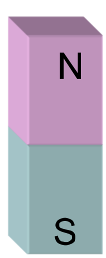
Chapter 28
Magnetic Fields

28-1 The magnetic Field
28-2 The Definition of the Magnetic Field
28-3 Crossed Fields
28-4 A Circulating Charged Particle
28-5 Magnetic Force on a Current-Carrying Wire
28-6 Torque on a Current Loop

Objective

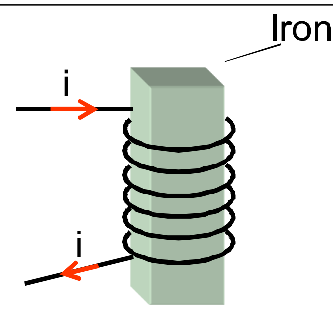
28-1 The magnetic Field
Ways to produce magnetic fields

How to produce magnetic fields?



Permanent magnets

An electron has an intrinsic magnetic field. In some materials, magnetic fields from the electrons add together to give a net magnetic field



Electromagnet

Need current

Moving charged particles (electrons) produce magnetic fields

28-2 The Definition of the Magnetic Field

Formula - Force on a charged particle due to a magnetic field

$$\boxed{\vec{F}_B = q\,\vec{v} \times \vec{B}}$$

$\vec{F}_B$: Force on a charged particle due to a magnetic field

q: Charge of the particle

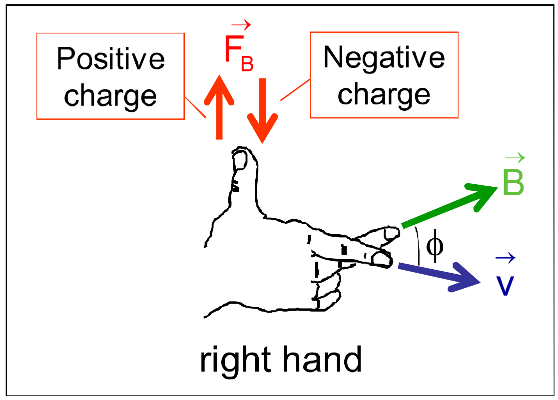
$\vec{v}$: Velocity of the particle

$\vec{B}$: Magnetic field

Direction: right-hand rule

Magnitude: $F_B = |q|\, v\, B \sin\phi$

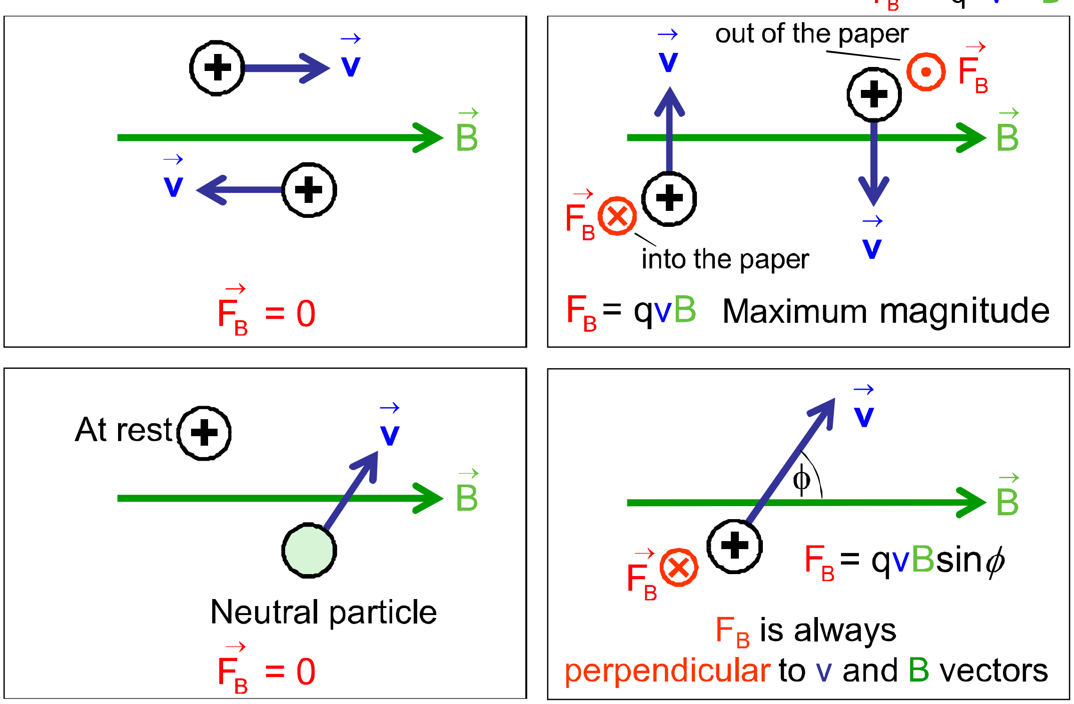
ϕ: The smaller angle between v and B vectors



28-2 The Definition of the Magnetic Field

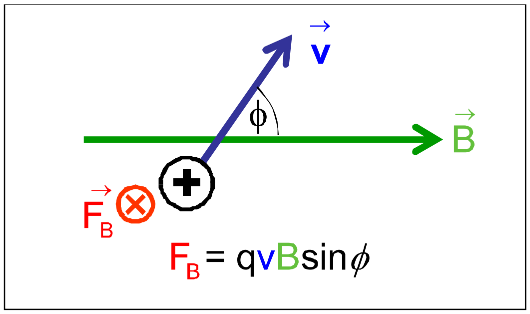
Illustration - magnetic force on a charged particle

$\vec{F}_B = q\,\vec{v} \times \vec{B}$



28-2 The Definition of the Magnetic Field
Magnetic force cannot do work

$\vec{F}_B = q\,\vec{v} \times \vec{B}$



F_B is always perpendicular to v vector

- → F_B does not have a component along v vector
- → F_B cannot change the speed of a particle
- → F_B cannot change the kinetic energy of a particle
- → F_B cannot do work on a particle
- → F_B can accelerate a particle only by changing its direction

28-2 The Definition of the Magnetic Field
SI unit for magnetic fields

SI unit for magnetic fields B

$\vec{F}_B = q\,\vec{v} \times \vec{B}$

Tesla

$$1 \text{ Tesla} = 1 \frac{\text{Newton}}{(\text{Coulomb})\ (\text{Meter/Second})}$$

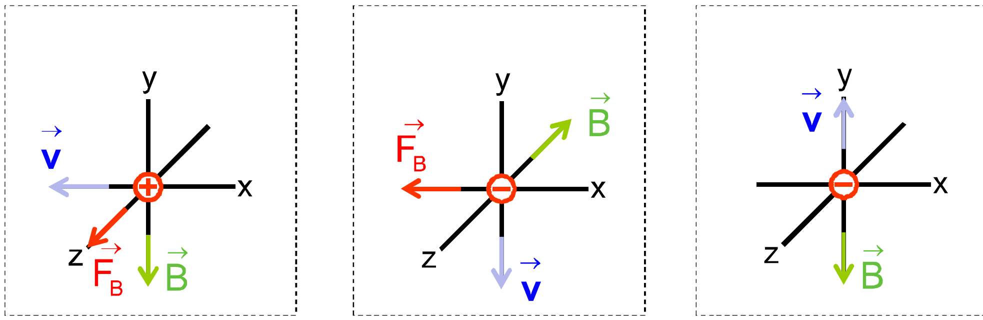
Gauss is another unit for measuring magnetic fields B

$\boxed{1 \text{ Tesla} = 10^4 \text{ Gauss}}$

Small bar magnets	10^{-2} T	100 G
Near Earth's surface	$0.5\text{X}10^{-4}$ T	0.5 G

28-2 The Definition of the Magnetic Field
Checkpoint 1

What is the direction of the magnetic force on the particle?



