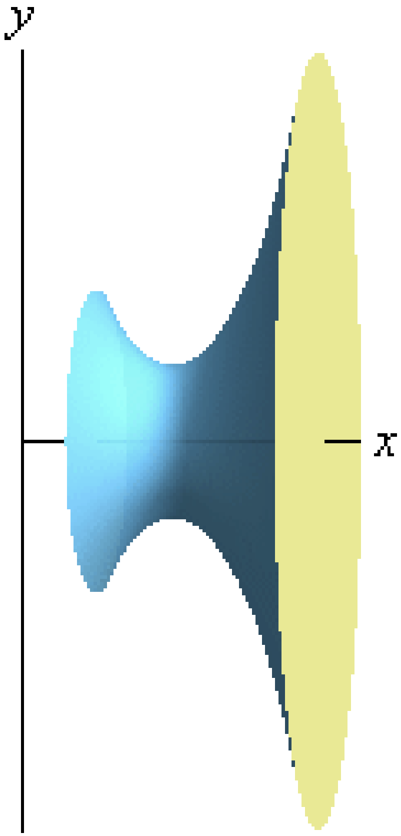
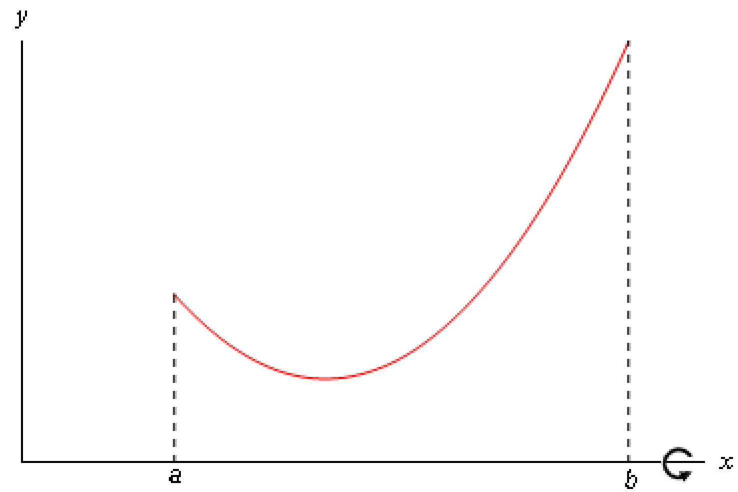
Volumes of Solids of Revolution / Method of Rings


y
x


y
a
b
x

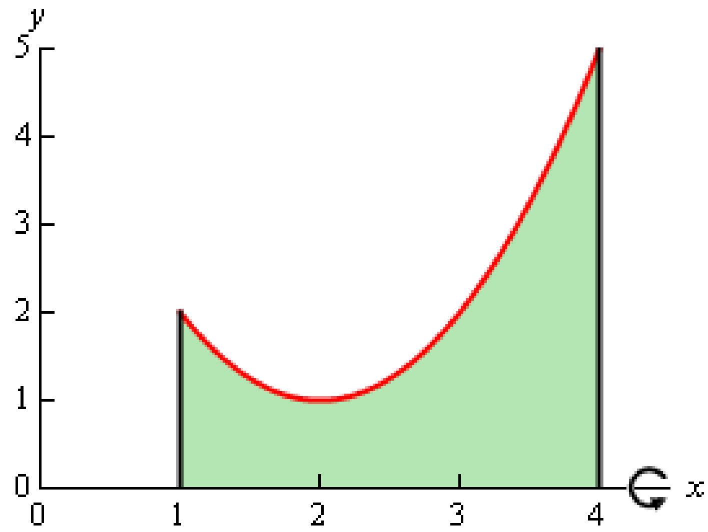
$$V = \int_a^b A(x)\,dx \qquad\qquad V = \int_c^d A(y)\,dy$$

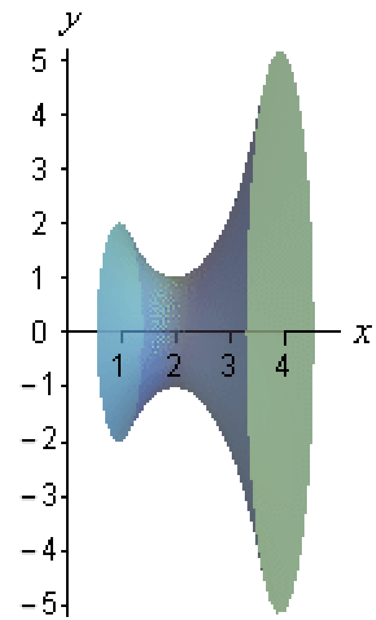
In the case that we get a ring the area is,

