# Chapter 10
# Rotational Dynamics

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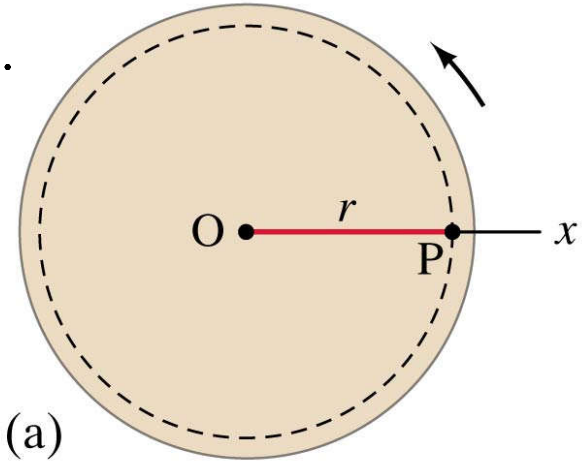
# 10 - 1 Angular Quantities: Position

In rotational motion, all points on the object move in circles around the axis of rotation ("$O$").

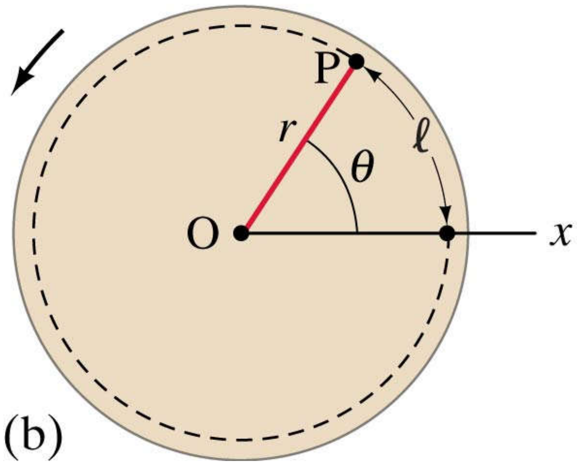
All points on a straight line drawn through the axis move through the same angle in the same time. The angle $\theta$ in radians is defined:

$$\theta = \frac{\ell}{r},$$

where $l$ is the arc length measured in radians.



(a)



(b)

# 10 - 2 Angular Quantities: Displacement and Velocity

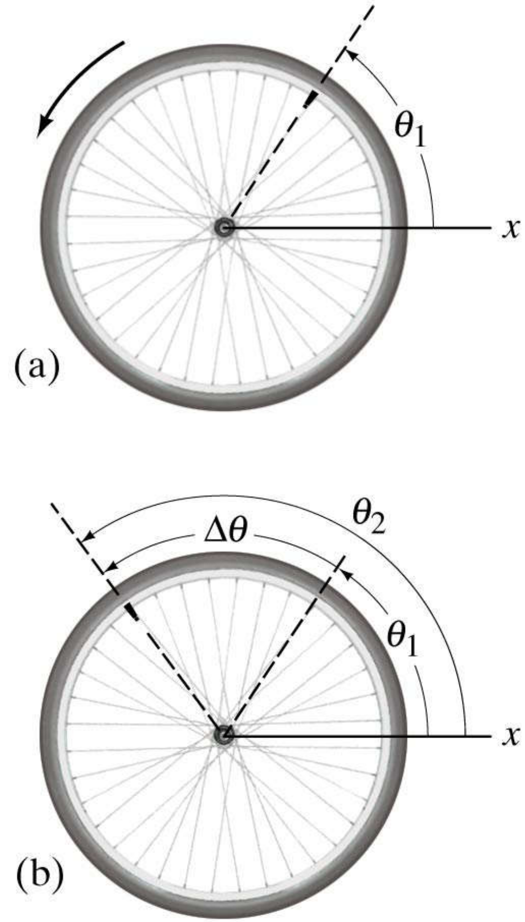
Angular displacement describes how much the object has rotated:

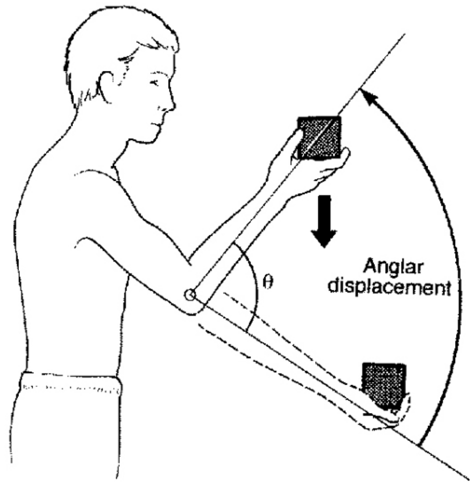
$$\Delta\theta = \theta_2 - \theta_1$$

The average angular velocity is defined as the total angular displacement divided by time:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t},$$

*Measured in Radians/s.*

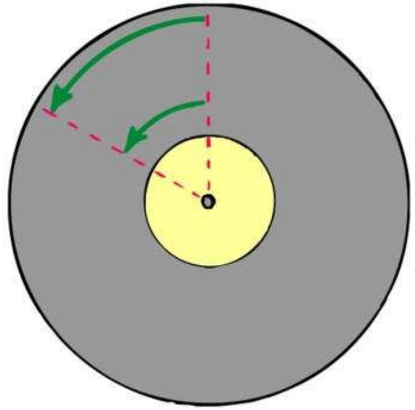


# 10 - 3 Angular Quantities: Acceleration

The angular acceleration is the rate at which the angular velocity changes with time:

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}$$



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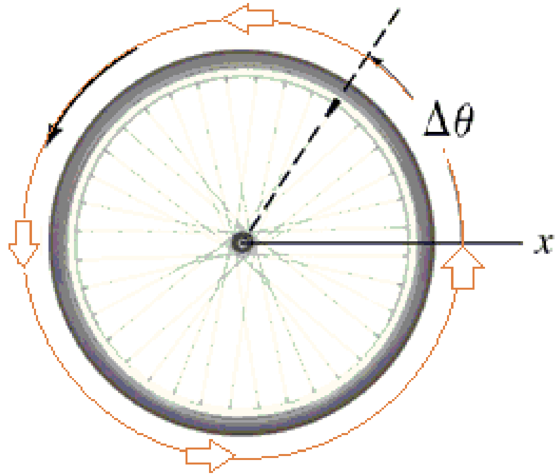
*Measured in Radians/s²*

- All points on a rotating rigid object have the same angular acceleration and the same angular speed as all other points on the object.

# 10 - 4 Angular Quantities: How do they relate?

**Sample question:**

If it takes a bike wheel 4 seconds to complete one revolution, what is the wheel's angular velocity?




Solution:

The definition of angular velocity is $\omega = \Delta\Theta \,/\, \Delta t$.

1 complete revolution : $\Delta\Theta = 2\pi$

$\Delta t = 4\ s$,

By putting this into the equation we can calculate the angular velocity:

$$\omega = 2\pi \,/\, 4\ s = 1.57\ \text{rad.}s^{-1}$$

# 10 - 4 Angular Quantities: How do they relate?

Here is the correspondence between linear and rotational quantities:

**TABLE 8–1 Linear and Rotational Quantities**

| Linear | Type | Rotational | Relation[‡] |
|---|---|---|---|
| $x$ | displacement | $\theta$ | $x = r\theta$ |
| $v$ | velocity | $\omega$ | $v = r\omega$ |
| $a_{\tan}$ | acceleration | $\alpha$ | $a_{\tan} = r\alpha$ |

[‡] You must use radians.

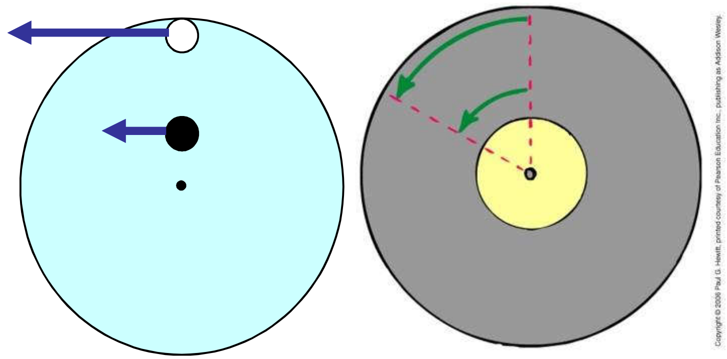
The equations of motion for constant angular acceleration are the same as those for linear motion, with the substitution of the angular quantities for the linear ones.

