Chapter 27
Circuits

Objective

27-1 Pumping Charges
Emf devices

Battery
Electric generator
Solar cell

Devices that maintain potential difference between their terminals by doing work on charge carriers.

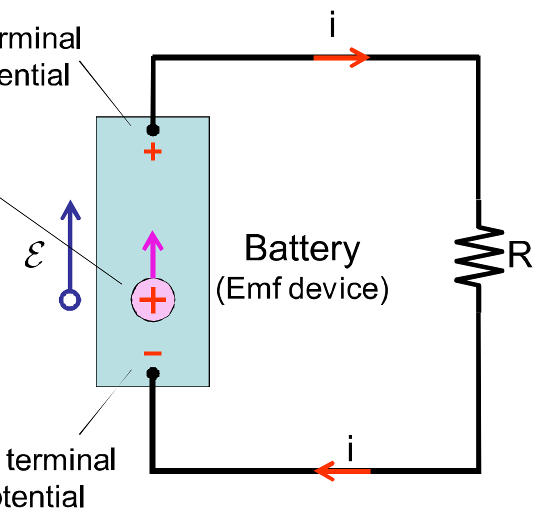
These devices are called emf devices

Emf is an outdated phrase which stands for electromotive force

An emf device provides an emf $\mathcal{E}$

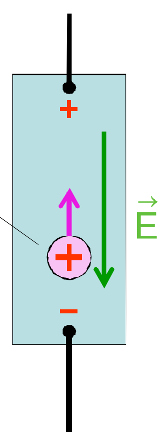
27-2 Work, Energy, and EMF
Battery

Internal chemistry of the battery causes a net flow of positive carriers from the negative terminal to the positive terminal.



27-2 Work, Energy, and EMF
Sources of energy in emf devices

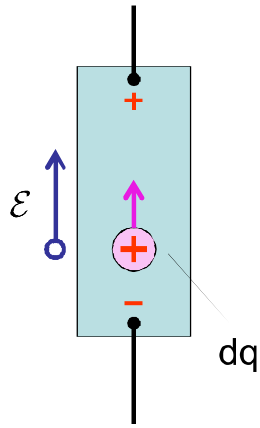
Motion is opposite to the electric field direction



There must be some source of energy within the source that enables it to do work on the charge carriers.

Battery	→ chemical energy
Electric generator	→ mechanical energy
Solar cell	→ light energy

27-2 Work, Energy, and EMF
Formula - Emf



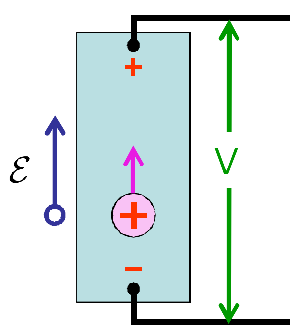
$$\mathcal{E} = \frac{dW}{dq}$$

The emf $\mathcal{E}$ of an emf device is the work per unit charge that an emf device does in moving charge from its lower-potential terminal to its higher-potential terminal.

$\mathcal{E}$ is measured in $\frac{\text{Joule}}{\text{Coulomb}}$ = Volt

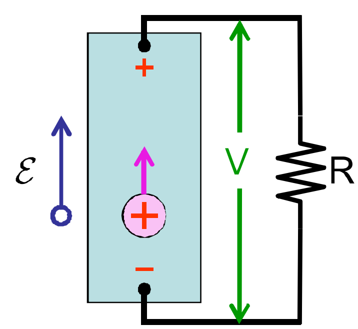
27-2 Work, Energy, and EMF
Ideal emf devices

Ideal emf devices
have no internal resistance to the movement of charge.



Not connected to a circuit

$V = \mathcal{E}$



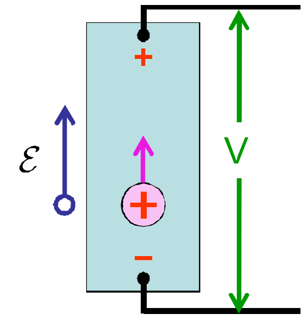
connected to a circuit

$V = \mathcal{E}$

Potential difference between the terminals = the emf of the device.

