

Area Using Integration

Rules of Integration:

$$1. \int 1 dx = x + C$$

$$2. \int a dx = ax + C$$

$$3. \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$4. \int \sin x dx = -\cos x + C$$

$$5. \int \cos x dx = \sin x + C$$

$$6. \int \sec^2 x dx = \tan x + C$$

$$7. \int \csc^2 x dx = -\cot x + C$$

$$8. \int \sec x(\tan x) dx = \sec x + C$$

$$9. \int \csc x(\cot x) dx = -\csc x + C$$

$$10. \int \frac{1}{x} dx = \ln |x| + C$$

$$11. \int e^x dx = e^x + C$$

$$12. \int a^x dx = \frac{a^x}{\ln a} + C, a > 0, a \neq 1$$

$$13. \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$14. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

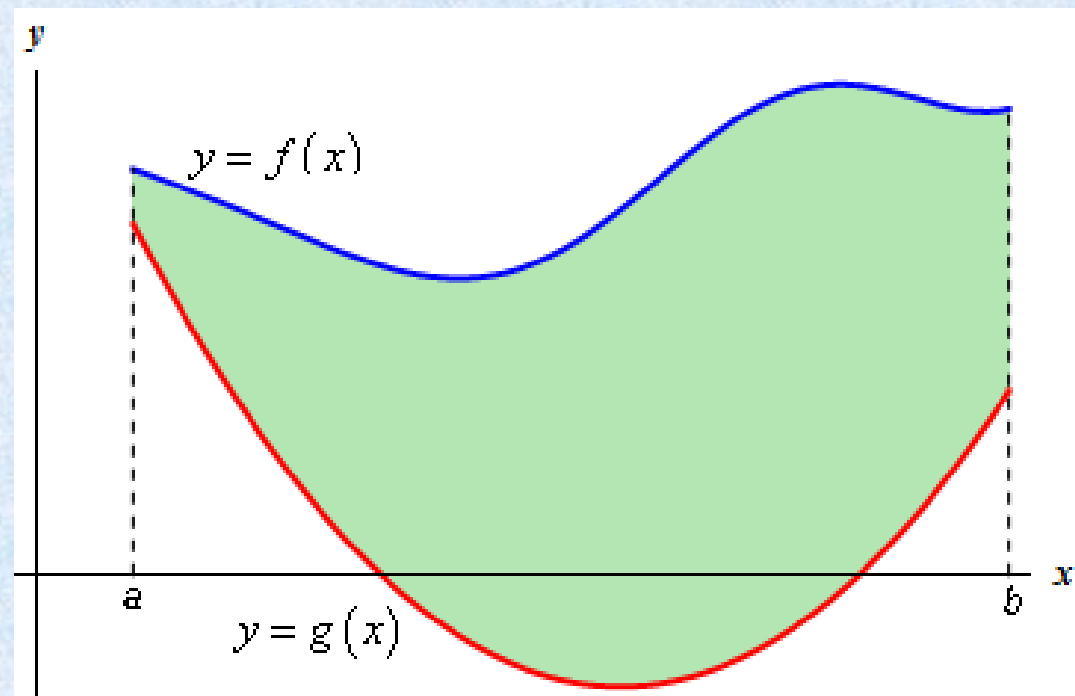
DEFINITION **Area Under a Curve as a Definite Integral**

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the area under the curve $y = f(x)$ over $[a, b]$ is the integral of f from a to b ,

$$A = \int_a^b f(x) \, dx.$$

Area Between Curves

In the first case we want to determine the area between $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$. We are also going to assume that $f(x) \geq g(x)$. Take a look at the following sketch to get an idea of what we're initially going to look at.

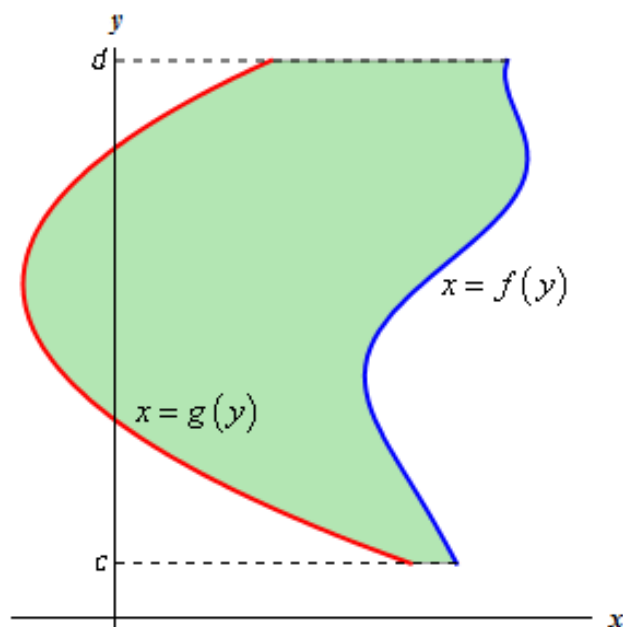


$$A = \int_a^b f(x) - g(x) dx$$

In the first case we will use,

$$A = \int_a^b \left(\begin{array}{c} \text{upper} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{lower} \\ \text{function} \end{array} \right) dx, \quad a \leq x \leq b \quad (3)$$

The second case is almost identical to the first case. Here we are going to determine the area between $x = f(y)$ and $x = g(y)$ on the interval $[c, d]$ with $f(y) \geq g(y)$.