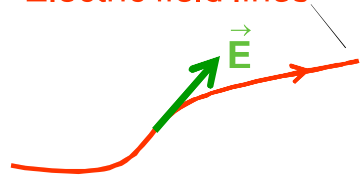
Solution

Along the positive z-axis	Along the negative x-axis	$\vec{F}_B = 0$

28-2 The Definition of the Magnetic Field
Magnetic field lines

Electric Field $\vec{E}$

Electric field lines

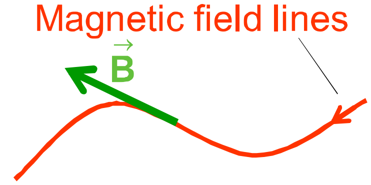


At any point, the tangent of an electric field line gives the direction of the electric field

Number of lines per unit area in a plane perpendicular to the electric field lines is proportional to the magnitude of the electric field

Magnetic Field $\vec{B}$

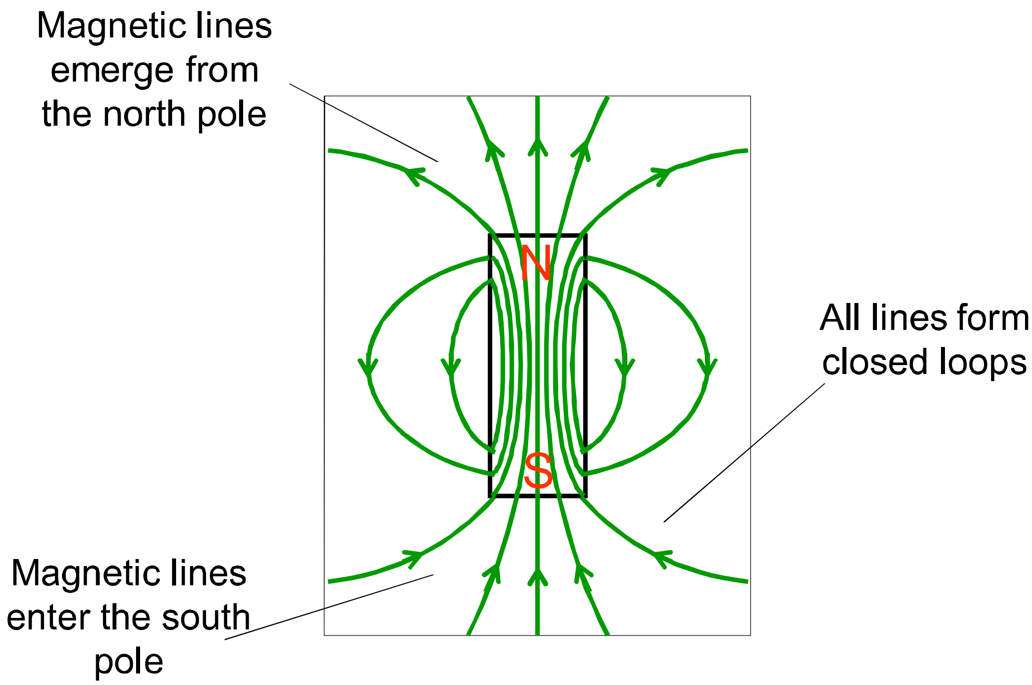
Magnetic field lines



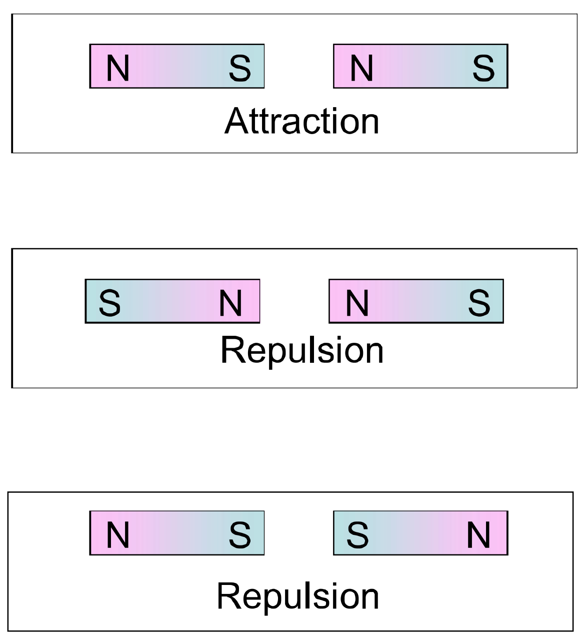
At any point, the tangent of a magnetic field line gives the direction of the magnetic field

Number of lines per unit area in a plane perpendicular to the magnetic field lines is proportional to the magnitude of the magnetic field

28-2 The Definition of the Magnetic Field
Properties of magnetic field lines

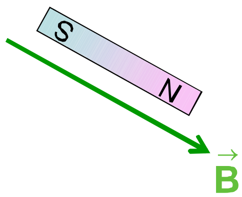


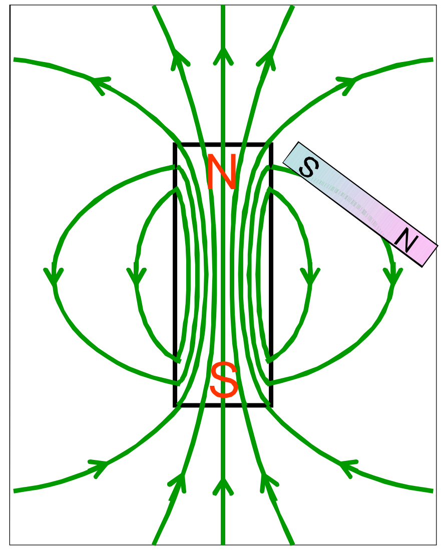
28-2 The Definition of the Magnetic Field
Attraction and repulsion of two magnets



28-2 The Definition of the Magnetic Field
Magnet bar in magnetic field

A magnet bar tries to align itself along the magnetic field

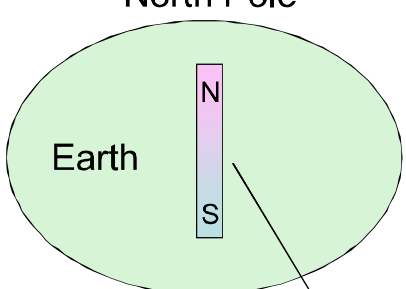




28-2 The Definition of the Magnetic Field
Earth's magnetic field

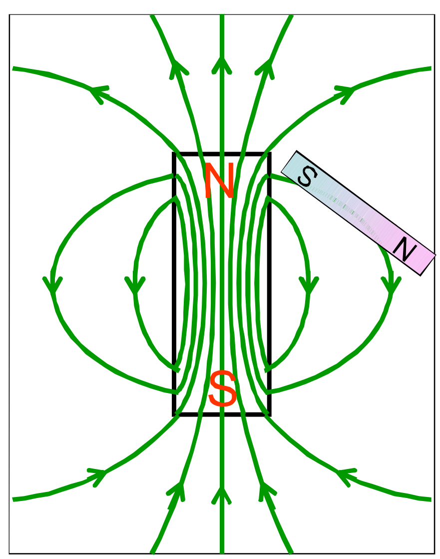
Earth's south magnetic pole

North Pole



South Pole

The north pole of a magnetic needle (compass) points towards Earth's north pole



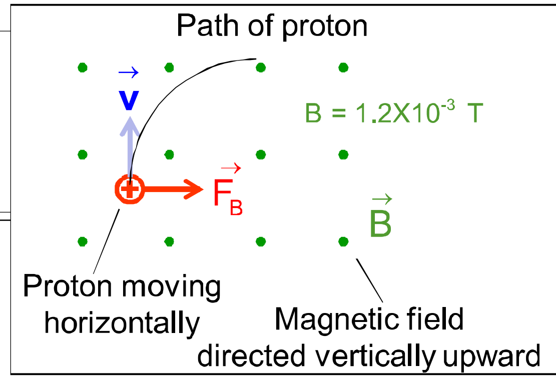
28-2 The Definition of the Magnetic Field

Example 1

Kinetic energy = 1.0 keV

Proton mass = 1.67×10^{-27} Kg

What magnetic deflecting force acts on the proton?