| Angular | Linear | |
|---|---|---|
| $\omega = \omega_0 + \alpha t$ | $v = v_0 + at$ | [constant $\alpha$, $a$] |
| $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $x = v_0 t + \frac{1}{2}at^2$ | [constant $\alpha$, $a$] |
| $\omega^2 = \omega_0^2 + 2\alpha\theta$ | $v^2 = v_0^2 + 2ax$ | [constant $\alpha$, $a$] |
| $\overline{\omega} = \dfrac{\omega + \omega_0}{2}$ | $\overline{v} = \dfrac{v + v_0}{2}$ | [constant $\alpha$, $a$] |

# 10 - 5 Linear and Angular Velocity

Suppose we have 2 horses on a carousel. The black horse is 1 meter from the center and the white horse is 2 meters from the center.

- Which horse has a greater <u>angular velocity</u>?
  - They have the same! They will each cover a full rotation ($360^o$) in the same amount of time.
- Which horse "feels" like they are going "faster"?
  - The white one!
- The white horse is going faster because it has a greater **<u>Linear Velocity</u>**.
  - It covers a greater distance (circumference) in the same amount of time.



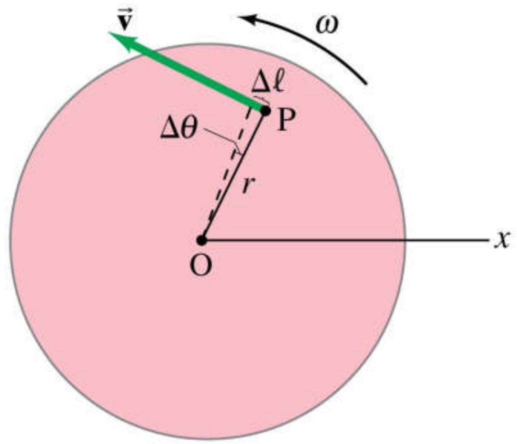
# Angular Quantities: Linear and Angular Velocity

Every point on a rotating body has an angular velocity $\omega$ and a linear velocity v. They are related:

$$v = \frac{\Delta \ell}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

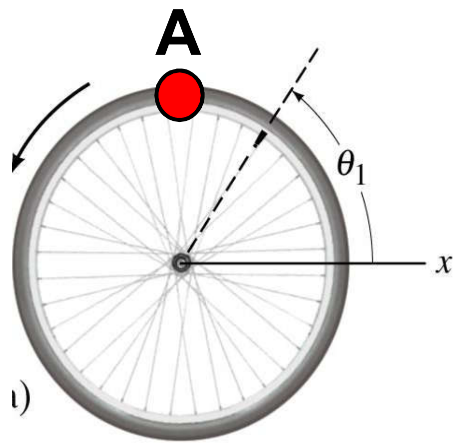
or (since $\Delta\theta / \Delta t = \omega$)

$$\boxed{v = r\omega.}$$




## Sample question:

The radius of a bike wheel is 70 cm, the angular velocity of the wheel is 2 rad/s, what is the linear velocity of the point A?




Solution:

$v = r \, . \, \omega$

$v = 0.7 \ m \ x \ 2 \ rad/s$

$v = 1.4 \ m/s$

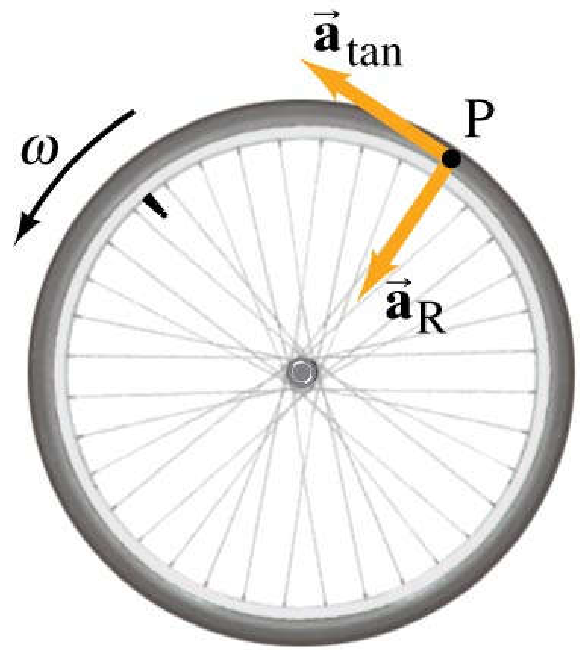
# Angular Quantities: Three Types of acceleration