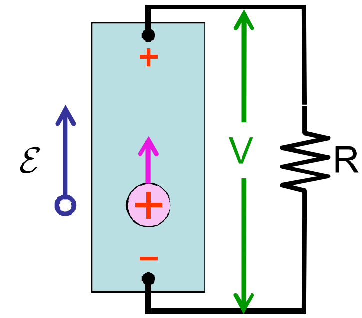
27-2 Work, Energy, and EMF
Real emf devices

Real emf devices
have internal resistance to the movement of charge.



Not connected to a circuit

$V = \mathcal{E}$

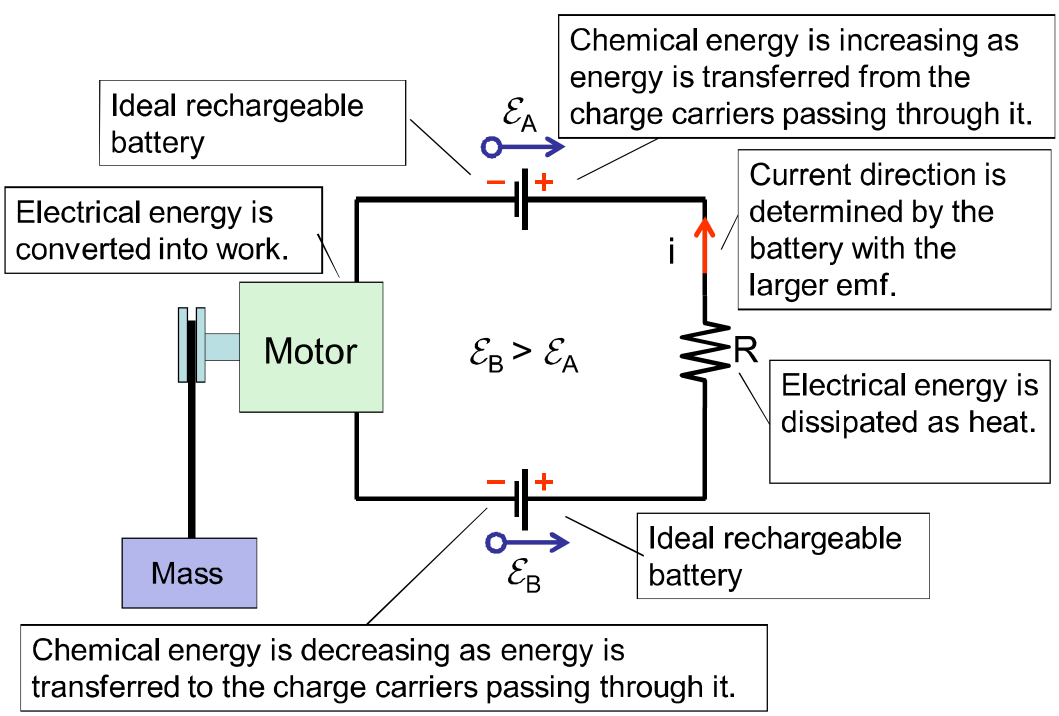


connected to a circuit

$V < \mathcal{E}$

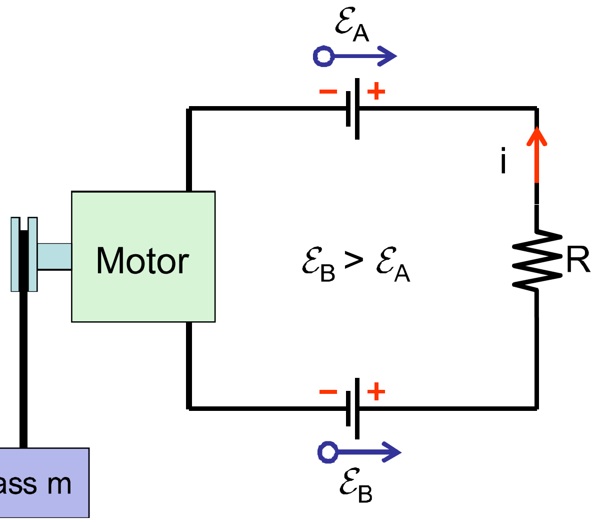
When connected to a resistor, the potential difference between the terminals < the emf of the device.