Solution

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

$$K = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.0X10^3 eV)(\frac{1.6X10^{-19} J}{eV})}{1.67X10^{-27} kg}} = 4.4X10^5 \text{ m/s}$$

$$F_B = q\,v\,B \sin\phi = (1.6X10^{-19})(4.5X10^5)(1.2X10^{-3})\sin 90^0 = 8.4X10^{-17} \text{ N}$$

Direction: in the horizontal plane perpendicular to the path of proton

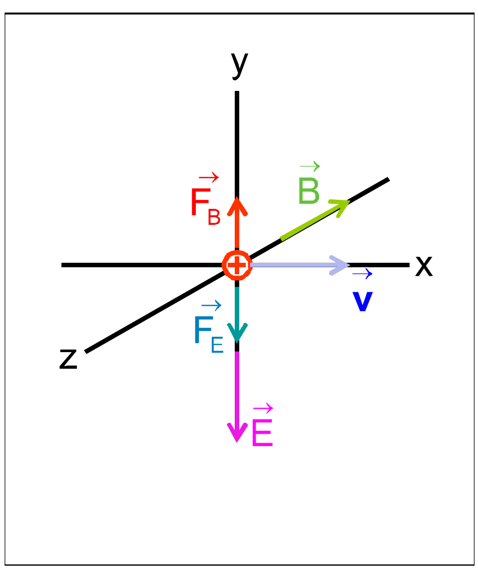
What is the acceleration of the proton?

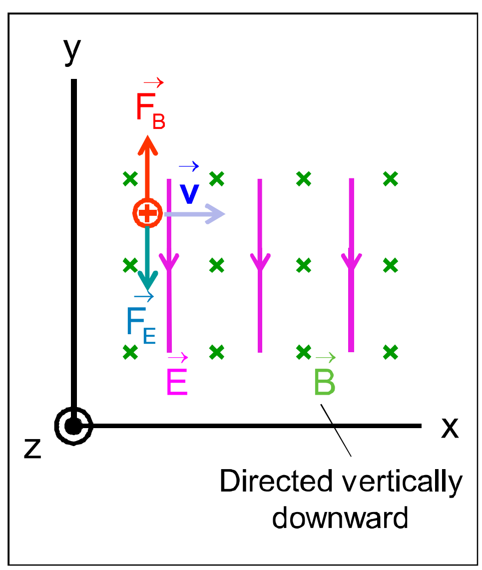
Solution $a = \frac{F_B}{m} = 5.0X10^{10}$ m/s^2