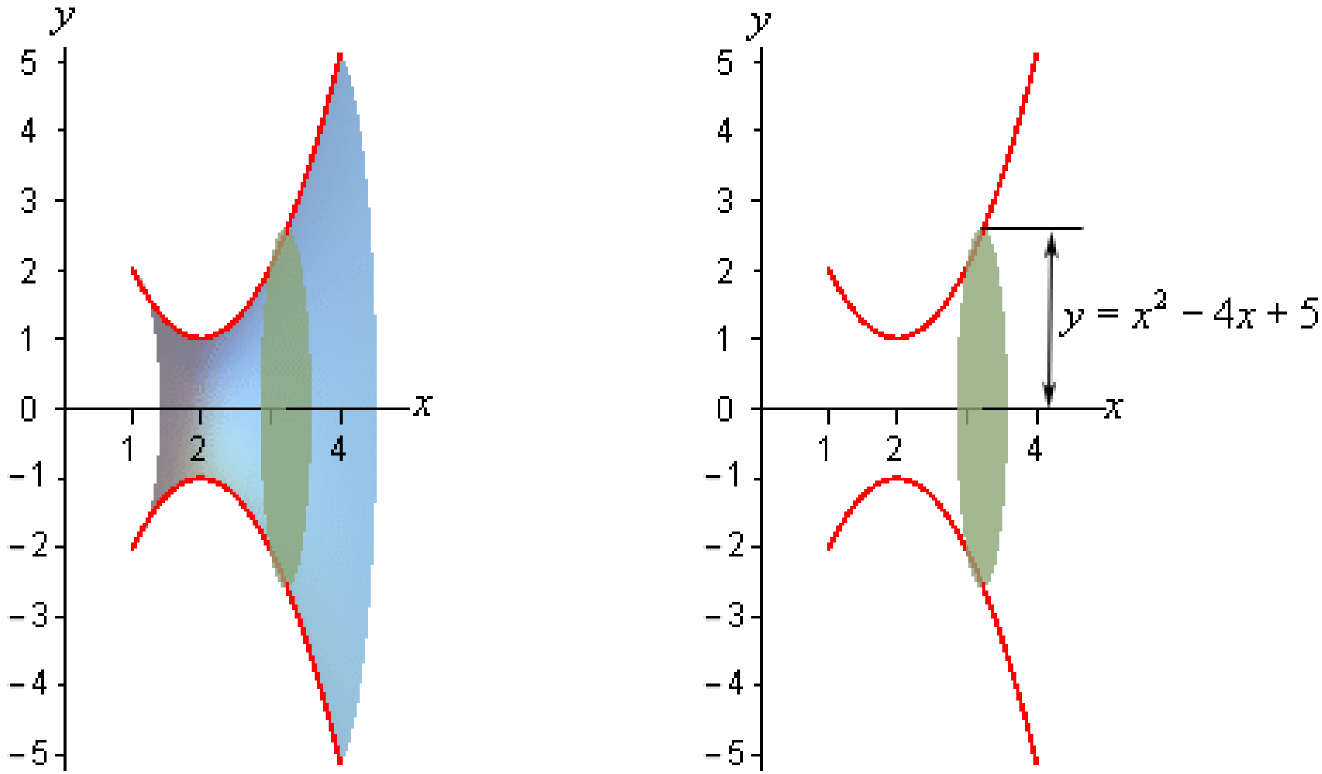
$$A = \pi\left(\left(\begin{array}{c}\text{outer}\\ \text{radius}\end{array}\right)^2 - \left(\begin{array}{c}\text{inner}\\ \text{radius}\end{array}\right)^2\right)$$

This method is often called the **method of disks** or the **method of rings**.