27-2 Work, Energy, and EMF
Energy conversion in an electric circuit



27-2 Work, Energy, and EMF

Energy conservation in an electric circuit



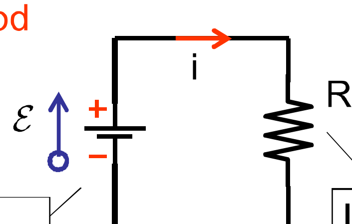
Chemical energy lost by battery B

- Thermal energy produced by resistance R
- Chemical energy stored in battery A
- Work done by motor on mass m

27-3 Calculating the Current in a Single-Loop Circuit

Energy method to analyze electric circuits

The energy method



In time dt,
the battery will do work of $dW = \mathcal{E}\, dq$ on dq

In time dt, thermal energy of $dE = i^2 R\, dt = i R\, dq$ will appear in the resistor

Energy is conserved

$$dW = dE$$

$$\mathcal{E}\, dq = i R\, dq$$

$$\boxed{\mathcal{E} = i R}$$

We will not use this method to analyze circuits

27-3 Calculating the Current in a Single-Loop Circuit
Potential method to analyze electric circuits

The potential method

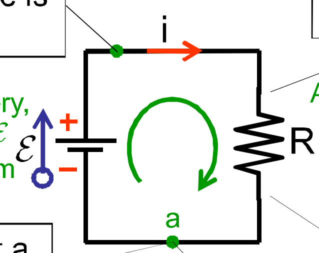
Find the change in the electric potential as you move around the circuit.

Let us choose the clockwise direction.

Thus the potential here is $V_a + \mathcal{E}$

In a resistor, the current flows from higher to lower potential.

As we move across the battery, the potential increases by $\mathcal{E}$ because we are moving from lower to higher potential.



Higher potential

As we move across the resistor, the potential decreases by i R because we are moving from higher to lower potential.

Lower potential

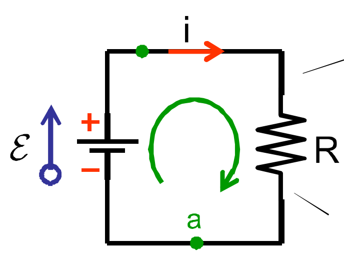
Let us start here at point a. Call the potential here V_a.

The potential here is $V_a + \mathcal{E} - i R$.
But this the same as V_a.
$V_a + \mathcal{E} - i R = V_a$

$+\mathcal{E} - i R = 0$

$\boxed{\mathcal{E} = i R}$

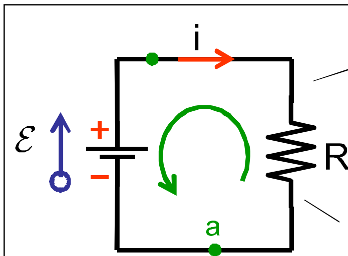
27-3 Calculating the Current in a Single-Loop Circuit
Direction does not affect the final result



Change in potential for clockwise direction
$V_a + \mathcal{E} - i R = V_a$

$+\mathcal{E} - i R = 0$

$\boxed{\mathcal{E} = i R}$



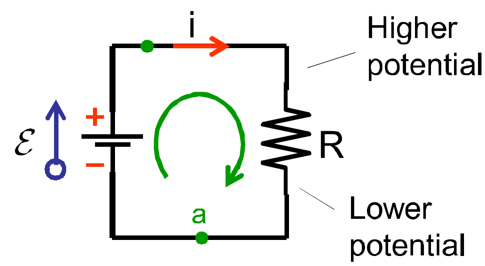
Change in potential for counterclockwise direction
$V_a + i R - \mathcal{E} = V_a$

$+ i R - \mathcal{E} = 0$

$\boxed{\mathcal{E} = i R}$

Direction does not affect the final result.

