Mechatronics Engineering
First Grade
Calculus

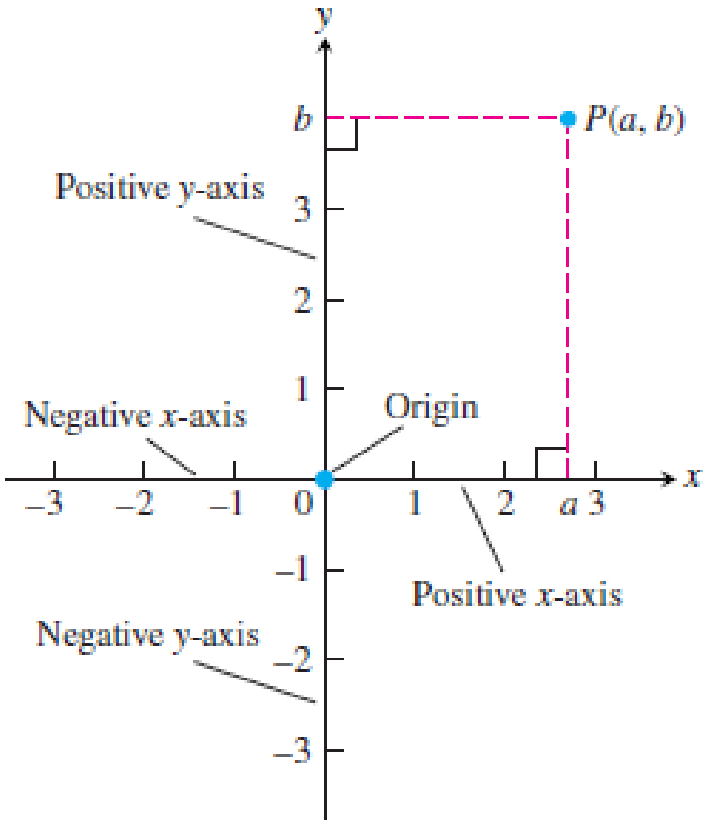
Lesson 1

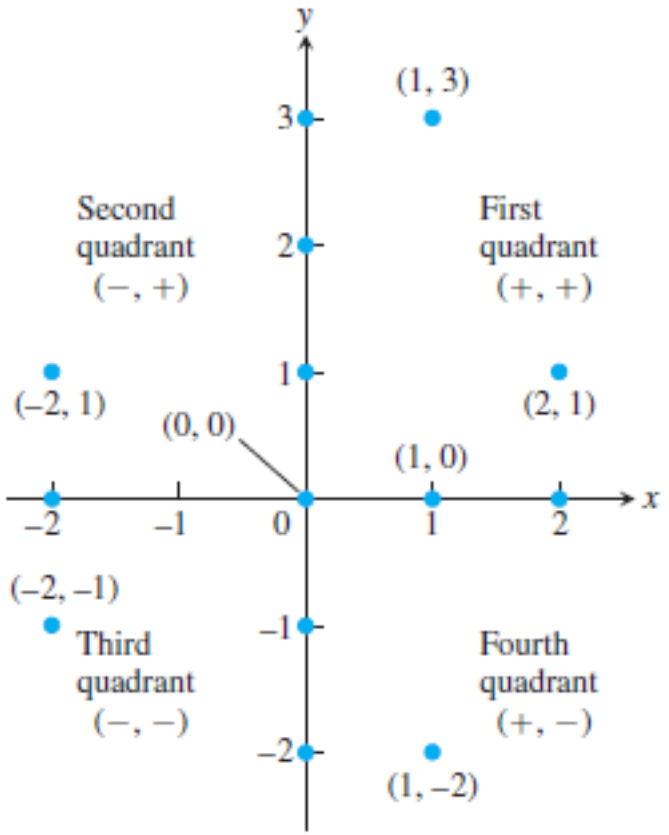
Lines & Circles

By: Rasha Alkabbanie (MSc)