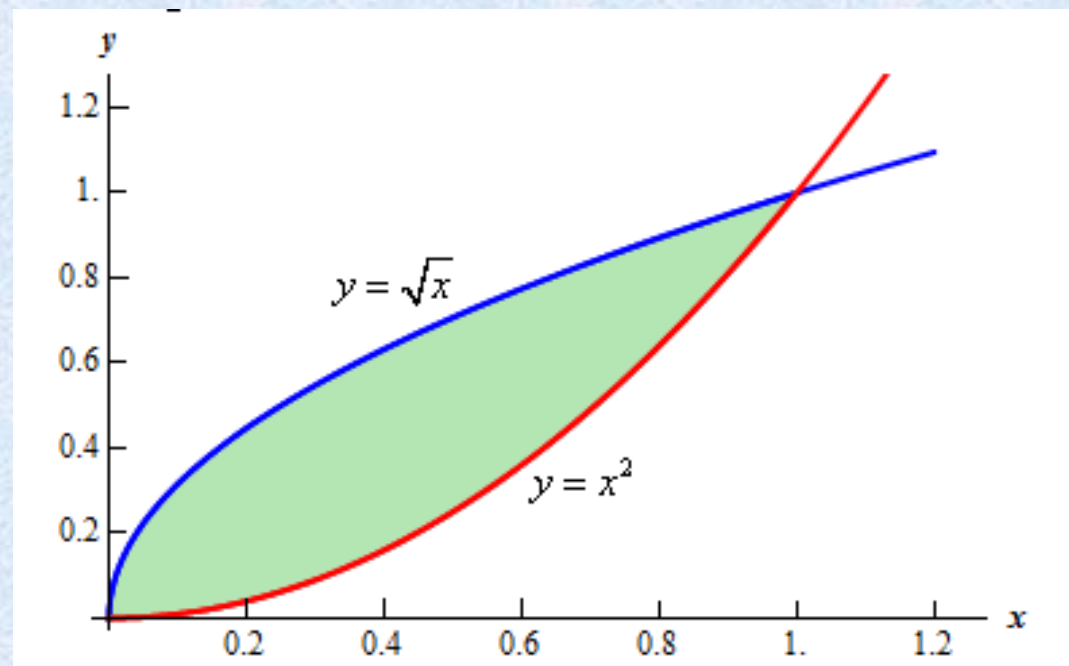


$$A = \int_c^d f(y) - g(y) dy$$

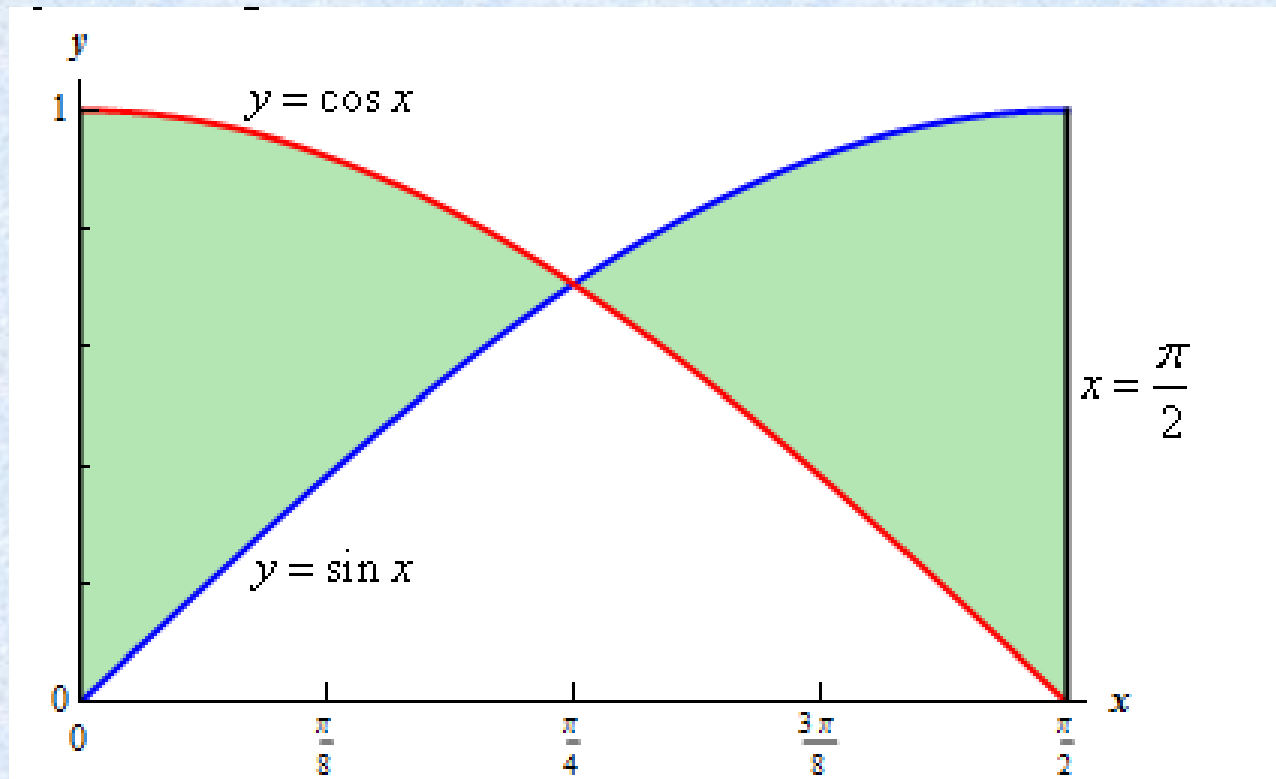
In the second case we will use,

$$A = \int_c^d \left(\begin{array}{c} \text{right} \\ \text{function} \end{array} \right) - \left(\begin{array}{c} \text{left} \\ \text{function} \end{array} \right) dy, \quad c \leq y \leq d \quad (4)$$

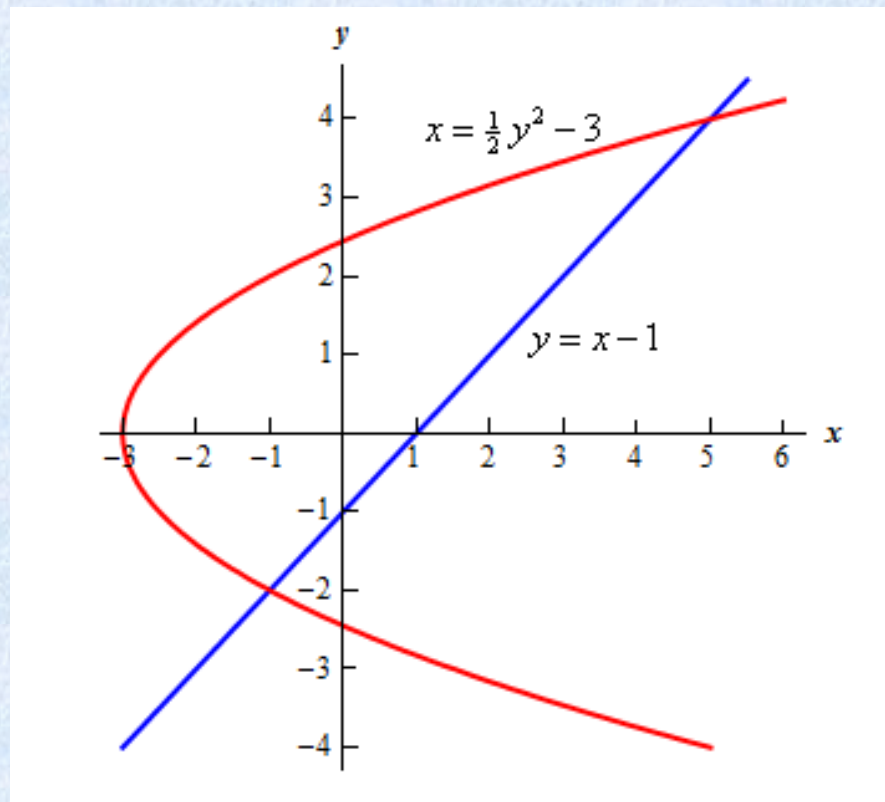
Example 1 Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$.