27-3 Calculating the Current in a Single-Loop Circuit
Loop rule

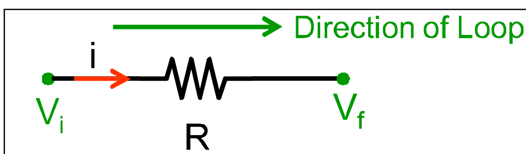


Loop rule (Kirchhoff's loop rule):
The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

$$\sum_{\text{loop}} \Delta V = 0$$

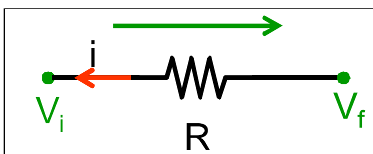
Conservation of energy

27-3 Calculating the Current in a Single-Loop Circuit
Signs convention

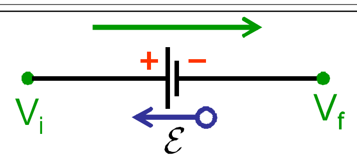


$V_f - V_i = - i R$

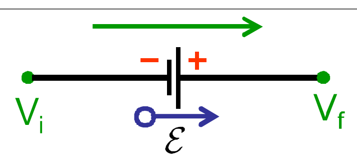
In a resistor, the current flows from higher to lower potential.