Cartesian Coordinates in the Plane

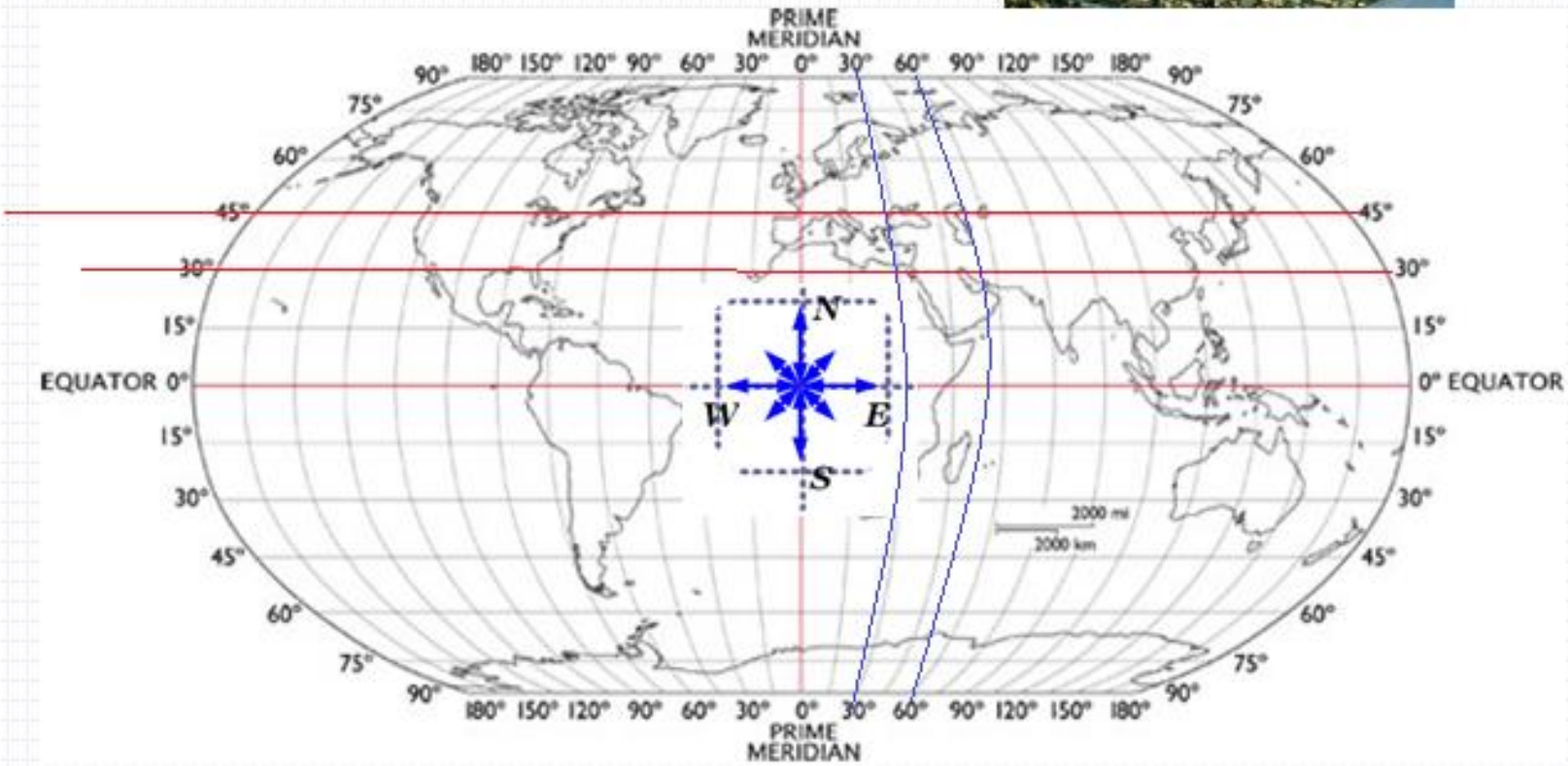
rectangular coordinate system





Coordinates of Erbil

36 N(y) , 44 E(x)

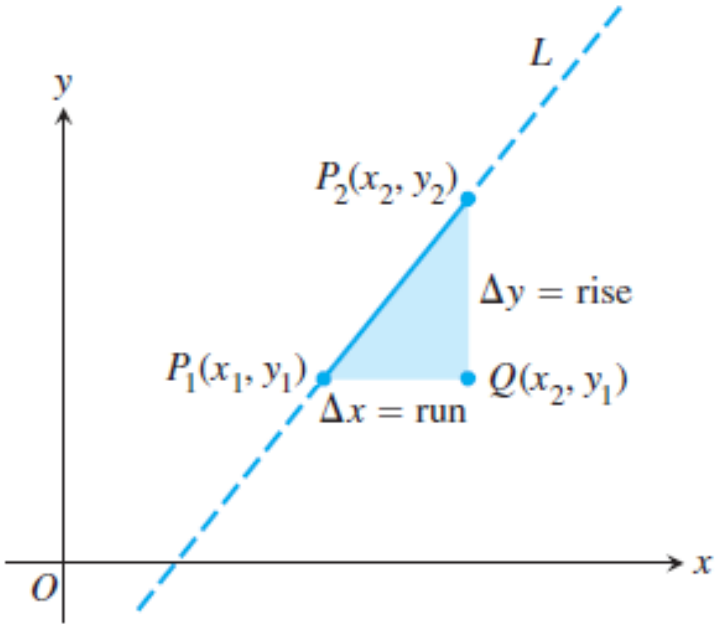


DEFINITION **Slope**

The constant

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

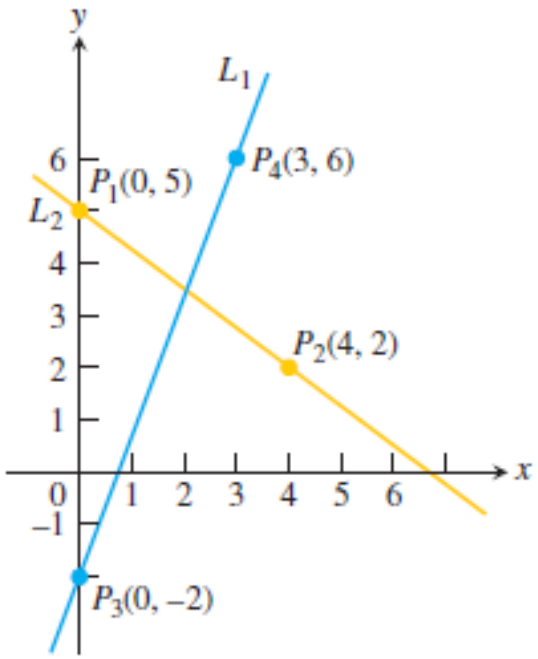
is the **slope** of the nonvertical line P_1P_2.



The slope tells us the direction (uphill, downhill)
and steepness of a line.