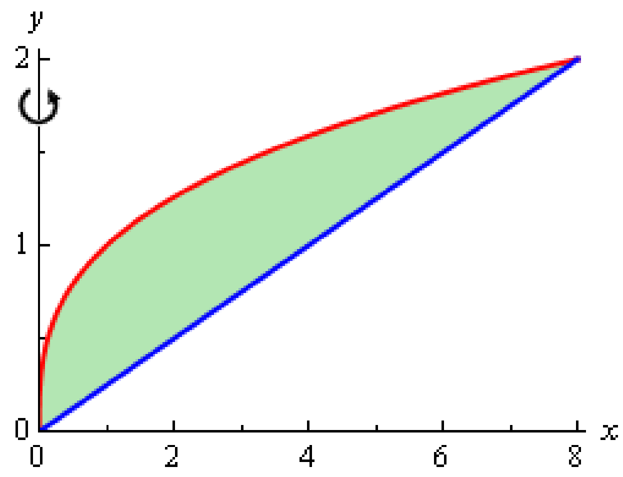
Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.

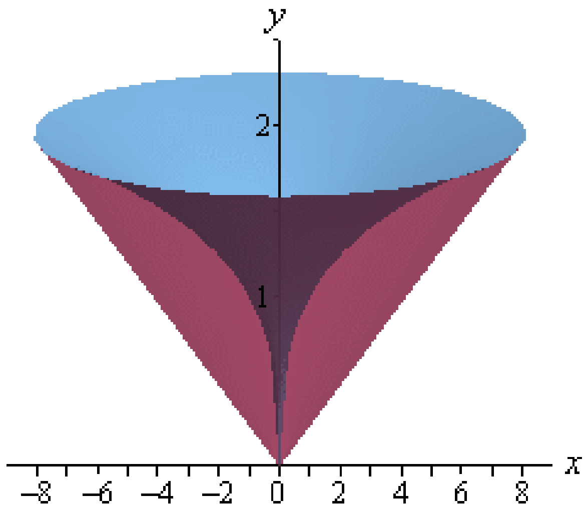


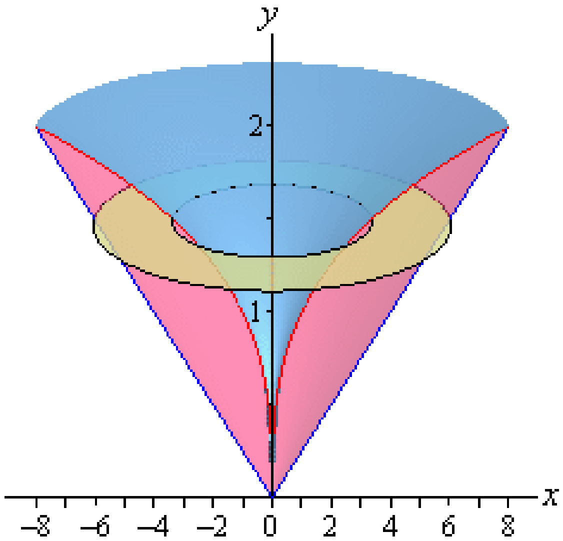



y
5
4
3
2
1
0
−1
−2
−3
−4
−5
1
2
4
x
y
5
4
3
2
1
0
−1
−2
−3
−4
−5
1
2
4
x
$y = x^2 - 4x + 5$

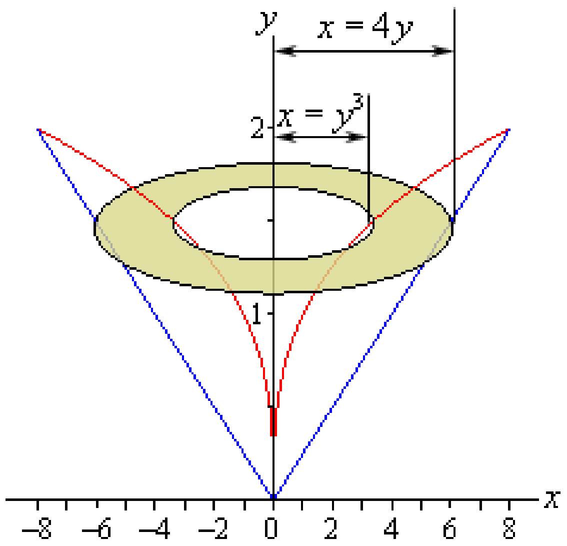
Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \dfrac{x}{4}$ that lies in the first quadrant about the y-axis.