$V_f - V_i = + i R$

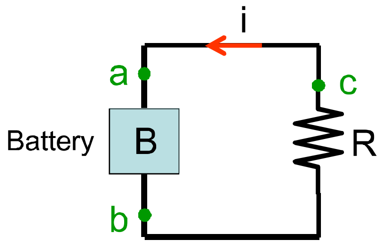


$V_f - V_i = - \mathcal{E}$



$V_f - V_i = + \mathcal{E}$

27-3 Calculating the Current in a Single-Loop Circuit
Checkpoint 1



Draw the emf arrow

At points a, b, and c rank …

the magnitude of current

the electric potential

the electric potential energy

Solution

All tie

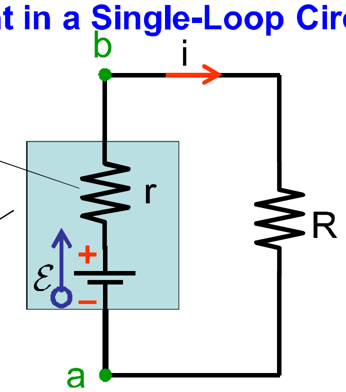
$V_b > V_a = V_c$

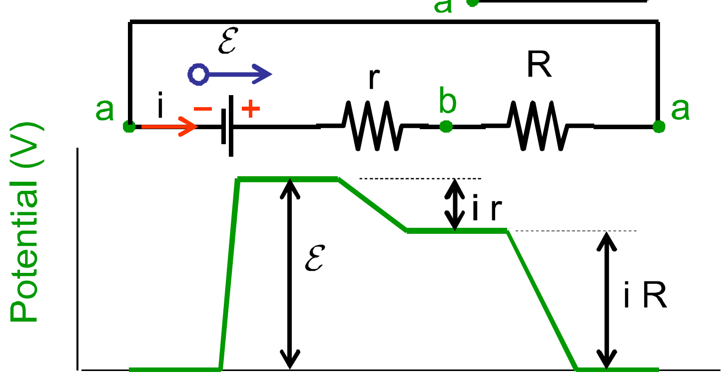
$U_b > U_a = U_c$

27-3 Calculating the Current in a Single-Loop Circuit
Real battery

Internal resistance

Real battery

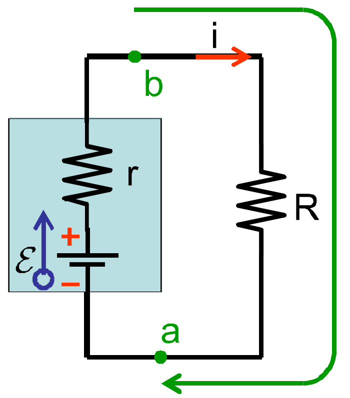




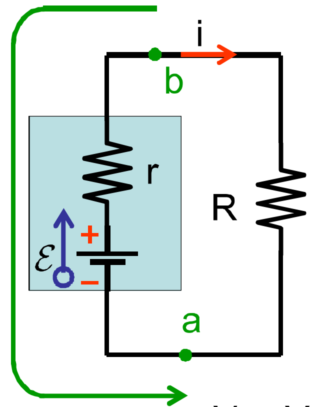
$$\mathcal{E} - i\,r - iR = 0$$

$$i = \frac{\mathcal{E}}{r + R}$$

27-4 Potential Differences
Potential difference does not depend on the path



$$V_b - iR = V_a$$
$$V_b - V_a = iR$$
$$V_b - V_a = \frac{\mathcal{E}}{r + R} R$$

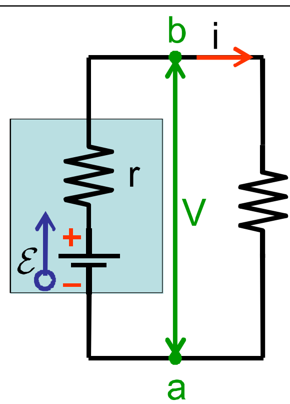


$$V_b + ir - \mathcal{E} = V_a$$
$$V_b - V_a = -ir + \mathcal{E}$$
$$= -\frac{\mathcal{E}}{r + R} r + \mathcal{E}$$
$$V_b - V_a = \frac{-\mathcal{E}r + \mathcal{E}r + \mathcal{E}R}{r + R}$$
$$V_b - V_a = \frac{\mathcal{E}}{r + R} R$$

The potential difference between any two points in a circuit does not depend on the path

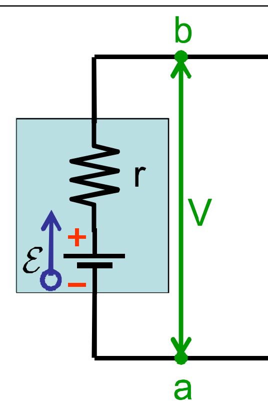
27-4 Potential Differences
Potential difference across real battery

$\mathcal{E}$ = 12 V	r = 2 Ω	R = 10 Ω



connected to a circuit

$$i = \frac{\mathcal{E}}{r + R} = \frac{12}{2 + 10} = 1\ \text{A}$$
$$V = \mathcal{E} - ir$$
$$= 12 - 1\,(2) = 10\ \text{V} < \mathcal{E}$$