A line with positive slope rises uphill to the right;

one with negative slope falls downhill to the right

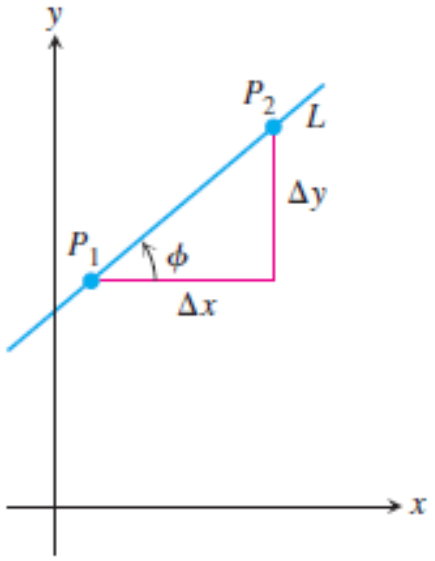


The slope of L_1 is

$$m = \frac{\Delta y}{\Delta x} = \frac{6 - (-2)}{3 - 0} = \frac{8}{3}.$$

That is, y increases 8 units every time x increases 3 units. The slope of L_2 is

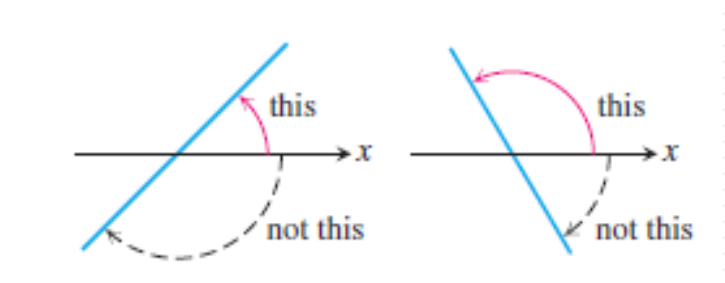
$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{4 - 0} = \frac{-3}{4}.$$