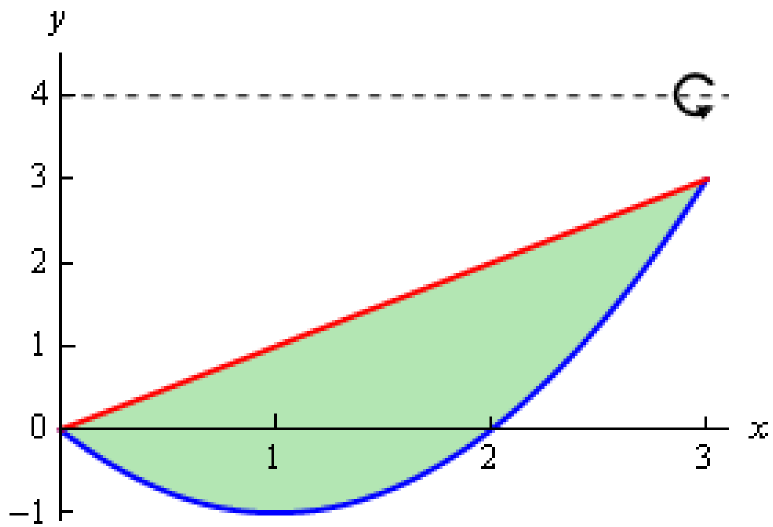


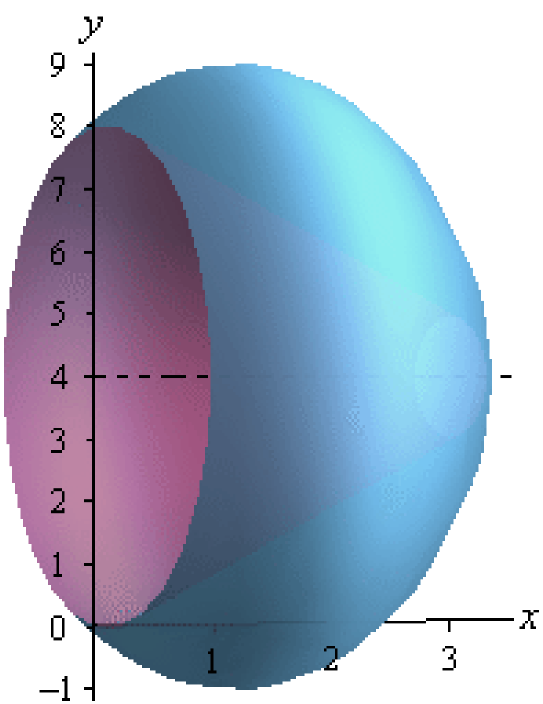
y
2
1
x
−8
−6
−4
−2
0
2
4
6
8