28-3 Crossed Fields

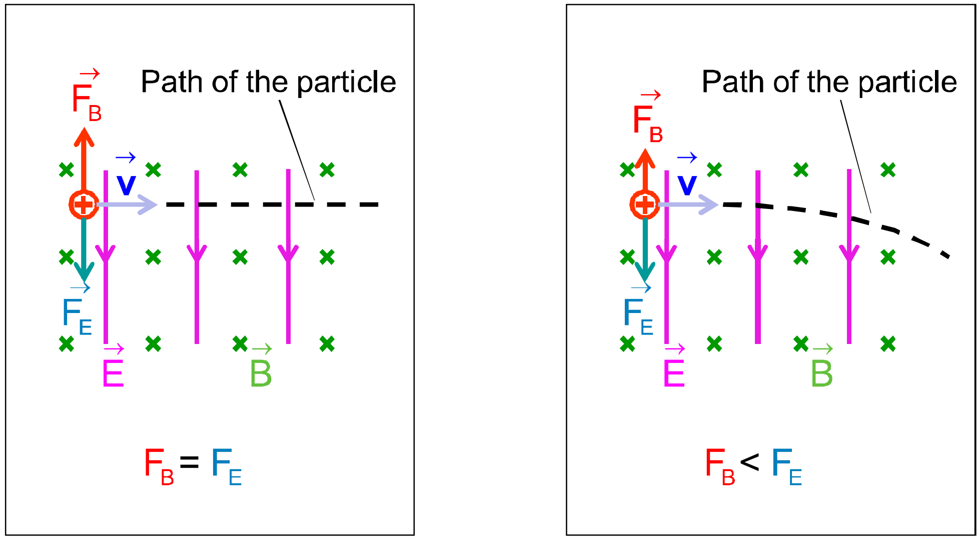
Magnetic field normal to electric field

Cross fields $\vec{B} \perp \vec{E}$

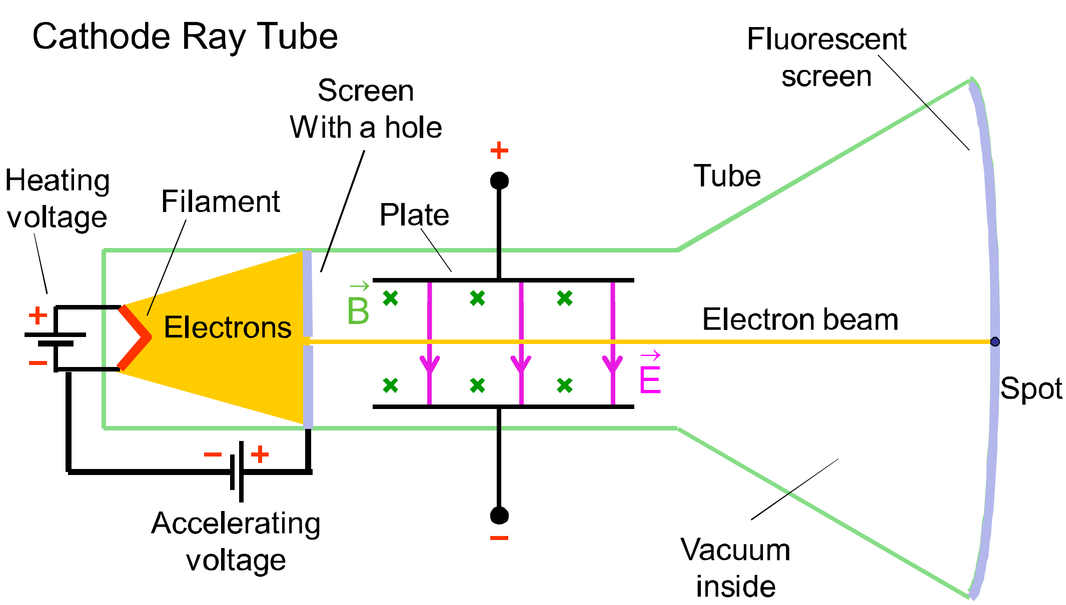




28-3 Crossed Fields
Motion of a charge particle in crossed fields



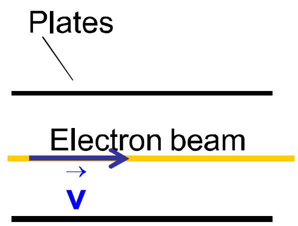
28-3 Crossed Fields
Cathode ray tube



28-3 Crossed Fields
Mass-to-charge ratio of an electron

Thomson measured m/q of an electron in 1897

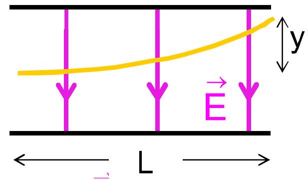
1



$\vec{E} = 0$

$\vec{B} = 0$

2



$\vec{E} \neq 0$

$\vec{B} = 0$

$F_E = q E = m a$

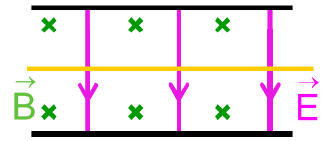
$$a = \frac{qE}{m}$$

$$y = \frac{1}{2} a t^2$$

$$y = \frac{1}{2}\frac{qE}{m}\left(\frac{L}{v}\right)^2$$

$$v^2 = \frac{1}{2}\frac{qE}{m}\frac{L^2}{y}$$

3



$\vec{E} \neq 0$

Increase B till

$F_B = F_E$

$|q|vB = |q|E$

$$\boxed{v = \frac{E}{B}}$$

$$\left(\frac{E}{B}\right)^2 = \frac{1}{2}\frac{qE}{m}\frac{L^2}{y}$$

$$\boxed{\frac{m}{q} = \frac{B^2L^2}{2Ey}}$$

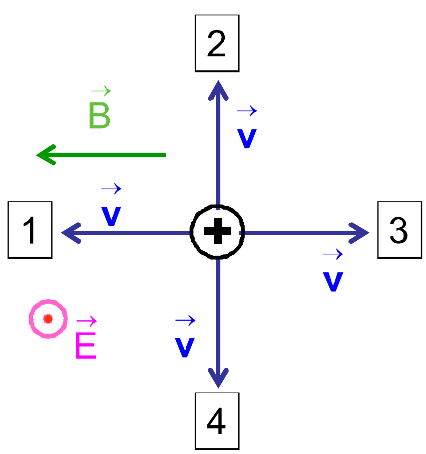
All quantities on the right can be measured

28-3 Crossed Fields
Checkpoint 2

Rank 1, 2, and 3 according to the net force on the particle, greatest first

Solution

2
1 and 3 tie



Which direction might result in a net force of zero?

Solution

4

28-4 A Circulating Charged Particle
Circular path

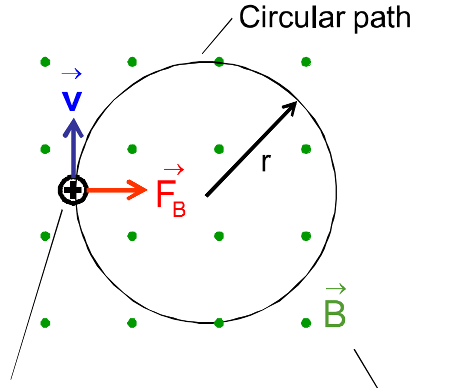
$$F_B = q\,v\,B \sin 90^0 = q\,v\,B$$

$$F_B = m\,a$$

For circular motion $a = \dfrac{v^2}{r}$

$$q\,v\,B = m\,\frac{v^2}{r}$$

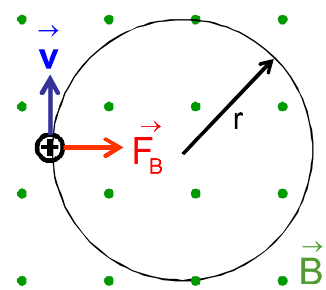
Radius $\boxed{r = \dfrac{m\,v}{q\,B}}$



The magnetic force changes the direction of the velocity vector. Since it cannot change the magnitude of the velocity vector, the particle moves in a circular path in a plane normal to the magnetic field.

28-4 A Circulating Charged Particle
Radius, period and frequency

Radius $\boxed{r = \dfrac{m\,v}{q\,B}}$



Period T (Time to make one revolution)

$T = \dfrac{2\pi\,r}{v}$ $\boxed{T = \dfrac{2\pi\,m}{q\,B}}$

Frequency (Number of revolutions per unit time)

$f = \dfrac{1}{T}$ $\boxed{f = \dfrac{q\,B}{2\pi\,m}}$

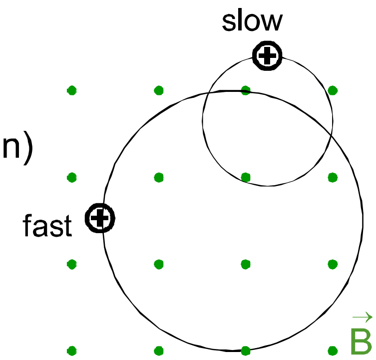
Angular frequency

$\omega = 2\,\pi\,f$ $\boxed{\omega = \dfrac{q\,B}{m}}$

28-4 A Circulating Charged Particle
Period does not depend on speed

Period T (Time to make one revolution)

$$T = \frac{2\pi m}{q B}$$



Periods do not depend on the speed of the particles

→ All particles with the same charge-to-mass ratio take the same time to make one revolution.

→ Fast particles move in big circles and slow particles move in small circles.

28-4 A Circulating Charged Particle
Checkpoint 3

An electron and a proton travel at the same speed in a uniform magnetic field

Which particle follows the smaller circle?

Solution

$$r = \frac{m v}{q B}$$

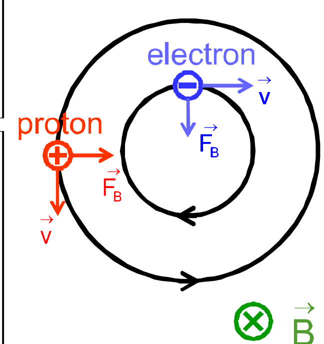
Proton charge = e

Electron charge = - e

Proton mass = 1.67×10^{-27} kg

Electron mass = 9.1×10^{-31} kg

Electron follows the smaller circle.



What is the travel direction of the particles?

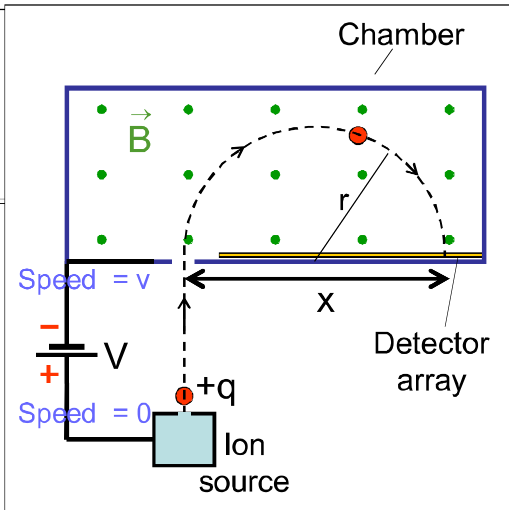
Solution

Electron follows clockwise direction.

Proton follows counterclockwise direction.

28-4 A Circulating Charged Particle
Example 2

Charge $q = 1.60 \times 10^{-19}$ C
$X = 1.63$ m
$B = 80.0$ mT
$V = 1.00$ kV
What is the mass of the ion?



Mass spectrometer
Device to measure mass of ions

Solution

$$r = \frac{m\,v}{q\,B} = \frac{X}{2}$$

To find the mass, we need to know the speed of the ion at the entrance of the chamber. To find the speed, compare the total energy at the ion source and at the entrance of the chamber.