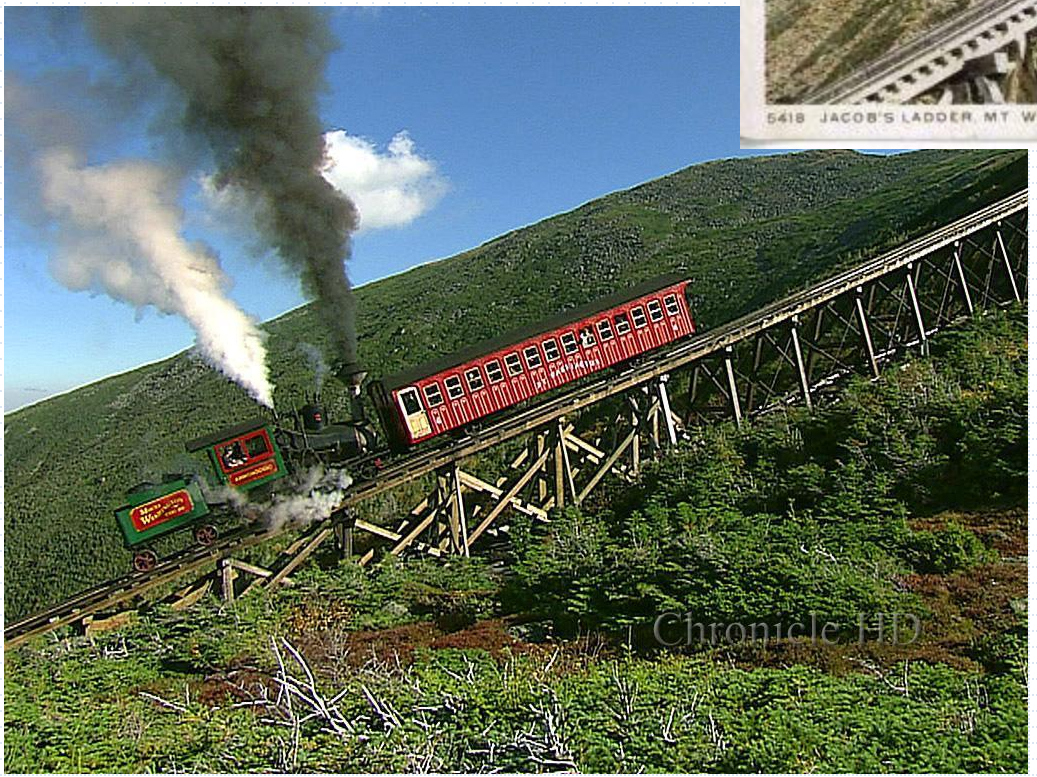
$$m = \tan\phi$$

The **angle of inclination** of a line that crosses the *x*-axis is the smallest counterclockwise angle from the *x*-axis to the line

$$0 \leq \phi < 180^\circ$$



Washington Cog Railway



The sloping part of the railway is approximately (480 km) long

(290 m) the height of the mountain

Slope??

Finding the Formula of a Line

Point-Slope Equation

We can write an equation for a nonvertical straight line L if we know its slope m and the coordinates of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is *any* other point on L, then we can use the two points P_1 and P to compute the slope,

$$m = \frac{y - y_1}{x - x_1}$$

so that

$$y - y_1 = m(x - x_1) \qquad \text{or} \qquad y = y_1 + m(x - x_1).$$

The equation

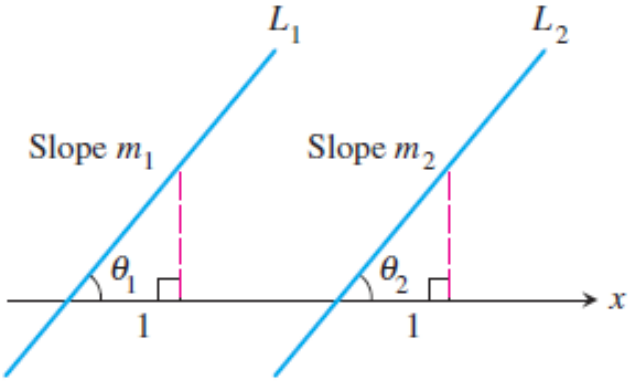
$$y = y_1 + m(x - x_1)$$

is the **point-slope equation** of the line that passes through the point (x_1, y_1) and has slope m.

EXAMPLE Write an equation for the line through the point $(2, 3)$ with slope $-3/2$.

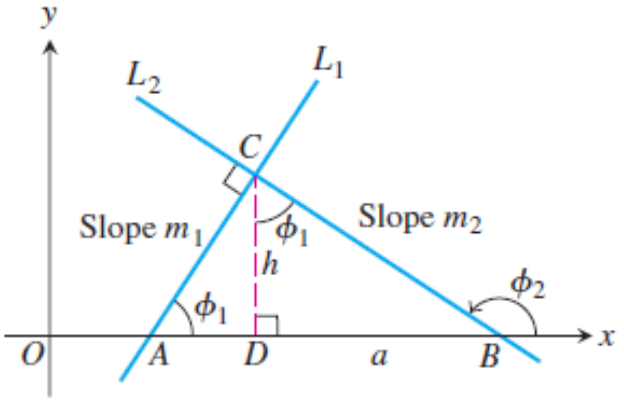
Write an equation for the line through $(-2, -1)$ and $(3, 4)$.