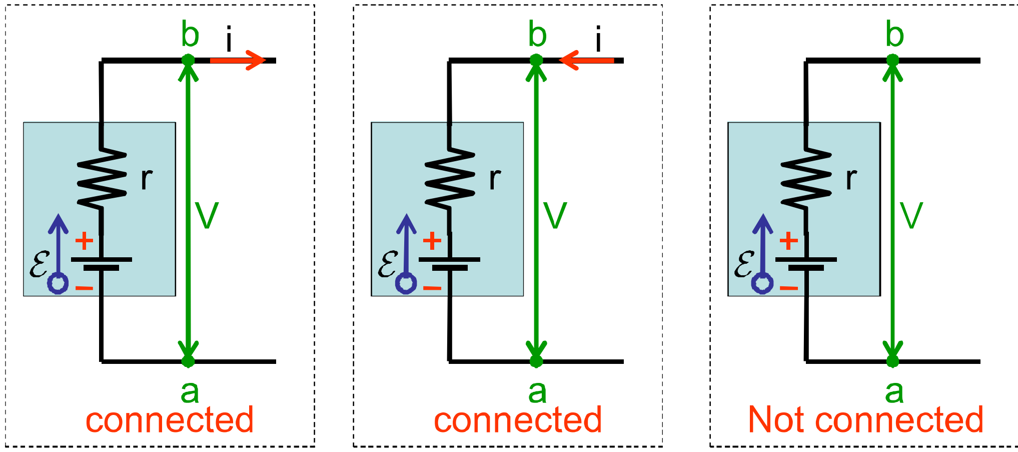


Not connected to a circuit

$$i = 0$$
$$V = \mathcal{E} - ir = \mathcal{E} = 12\ \text{V}$$

27-4 Potential Differences
Checkpoint 2

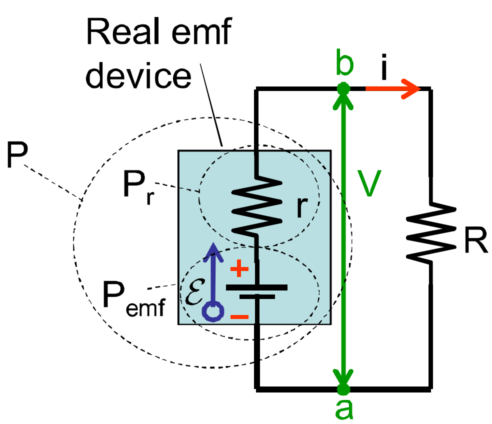
Compare $\mathcal{E}$ and V



Solution

$V = +\mathcal{E} - i\,r$	$V = +\mathcal{E} + i\,r$	$V = +\mathcal{E} + 0(r)$
$V < \mathcal{E}$	$V > \mathcal{E}$	$V = \mathcal{E}$

27-4 Potential Differences
Power from real emf devices



Rate of energy transfer from the real emf device to the charge carriers

Electrons

$$P = i\,V$$
$$P = i\,(\mathcal{E} - i\,r)$$
$$P = i\,\mathcal{E} - i^2\,r$$
$$P = P_{emf} - P_r$$

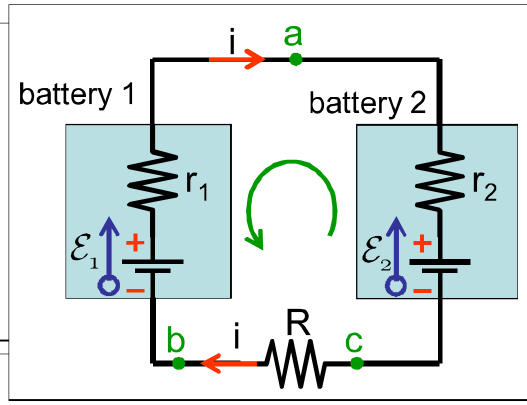
Rate of energy transfer form the emf device to the charge carriers and to internal thermal energy

Rate of energy transfer to internal thermal energy

27-4 Potential Differences
Example 1

$\mathcal{E}_1 = 4.4$ V, $\mathcal{E}_2 = 2.1$ V,
$r_1 = 2.3\ \Omega$, $r_2 = 1.8\ \Omega$, $R = 5.5\ \Omega$.

What is the current in the circuit?



Solution

Pick any direction for the current.

Pick any direction for the loop rule.

If your guess is wrong, your current will be negative.