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv^2 - 0 = -(q(-V))$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{m}{q\,B}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = \frac{X}{2}$$

28-4 A Circulating Charged Particle
Example 2

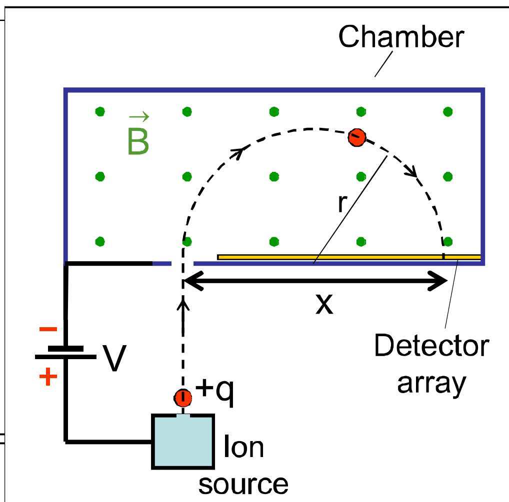
Solution

$$\frac{X}{2} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$m = \frac{B^2qX^2}{8V}$$

$$= \frac{(80\text{x}10^{-3})^2(1.6\text{x}10^{-19})(1.63)^2}{8(10^3)}$$

$$= 3.39\text{x}10^{-25}\ \text{kg}$$



Mass spectrometer
Device to measure mass of ions

What is the mass in atomic mass unit u ($1\text{u} = 1.67 \text{ x}10^{-27}$ kg)?

Solution

$$m = 3.39\text{x}10^{-25}\ \text{kg}\ \frac{1\ \text{u}}{1.67 \times 10^{-27}\text{kg}} = 204\ \text{u}$$

28-4 A Circulating Charged Particle
When velocity is not normal to the magnetic field

$$\vec{F}_B = q\ \vec{v} \times \vec{B}$$

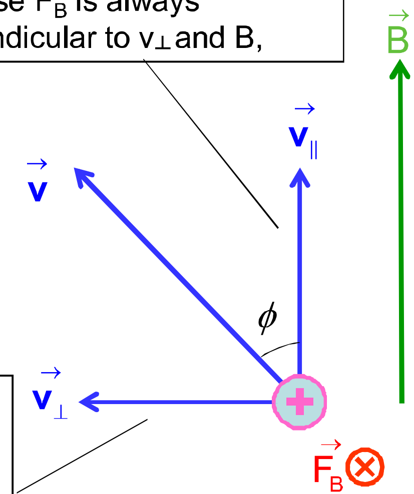
$$F_B = q\,v\,B\,\sin\phi$$

$$F_B = q\,v\,\sin\phi\,B$$

$$F_B = q\,v_\perp B$$

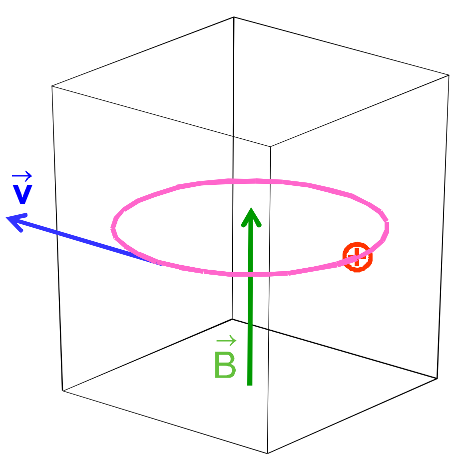
All the force on the particle is produced because of $v_\perp$.

$v_\parallel$ does not change in magnitude or direction, because F_B is always perpendicular to $v_\perp$ and B,

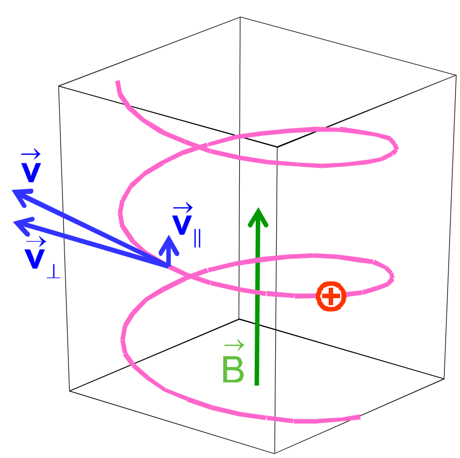


F_B changes the direction of $v_\perp$ but not its magnitude, because F_B is always perpendicular to $v_\perp$ and B.

28-4 A Circulating Charged Particle
Helical paths

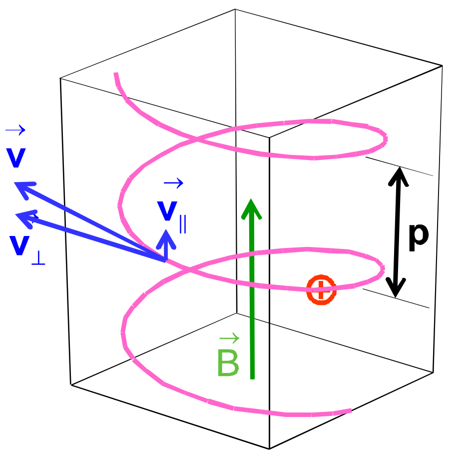


v is perpendicular to B
Circular path



v has a component along B
Helical path

28-4 A Circulating Charged Particle
Pitch of the helix



v has a component along B

Pitch p of the helix

$$p = v_{\|}\,T$$

Period T (Time to make one revolution)

$$p = v_{\|}\,\frac{2\pi\, m}{q\,B}$$

28-4 A Circulating Charged Particle
Example 3

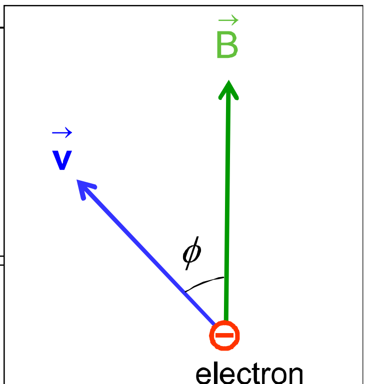
Kinetic energy = 22.5 eV

B = 0.455 mT

ϕ =65.5^0

1 eV =1.6x10^{-19} J

What is the pitch of the helical path taken by the electron?



Solution

$$p = v_{\|}\,\frac{2\pi\, m}{q\,B}$$

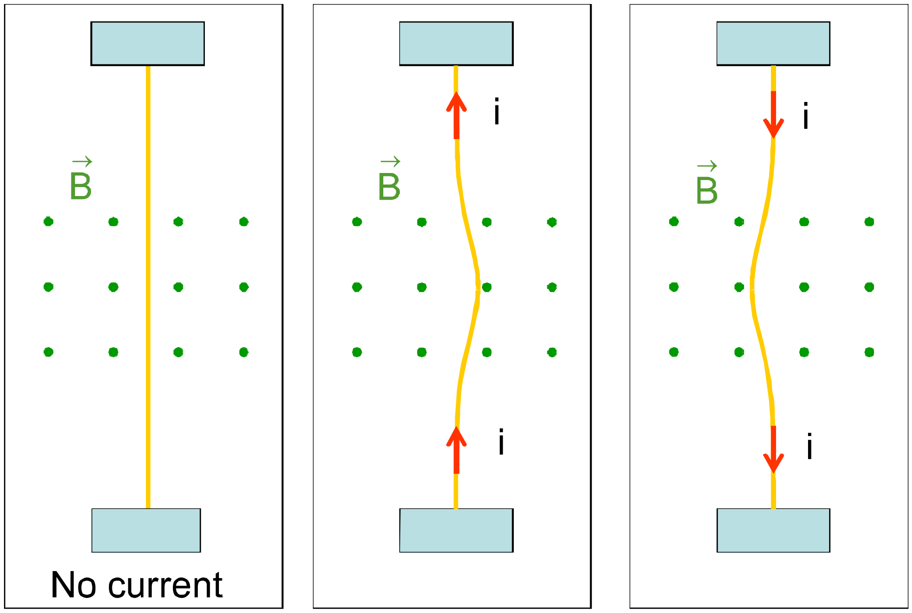
$$p = v\cos\phi\,\frac{2\pi\, m}{q\,B}$$

$$K = \frac{1}{2}mv^2$$

$$p = \sqrt{\frac{2K}{m}}\cos\phi\,\frac{2\pi\, m}{q\,B} = \sqrt{2K\,m}\,\cos\phi\,\frac{2\pi}{q\,B}$$