Parallel Lines



$$m_1 = m_2$$

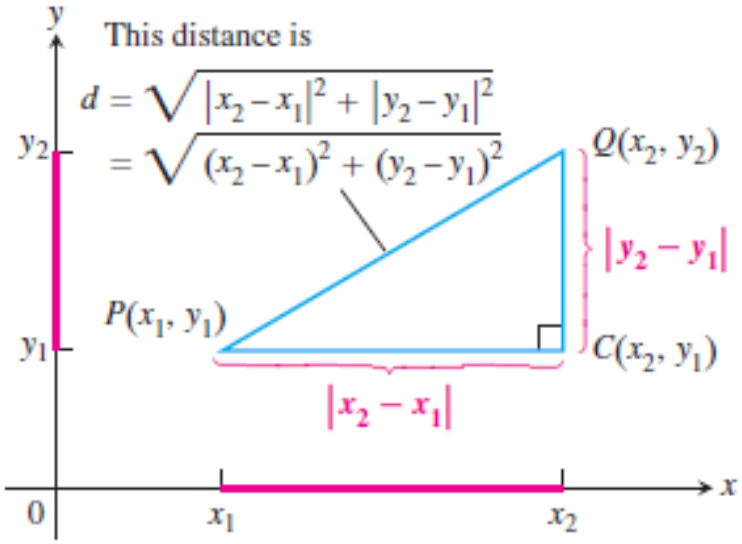
Perpendicular Lines



$$m_1 m_2 = -1$$

Do exercises on your notebook

Distance between two points

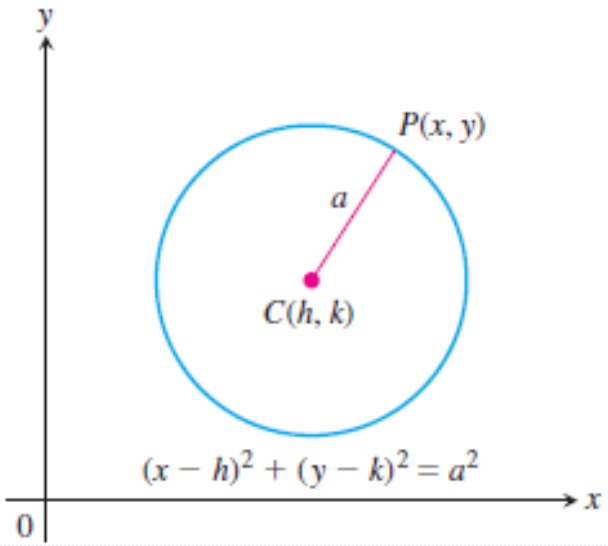


The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example : (a) The distance between $P(-1, 2)$ and $Q(3, 4)$ is

The equation of the circle :



the **standard equation** of a circle with centre (h, k) and radius a

$$(x - h)^2 + (y - k)^2 = a^2. \qquad (1)$$

EXAMPLE 7 **Find the centre and radius of the circle**

$$x^2 + y^2 + 4x - 6y - 3 = 0.$$

write an equation for each line described.

18. Passes through $(2, -3)$ with slope $1/2$

19. Passes through $(3, 4)$ and $(-2, 5)$

20. Passes through $(-8, 0)$ and $(-1, 3)$

21. Has slope $-5/4$ and y-intercept 6

22. Has slope $1/2$ and y-intercept -3

23. Passes through $(-12, -9)$ and has slope 0

24. Passes through $(1/3, 4)$, and has no slope

25. Has y-intercept 4 and x-intercept -1

26. Has y-intercept -6 and x-intercept 2

27. Passes through $(5, -1)$ and is parallel to the line $2x + 5y = 15$

28. Passes through $\left(-\sqrt{2}, 2\right)$ parallel to the line $\sqrt{2}x + 5y = \sqrt{3}$

29. Passes through $(4, 10)$ and is perpendicular to the line $6x - 3y = 5$

30. Passes through $(0, 1)$ and is perpendicular to the line $8x - 13y = 13$

Thanks