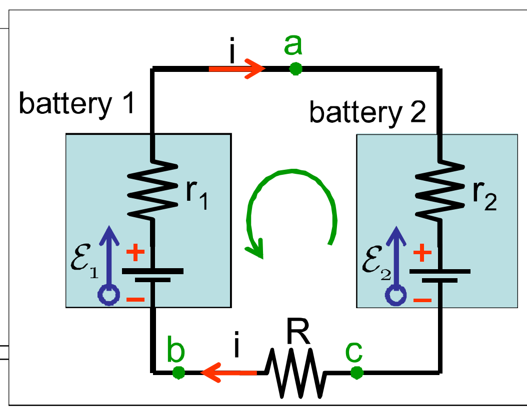
$$i\,r_1 - \mathcal{E}_1 + i\,R + \mathcal{E}_2 + i\,r_2 = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + R + r_2} = 0.240 \text{ A} = 240 \text{ mA}$$

27-4 Potential Differences
Example 2

$\mathcal{E}_1 = 4.4$ V, $\mathcal{E}_2 = 2.1$ V,
$r_1 = 2.3\ \Omega$, $r_2 = 1.8\ \Omega$, $R = 5.5\ \Omega$,
$i = 240$ mA.

What is the potential difference between the terminals of battery 1?



Solution

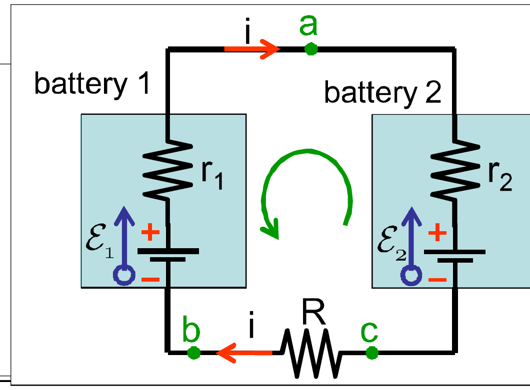
$$V_a + i\,r_1 - \mathcal{E}_1 = V_b$$

$$V_a - V_b = -\,i\,r_1 + \mathcal{E}_1 = 3.84 \text{ V}$$

27-4 Potential Differences
Example 3

$\mathcal{E}_1$ = 4.4 V, $\mathcal{E}_2$= 2.1 V,
r_1 = 2.3 Ω, r_2 = 1.8 Ω, R = 5.5 Ω,
i = 240 mA.

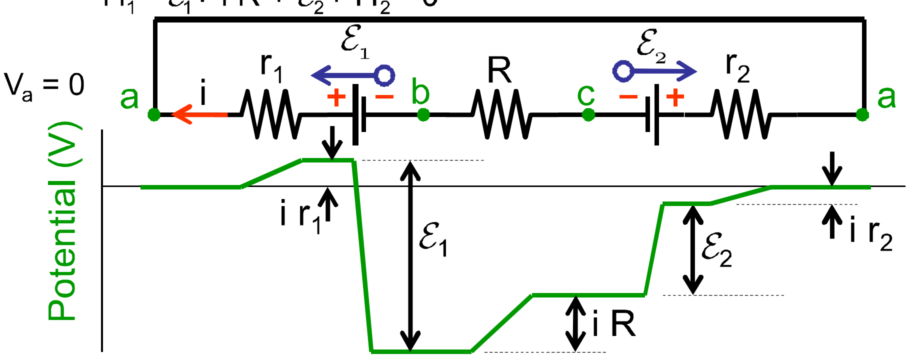
Redraw the circuit such that all the components are located on a horizontal line then plot the potential as a function of the location on the line.



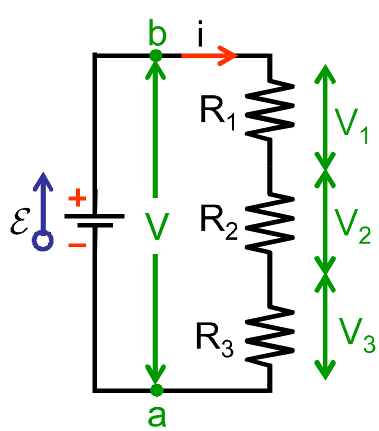
Solution

$$i\,r_1 - \mathcal{E}_1 + i\,R + \mathcal{E}_2 + i\,r_2 = 0$$

Let $V_a = 0$



27-5 Resistances in Series and in Parallel
Resistances in series



Resistances in series

Resistances are wired one after another and a potential difference is applied to the two ends of the series.