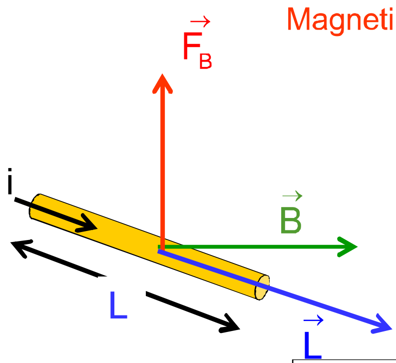
$$= \sqrt{2(22.5\text{x}1.6\text{x}10^{-19})(9.11\text{x}10^{-31})}\,\cos(65.5^0)\,\frac{2\pi}{(1.6\text{x}10^{-19})(45.5\text{x}10^{-3})} = 9.17\text{ cm}$$

28-5 Magnetic Force on a Current-Carrying Wire
Illustration - Force on a wire in a magnetic field



Flexible wire fixed at both ends

28-5 Magnetic Force on a Current-Carrying Wire
Formula - Magnetic force on a wire



Magnetic force of a wire of length L

Current through the wire

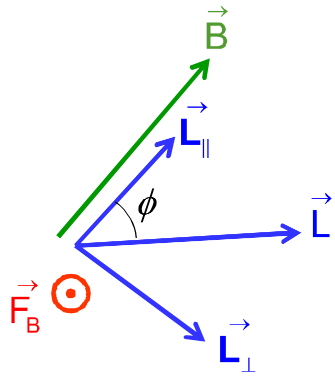
$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

Length vector

Magnitude: Length of the wire.
Direction : Along the wire segment in the direction of conventional current.

F_B is perpendicular to L and B vectors

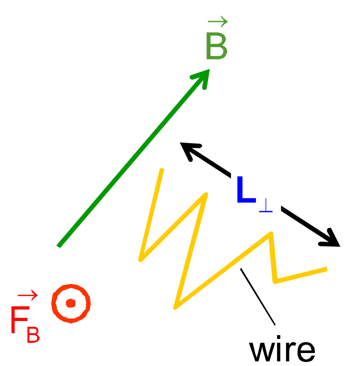
28-5 Magnetic Force on a Current-Carrying Wire
Force is due to the length normal to the magnetic field



$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$
$$F_B = i\,L\,B\,\sin\phi$$
$$F_B = i\,L\,\sin\phi\,B$$
$$F_B = i\,L_\perp B$$

Only the length perpendicular to the magnetic field $L_\perp$ contributes to the force.

28-5 Magnetic Force on a Current-Carrying Wire
Illustration - Length normal to the magnetic field

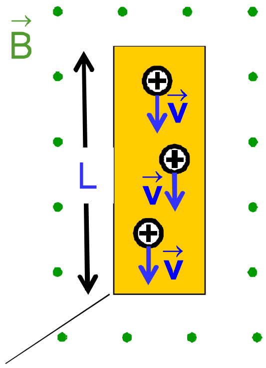


$$F_B = i\,L_\perp B$$

Only the length perpendicular to the magnetic field $L_\perp$ contributes to the force.

28-5 Magnetic Force on a Current-Carrying Wire
Derivation - Magnetic force on a wire

Derivation of $\vec{F}_B = i\,\vec{L} \times \vec{B}$



Section of a wire

All the "moving" ions in a section of length L enter the section during time

$$dt = \frac{L}{v}$$

The charge in length L is

$$q = i\,dt = i\,\frac{L}{v}$$

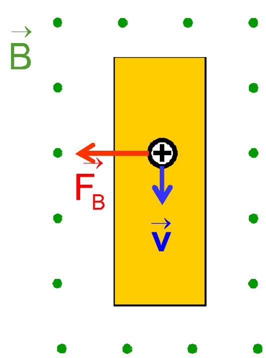
Magnetic force on the section

$$F_B = q\,v\,B \sin\phi = i\,\frac{L}{v}\,v\,B\sin\phi$$

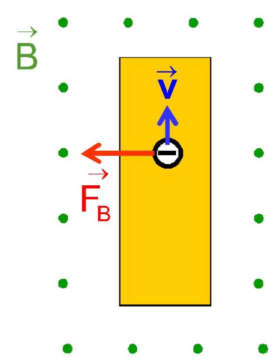
$$= i\,L\,B\sin\phi = i\left|\vec{L} \times \vec{B}\right|$$

28-5 Magnetic Force on a Current-Carrying Wire
Convention and real charge carriers

The direction of the magnetic force is the same whether we consider moving ions (convention) or moving electrons (real).



Conventional picture
Moving ions



Real situation
Moving electrons

$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

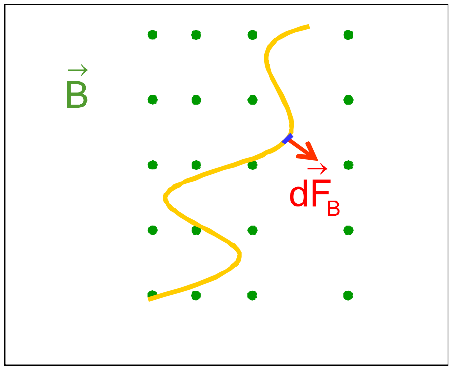
28-5 Magnetic Force on a Current-Carrying Wire
Non-uniform magnetic field

If the wire is not straight or the magnetic field is not uniform,
We can imagine breaking the wire into small segments and apply

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$$

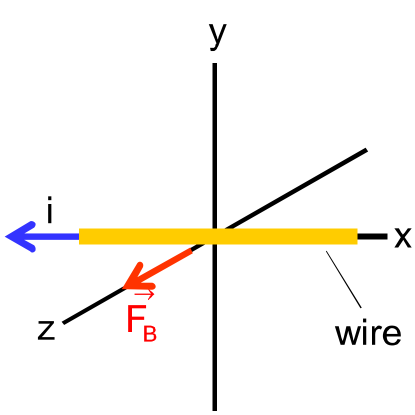
The total force on the wire is

$$\vec{F}_B = \int d\vec{F}_B$$

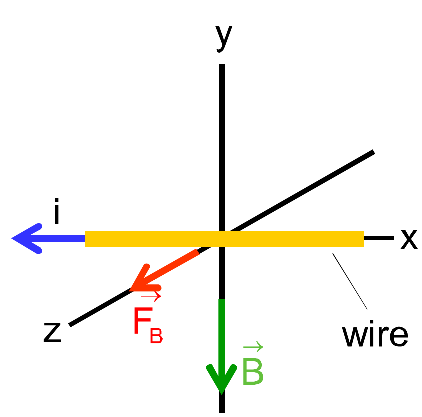


28-5 Magnetic Force on a Current-Carrying Wire
Checkpoint 4

What is the direction of the field?



Solution



28-5 Magnetic Force on a Current-Carrying Wire
Example 4

i = 28 A
Wire linear mass density is λ = 46.6 g/m
What is the magnitude and direction of the minimum magnetic field to balance the gravitational force on the wire?

