Mechatronics Engineering
First Grade
Calculus

Lesson 2

Preliminaries

By: Rasha Alkabbanie (MSc)

WE WILL LERN TODAY ABOUT :

Real numbers

Inequalities,

Absolute values

Real numbers

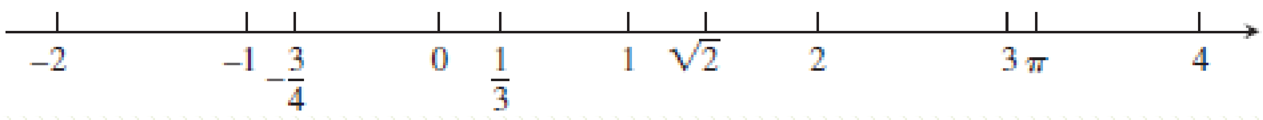
are numbers that can be expressed as decimals, such as

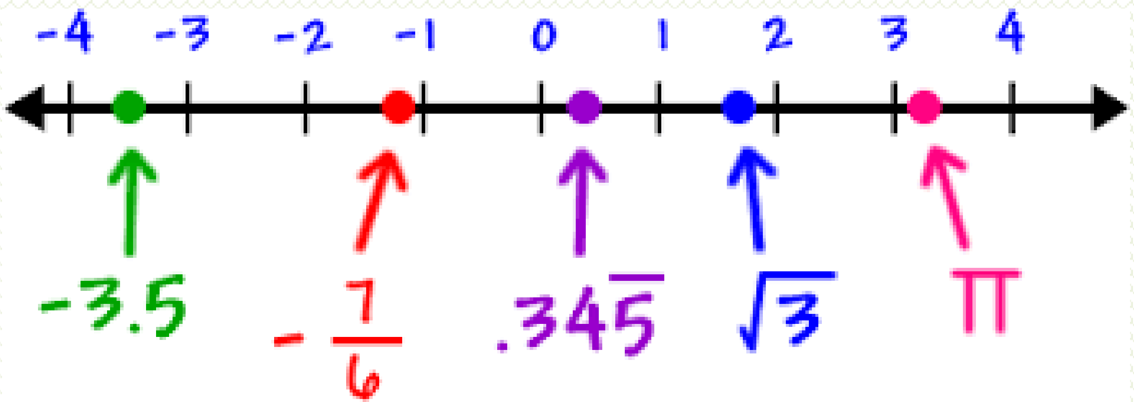
$$-\frac{3}{4} = -0.75000\ldots$$

$$\frac{1}{3} = 0.33333\ldots$$

$$\sqrt{2} = 1.4142\ldots$$

The real numbers can be represented geometrically as points on a number line called **The Real Line**.





The properties of the real number system fall into three categories

Algebraic Properties

Completeness

Order Properties

Firstly : Algebraic properties

real numbers can be added, subtracted, multiplied, and divided to make other real numbers .

You can never divide by 0

Secondly: Completeness Property

there are enough real numbers to "complete" the real number line, in the sense that there are no "holes" or "gaps" in it

We distinguish Four special subsets of real numbers.

1. **The natural numbers**, namely $1, 2, 3, 4, \ldots$

2. **The integers**, namely $0, \pm 1, \pm 2, \pm 3, \ldots$

3. The **rational numbers**, namely the numbers that can be expressed in the form of a fraction m/n, where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{3}, \quad -\frac{4}{9} = \frac{-4}{9} = \frac{4}{-9}, \quad \frac{200}{13}, \quad \text{and} \quad 57 = \frac{57}{1}.$$

They are either

(a) terminating (ending in an infinite string of zeros), for example,

$$\frac{3}{4} = 0.75000\ldots = 0.75 \qquad \text{or}$$

A terminating decimal is a decimal that looks like it goes on and on, but at some point has an end

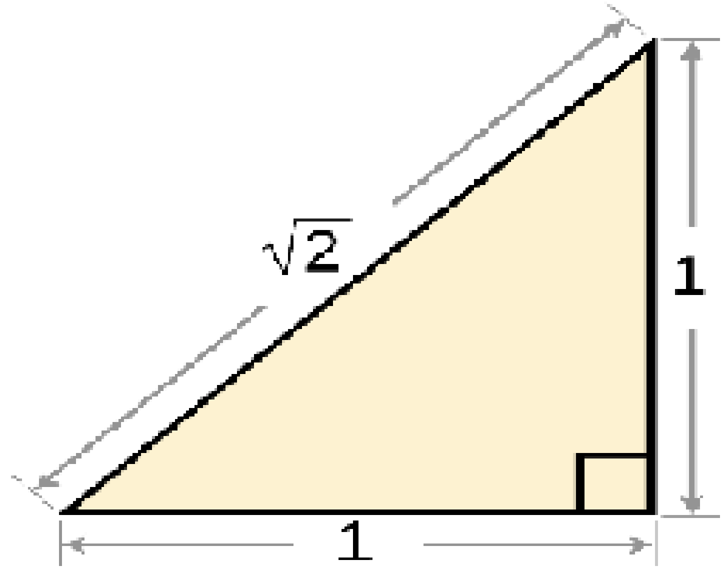
(b) eventually repeating (ending with a block of digits that repeats over and over), for example

$$\frac{23}{11} = 2.090909\ldots = 2.\overline{09}$$

The bar indicates the block of repeating digits.

