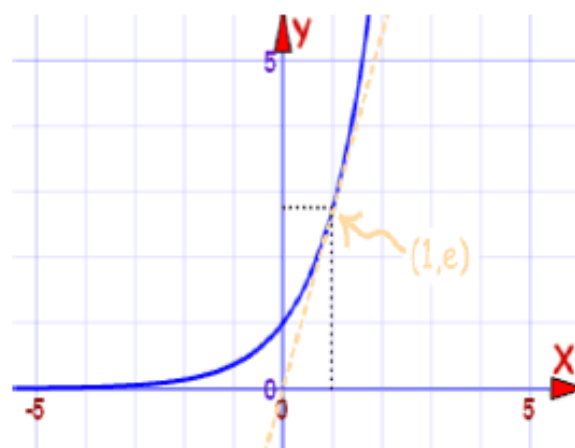
$$f(x) = 1/x$$

Logarithmic Function:



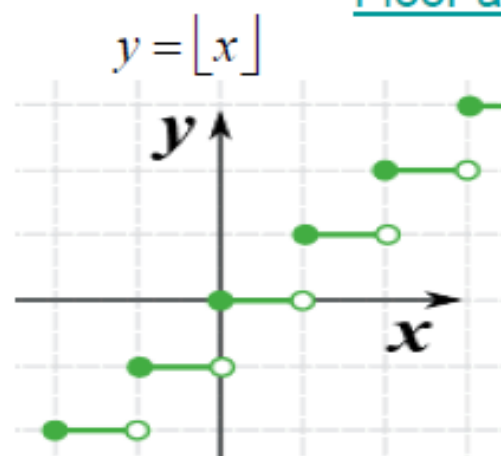
$$f(x) = \ln(x)$$

Exponential Function:

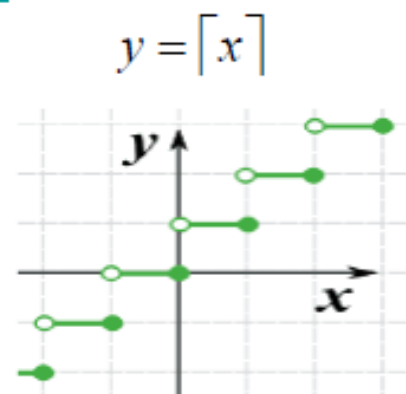


$$f(x) = e^x$$

Floor and Ceiling Function:

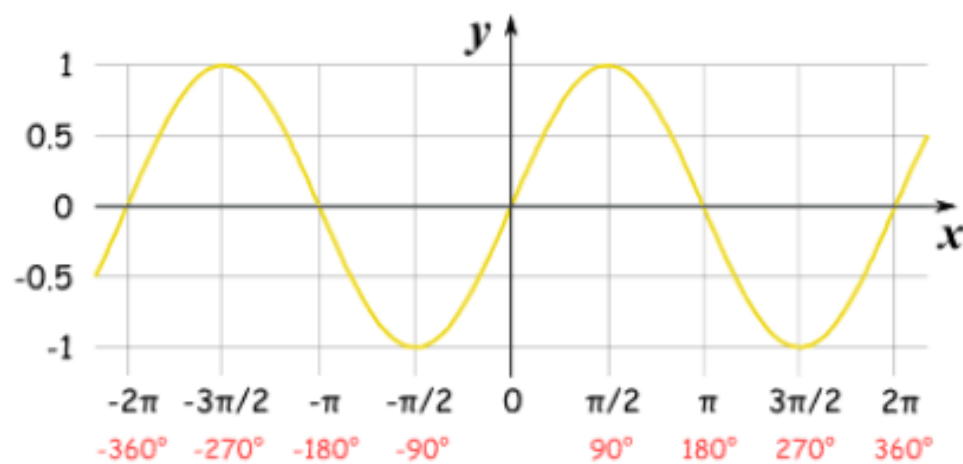


The Floor Function

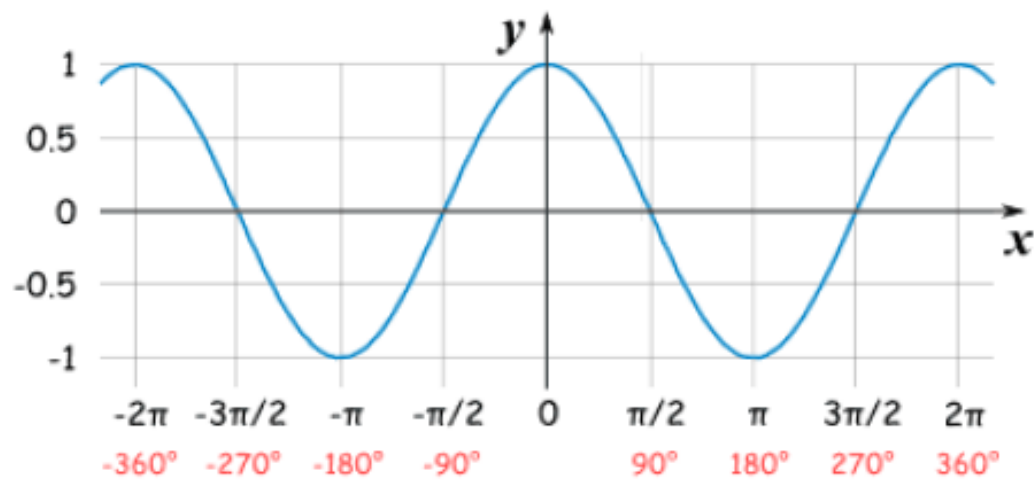


The Ceiling Function

Sine Function:

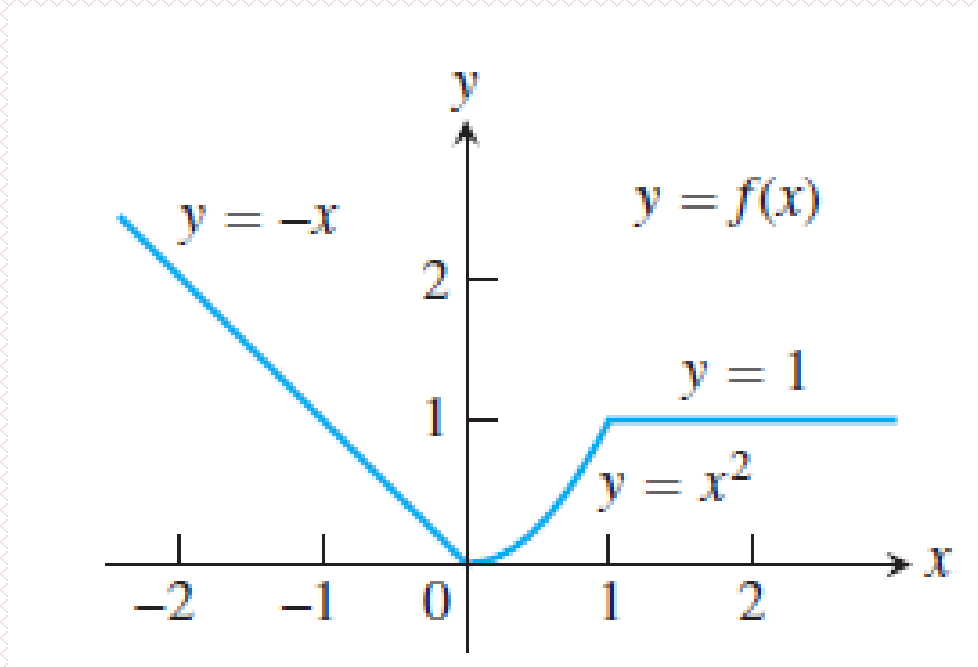


Cosine Function:

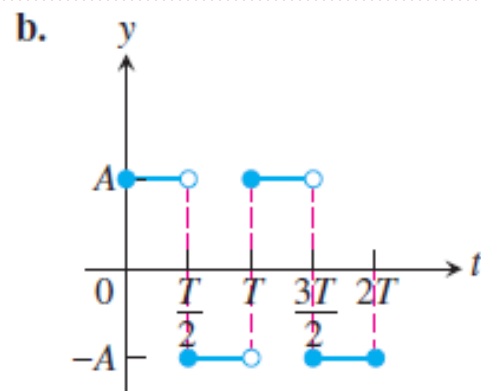
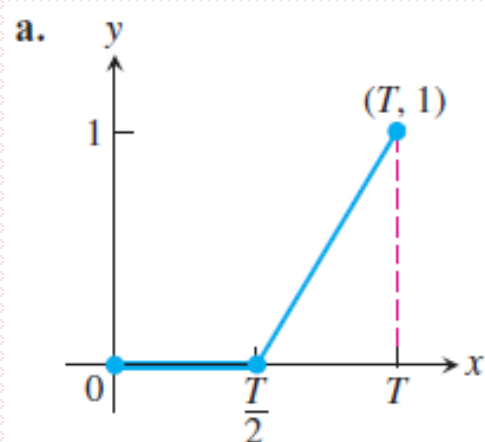
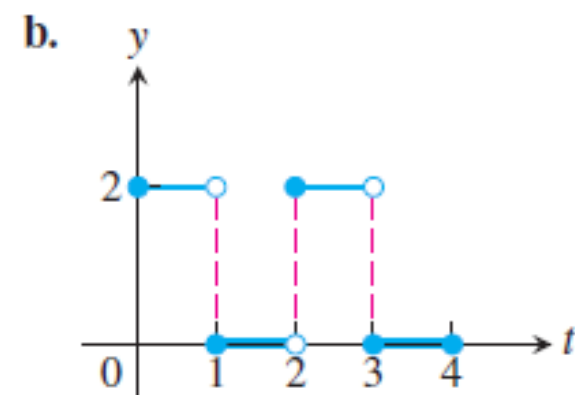
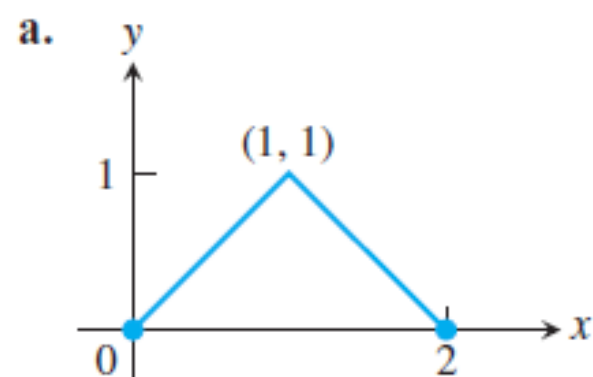
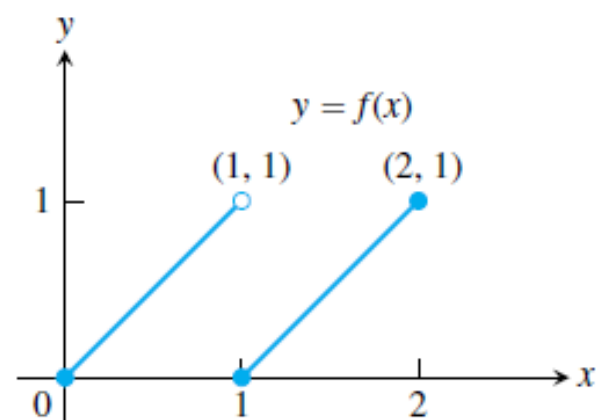


Piecewise-Defined Functions

Sometimes a function is described by using different formulas on different parts of its domain.



Find a formula for each function graphed.



Thanks