Solution

$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

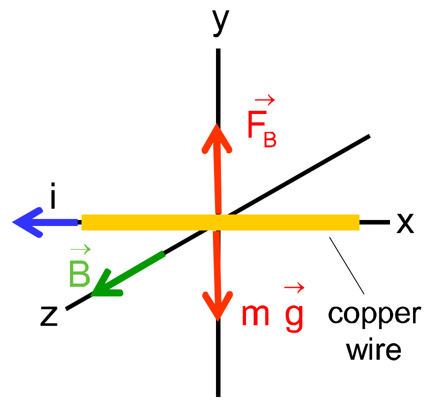
$$F_B = i\,L\,B\,\sin\phi = m\,g$$

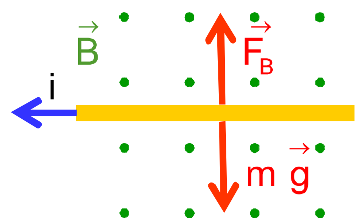
Minimum B
→ Maximum $\sin\phi = 1$

$$i\,L\,B = m\,g = \lambda\,L\,g$$

$$B = \frac{\lambda g}{i} = \frac{(46.6 \times 10^{-3})(9.8)}{28}$$

$$= 1.6 \times 10^{-2}\ \text{T}$$

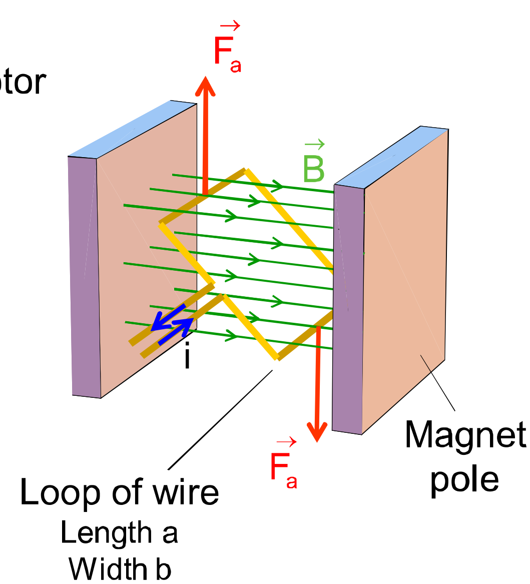




Two-dimensional view

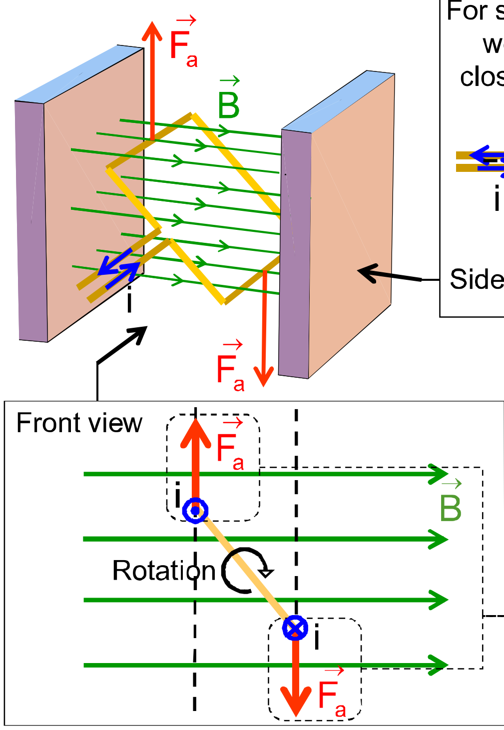
28-6 Torque on a Current Loop
Electric motor

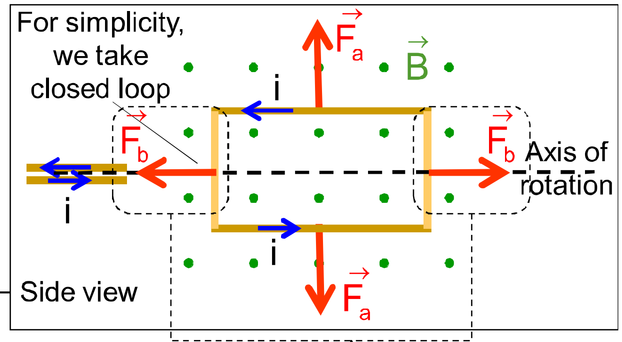
Electric motor



28-6 Torque on a Current Loop

Forces on a current loop



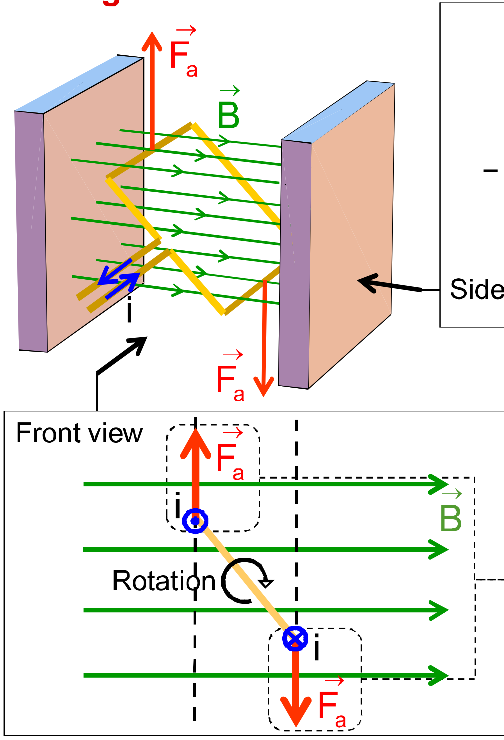


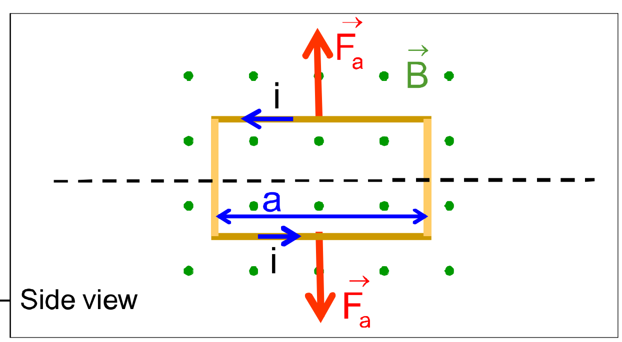
The sum of these two forces is zero.
They produce **no torque** because they share the same line of action.

The sum of these two forces is zero.
But they produce a torque because they do not share the same line of action.

28-6 Torque on a Current Loop

Rotating forces





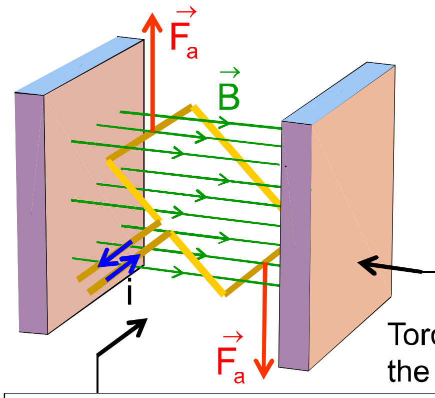
$$\vec{F}_a = i\,\vec{L} \times \vec{B}$$

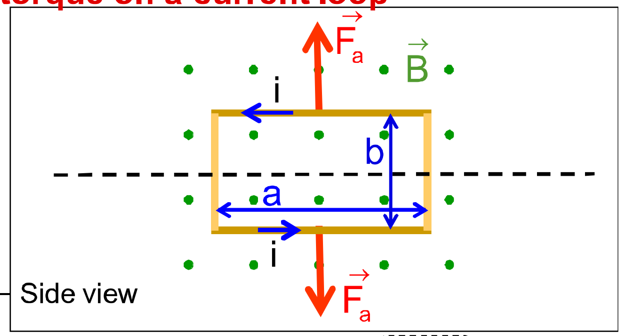
$$F_a = i\,a\,B \sin 90^0$$

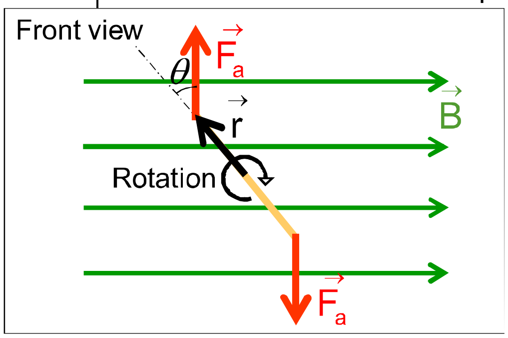
$$F_a = i\,a\,B$$

These wires are always perpendicular to the magnetic field.