If the angular velocity of a rotating object changes, it has a tangential acceleration:

$$a_{\tan} = \frac{\Delta v}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

or (using Eq. 8–3)

$$a_{\tan} = r\alpha.$$




Even if the angular velocity is constant, each point on the object has a centripetal acceleration:

$$a_R = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r.$$

# Angular Quantities: Three Types of acceleration

## Sample question:

An object moves at a constant speed of 9.0 *m/s* in a circular path of radius of 1.5 m. What is the angular acceleration of the object?

**Solution:**

The relationship between angular acceleration and linear acceleration is:

$a = \alpha . r$

We can find linear acceleration from this equation.

$a = v^2 / r$ $a = 9^2 / 1.5$ $a = 54\ m/s^2$

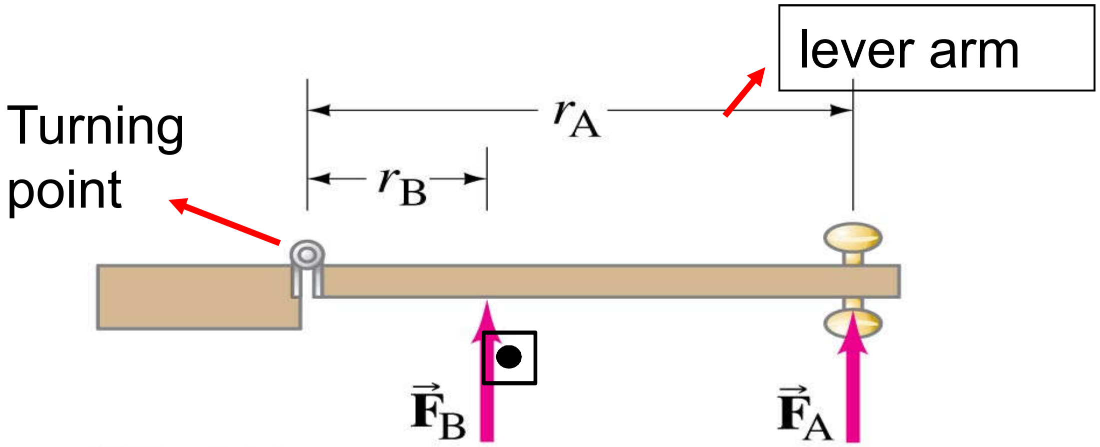
Then the angular acceleration is:

$$\alpha = \frac{a}{r} = \frac{54\ \frac{m}{s^2}}{1.5} = 36\ \frac{rad}{s^2}$$

# 10 - 6 Torque

To rotate an object, a force is needed; the position and direction of the force is also important.

Lever arm: The perpendicular distance from the turning point to the line along which the force acts.

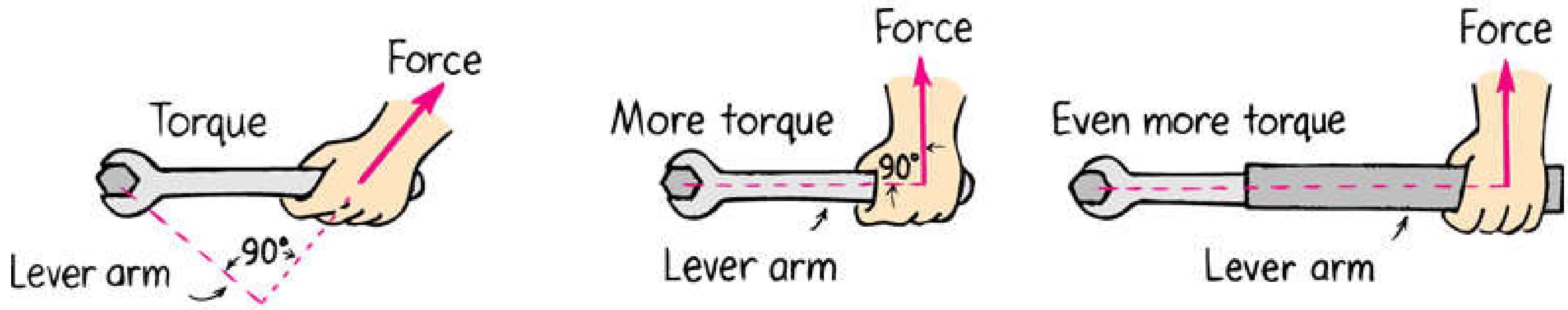



Torque: is a physical quantity used to measure the turning effect of a force.