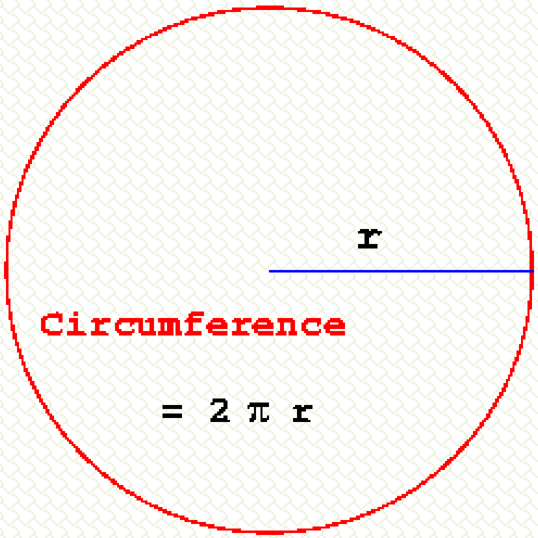
4. Irrational Numbers

This is the last type of number that is a decimal, but is NOT a rational number. It is called an *irrational number*. An irrational number is a decimal that does not end and has no repetition. It goes on and on and on. Irrational numbers cannot be represented as fractions.



Real numbers that are not rational are called **irrational numbers**. They are characterized by having nonterminating and nonrepeating decimal expansions. Examples are π, $\sqrt{2}$, $\sqrt[3]{5}$, and $\log_{10} 3$.

Lets say you want to make an enclosure around a circular park and you want to figure out the length of this enclosure (you have the radius of the park)



3.14159265358

Example

Is $\frac{23}{4}$ *rational or irrational?*

Because the number is written as a fraction, it must be **a rational number.**

Example

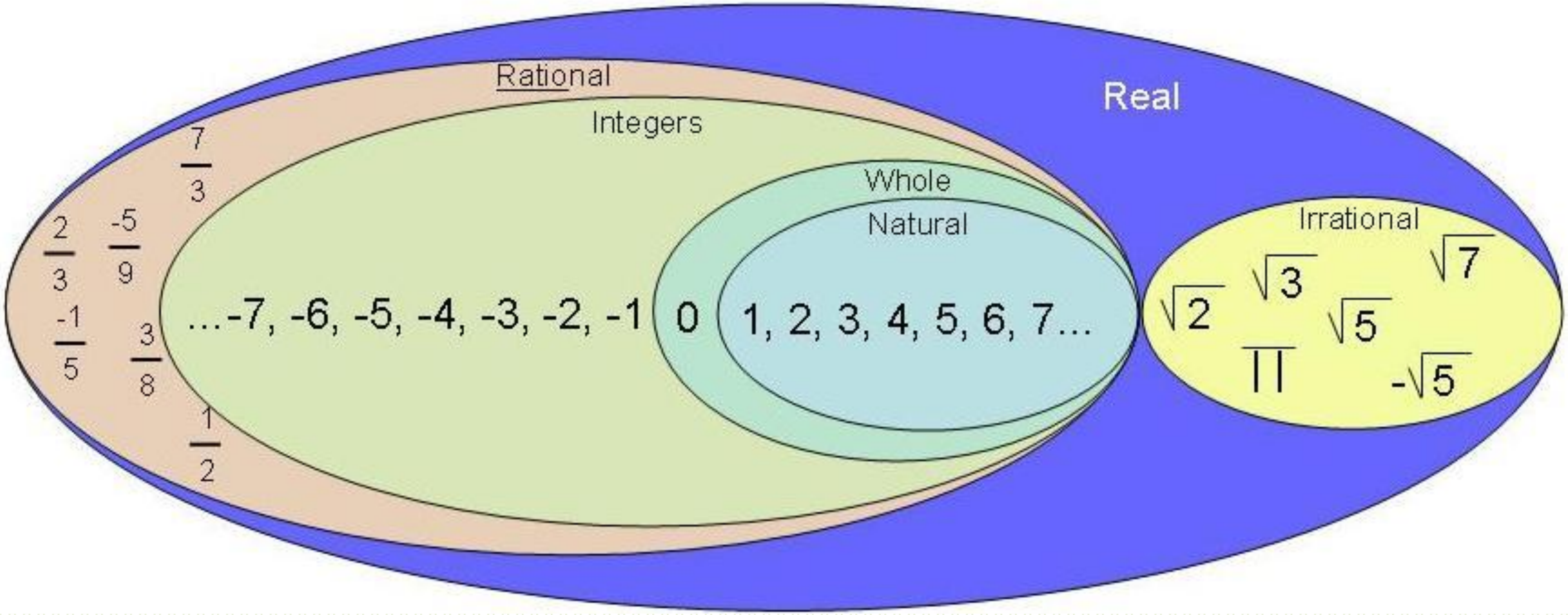
Is $\sqrt{7}$ *a rational or an irrational number?*

To figure this out, we convert this fraction to a decimal on our calculator.