28-6 Torque on a Current Loop
Formula - Magnitude of the torque on a current loop







Torque on the loop $\vec{\tau} = \boxed{\vec{r}} \times \vec{F}$; $r = \frac{b}{2}$

$$\tau = \frac{b}{2} F_a \sin\theta + \frac{b}{2} F_a \sin\theta$$

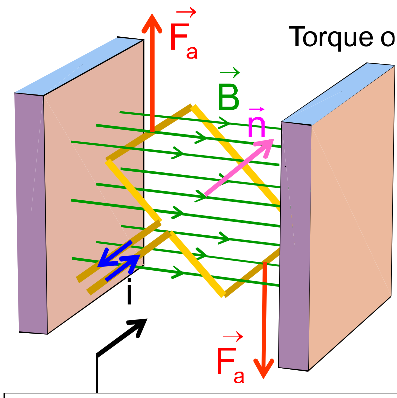
$$\tau = b \boxed{F_a} \sin\theta \qquad \boxed{F_a = i\,a\,B}$$

$$\tau = i\,a\,b\,B \sin\theta$$

$$\boxed{\tau = i \text{ area } B \sin\theta}$$

Area of the current loop

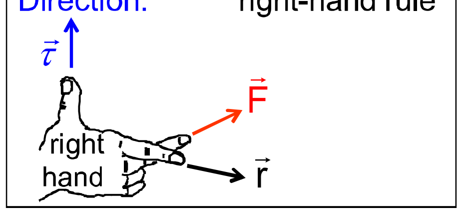
28-6 Torque on a Current Loop
Direction of the torque



Torque on the loop $\vec{\tau} = \vec{r} \times \vec{F}$

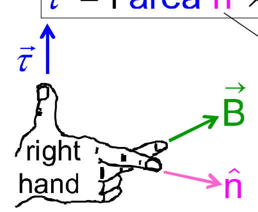
Magnitude: $\tau = i \text{ area } B \sin\theta$

Direction: right-hand rule

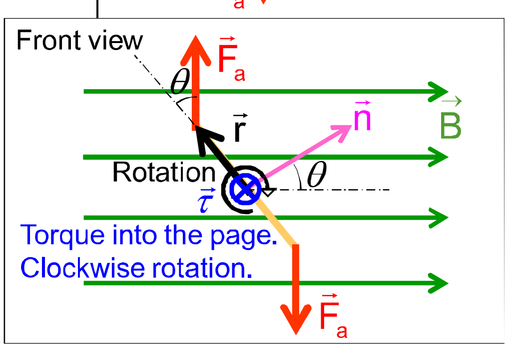


We may rewrite the torque vector as

$$\boxed{\vec{\tau} = i \text{ area } \hat{n} \times \vec{B}}$$

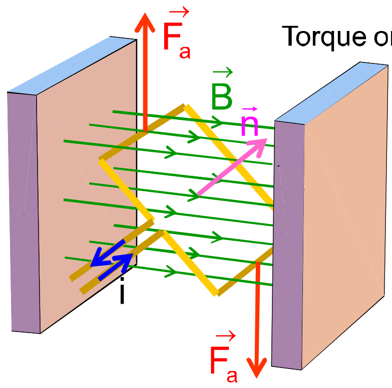


Normal vector has a magnitude of one and is normal to the plane of the loop.



Torque tries to align the normal vector with the direction of the magnetic field

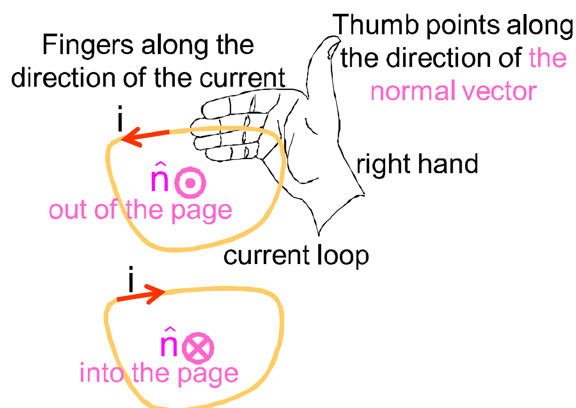
28-6 Torque on a Current Loop
Direction of the normal vector



Torque on the loop $\vec{\tau} = i \text{ area } \hat{n} \times \vec{B}$

True for any loop shape

The normal vector is a unit vector normal to the plane of the loop.



For an N-turn loop

$\vec{\tau} = N\, i \text{ area } \hat{n} \times \vec{B}$

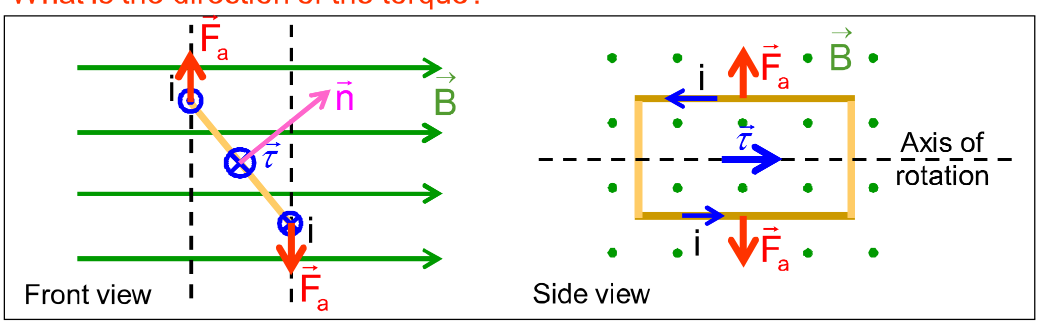
Number of turns

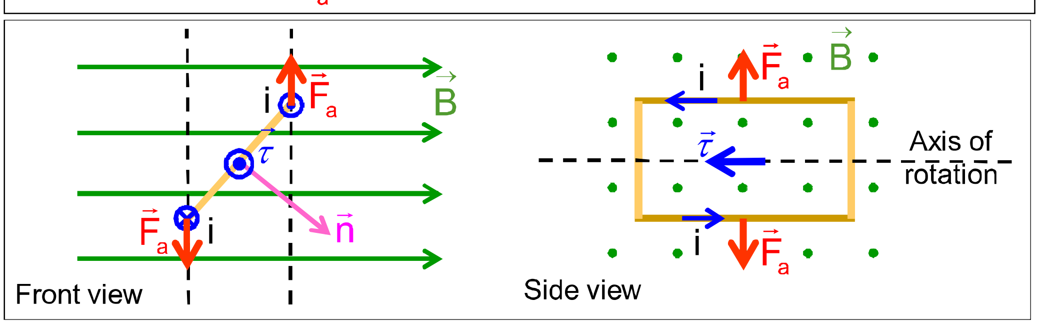
Torque tries to align the normal vector with the direction of the magnetic field

28-6 Torque on a Current Loop
Checkpoint 5

What is the direction of the torque?

Solution





28-7 The magnetic dipole moment
Formula

A current-carrying coil behaves like a bar maget