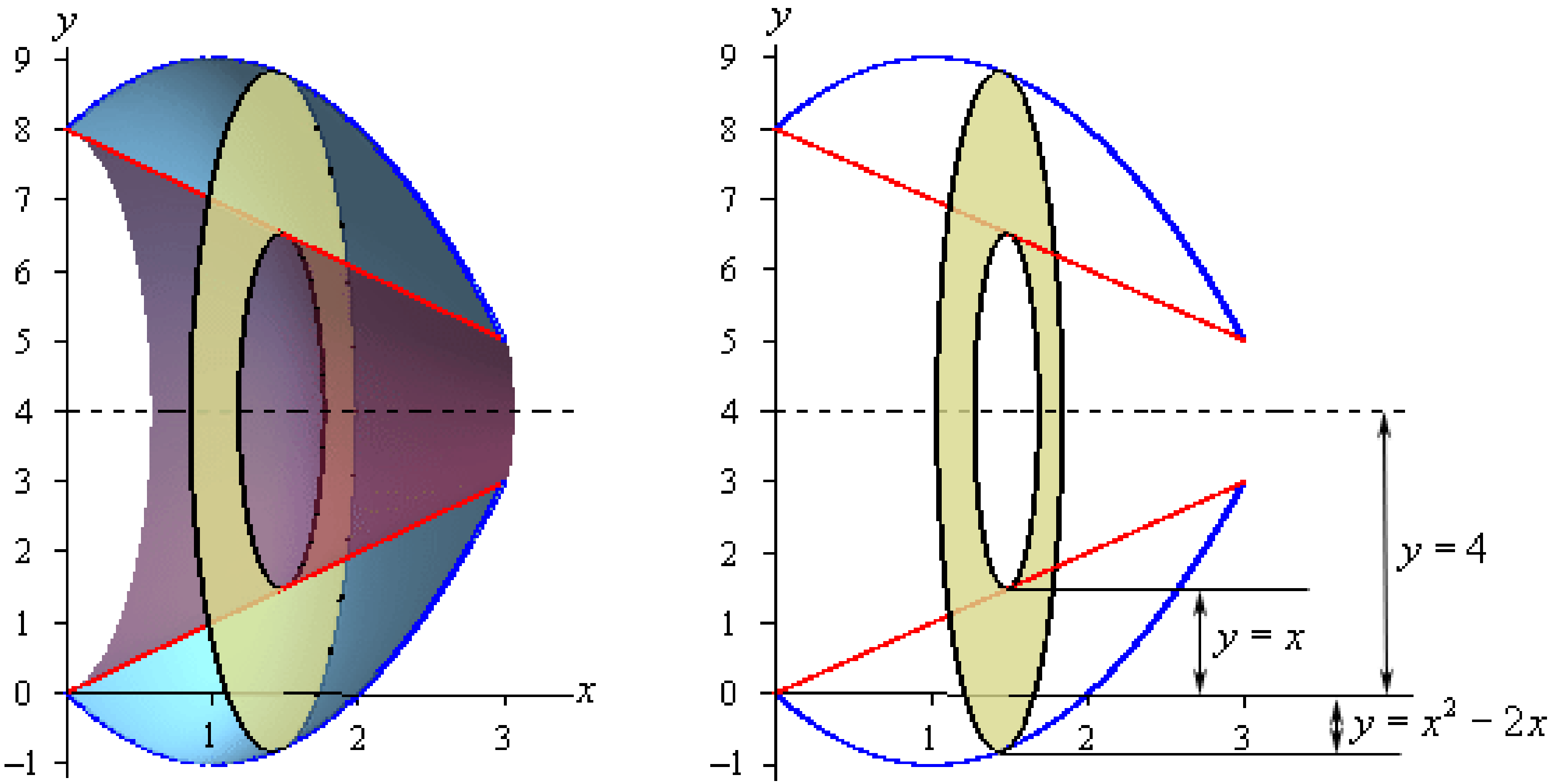


y
x = 4y
x = y³
2
1
x
−8
−6
−4
−2
0
2
4
6
8

Example 3 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$.