Torque = lever arm x force

# 10 - 6 Torque

Example: Turning a bolt with longer lever arm is easier.

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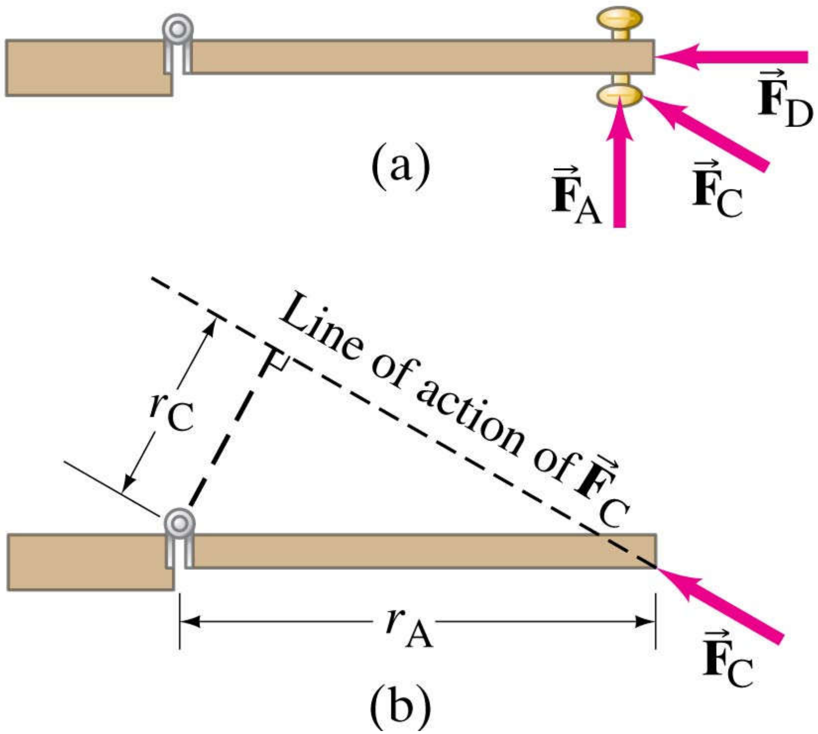
A longer lever arm is very helpful in rotating objects.


(a) (b)