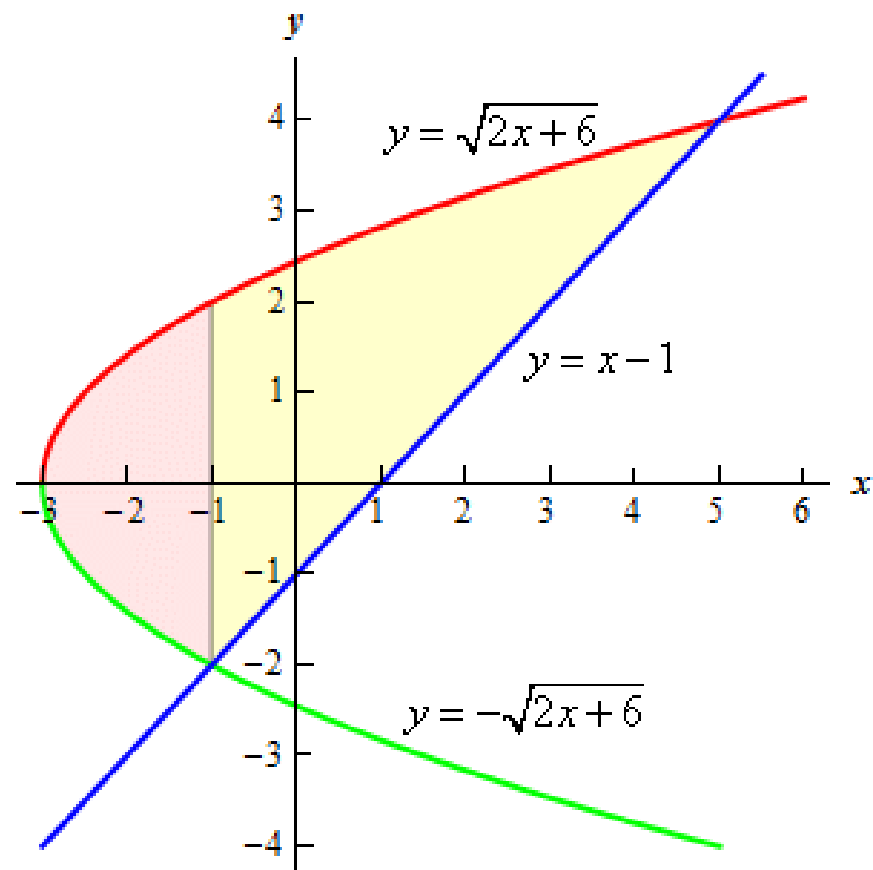


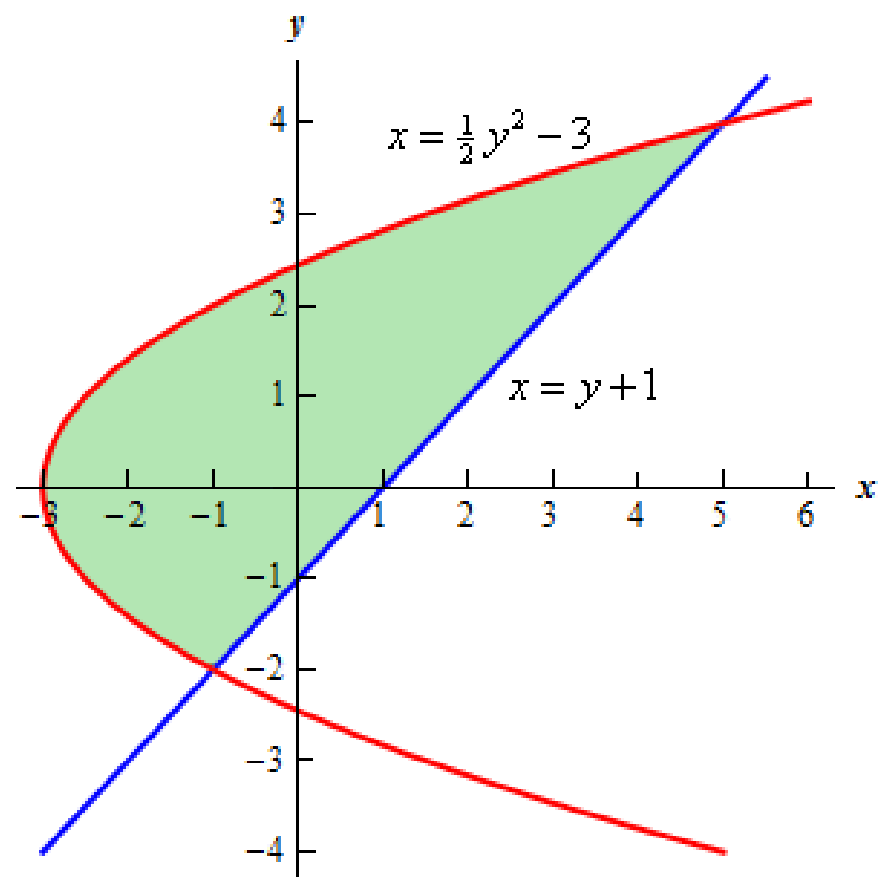
Example 5 Determine the area of the region enclosed by $y = \sin x$, $y = \cos x$, $x = \frac{\pi}{2}$, and the y -axis.



Example 6 Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.







Thanks