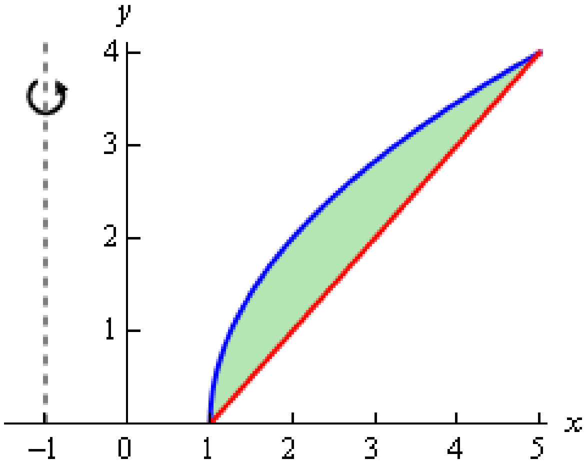


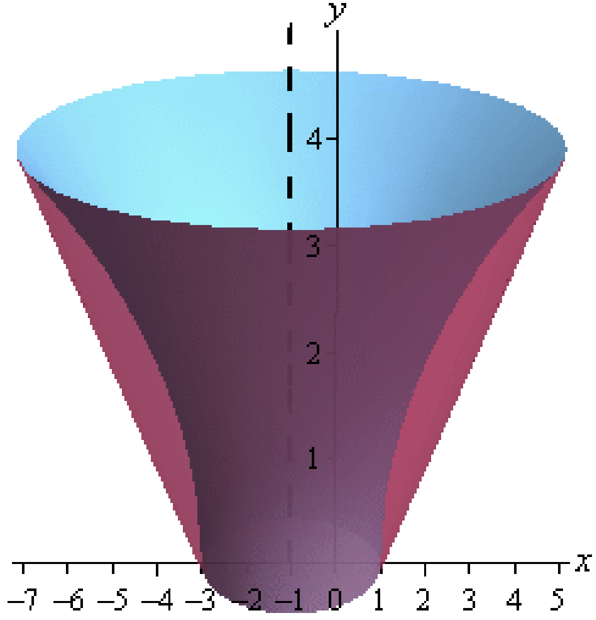


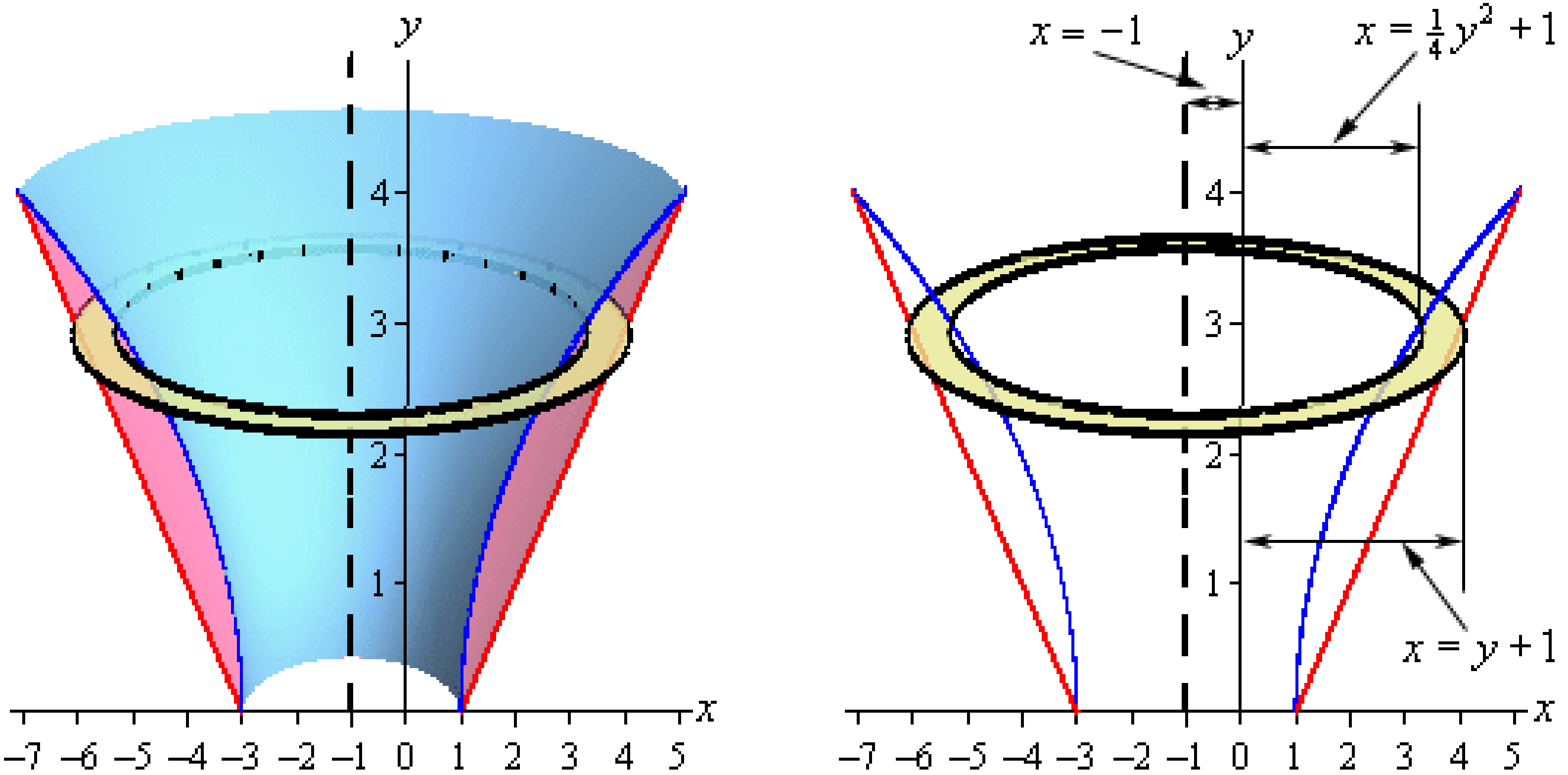


y
x
9
8
7
6
5
4
3
2
1
0
−1
1
2
3
y = 4
y = x
y = x² − 2x

Example 4 Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$.







y
x
−7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5
1 2 3 4
$x = -1$
$x = \frac{1}{4}y^2 + 1$
$x = y + 1$

Thanks