# 10 - 6 Torque

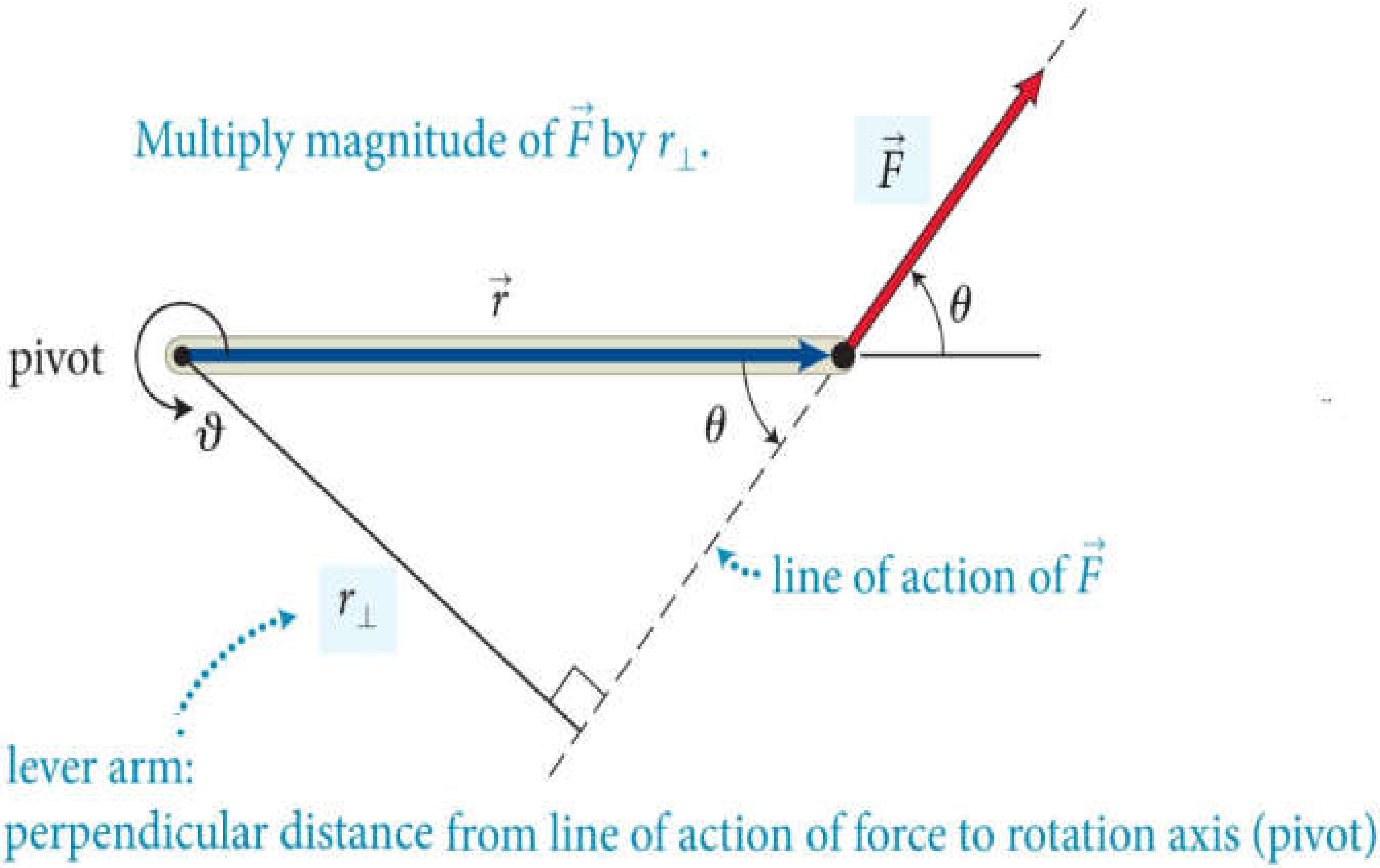
Here, the lever arm for $F_A$ is the distance from the knob to the turning point; the lever arm for $F_D$ is zero; and the lever arm for $F_C$ is as shown.

The torque is defined as: $\tau = r_{\perp} F$



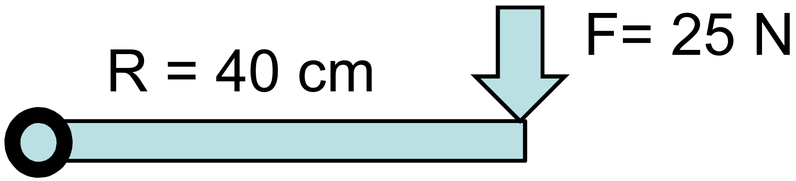
## 10 - 6 Torque

**Torque:** $\tau$ **= F** x **r** x **Sinθ**  θ = angle of rotation



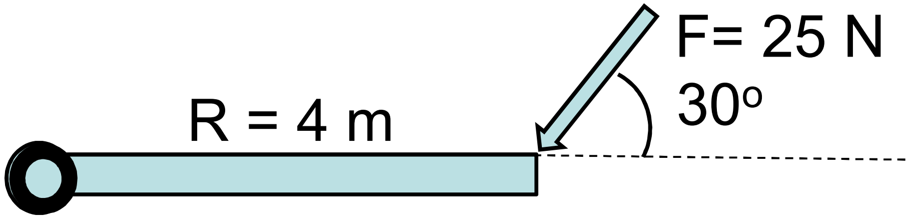
# 10 - 6 Torque

**Sample question:** Calculate the torque on the object below.




**Solution:** $\tau$ **= F** x **r =** 25 N x 0.4 m = 10 Nm

**Sample question:** Calculate the torque on the object below.




**Solution:**

$\tau$ = F x r x Sinθ = 25 x 4 x Sin30 = 25 x 4 x sin30

$\tau$ = 100 x 0.5 = 50 Nm