$\sqrt{7} = 2.645751311...$

This decimal doesn't end or repeat. **This is an irrational number.**

Real Number System



Notes

If S and T are two different sets then:

$S \cap T$ is their intersection,

$S \cup T$ is their union

The empty set ϕ is the set that contain no element

Intervals

A subset of the real line is called an **interval** if it contains at least two numbers and contains all the real numbers lying between any two of its elements.

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \le x \le b\}$	Closed	
	$[a, b)$	$\{x \mid a \le x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \le b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \ge a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \le b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

Secondly : Order Properties

Rules for Inequalities

If a, b, and c are real numbers, then:

1. $a < b \Rightarrow a + c < b + c$
2. $a < b \Rightarrow a - c < b - c$
3. $a < b$ and $c > 0 \Rightarrow ac < bc$
4. $a < b$ and $c < 0 \Rightarrow bc < ac$
   Special case: $a < b \Rightarrow -b < -a$
5. $a > 0 \Rightarrow \frac{1}{a} > 0$
6. If a and b are both positive or both negative, then $a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$

Problem

Kevin is loading a truck with some boxes of ceramic tiles, which weigh 40kg, and some boxes of wooden floorboard, which weigh 10kg. He can fit at most 50 boxes in the truck. The maximum capacity of the truck is 1100 kg. Which system of inequalities best represents this situation?

$$\text{If } a < b, \text{ then } a + c < b + c$$

Example: Alex has less coins than Billy.

If both Alex and Billy get 3 more coins each, Alex will still have less coins than Billy.

Example: Alex's score of 3 is **lower than** Billy's score of 7.

$$a < b$$

If both Alex and Billy manage to **double** their scores ($\times 2$), Alex's score will still be lower than Billy's score.

$$2a < 2b$$

But when multiplying by a negative the opposite happens:

But if the scores become **minuses**, then Alex **loses 3** points and Billy **loses 7** points

So Alex has now done **better** than Billy!

$$-a > -b$$

Example: Alex and Billy both complete a journey of 12 kilometers.

Alex runs at **6 km/h** and Billy walks at **4 km/h**.

Alex's speed is greater than Billy's speed

$$6 > 4$$

But Alex's time is less than Billy's time:

$$12/6 < 12/4$$

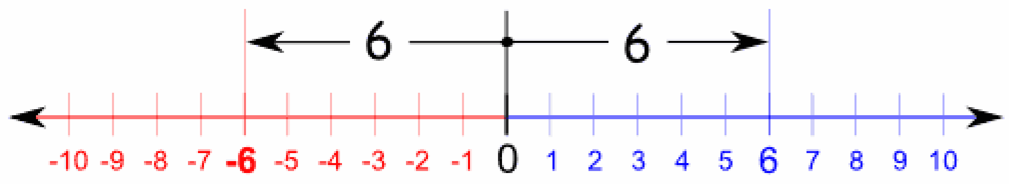
$$2 \text{ hours} < 3 \text{ hours}$$

Absolute Value

The **absolute value** of a number x, denoted by $|x|$, is defined by the formula

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Geometrically, the absolute value of x is the distance from x to 0 on the real number line. Since distances are always positive or 0, we see that $|x| \geq 0$ for every real number x, and $|x| = 0$ if and only if $x = 0$. Also,



$$|x - y| = \text{the distance between } x \text{ and } y$$

It is important to remember that $\sqrt{a^2} = |a|$. Do not write $\sqrt{a^2} = a$ unless you already know that $a \geq 0$.

Absolute Value Properties

1.	$\lvert -a \rvert = \lvert a \rvert$	A number and its additive inverse or negative have the same absolute value.
2.	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	The absolute value of a product is the product of the absolute values.
3.	$\left\lvert \dfrac{a}{b} \right\rvert = \dfrac{\lvert a \rvert}{\lvert b \rvert}$	The absolute value of a quotient is the quotient of the absolute values.
4.	$\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values.

Absolute Values and Intervals

If a is any positive number, then

5. $|x| = a$ if and only if $x = \pm a$
6. $|x| < a$ if and only if $-a < x < a$
7. $|x| > a$ if and only if $x > a$ or $x < -a$
8. $|x| \leq a$ if and only if $-a \leq x \leq a$
9. $|x| \geq a$ if and only if $x \geq a$ or $x \leq -a$

Problem

Manufacturing The ideal width of a certain conveyor belt for a manufacturing plant is 50 in. An actual conveyor belt can vary from the ideal by at most $\frac{7}{32}$ in. Find the acceptable widths for this conveyor belt.

Thank you