The applied potential difference V is equal to the sum of the potential differences across all the resistances.

$$V = V_1+V_2+V_3$$

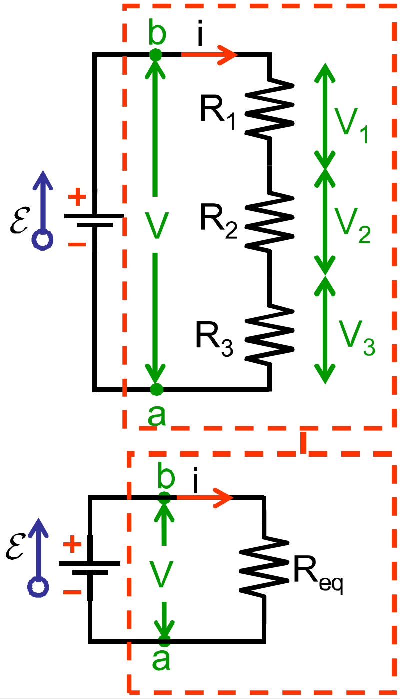
For steady flow of charge

Since the charge is conserved, all the resistances have the same current i.

$$i = i_1= i_2 = i_3$$

27-5 Resistances in Series and in Parallel

Formula - Resistances in series



The equivalent resistance R_{eq} has the same current

$i = i_1 = i_2 = i_3$

and the same total potential difference

$V = V_1+V_2+V_3$

as the actual resistances.

Equivalent resistance

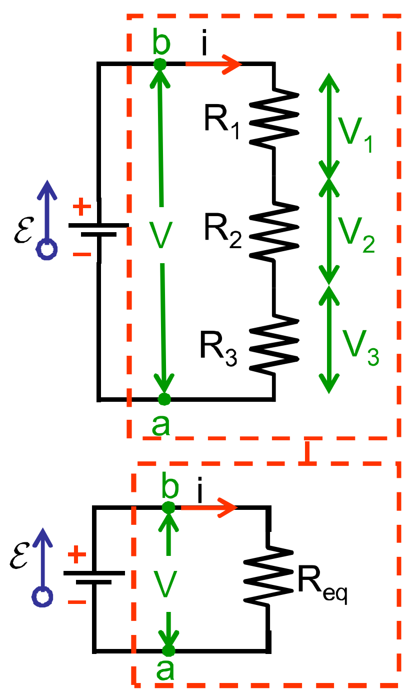
$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

$$\boxed{R_{eq} = \sum_{j=1}^{n} R_j}$$

For n resistances in series

27-5 Resistances in Series and in Parallel

Derivation - Resistances in series



Derivation of $\boxed{R_{eq} = R_1 + R_2 + R_3}$

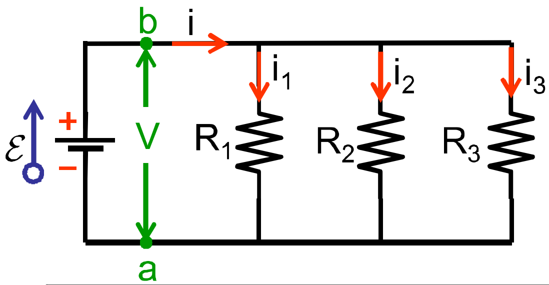
$$V = V_1 + V_2 + V_3$$

$$i R_{eq} = i R_1 + i R_2 + i R_3$$

$$R_{eq} = R_1 + R_2 + R_3$$

27-5 Resistances in Series and in Parallel

Resistances in parallel



Resistances in parallel

Resistances are directly wired together on one side and directly wired together on the other side and a potential difference is applied across the pair of connected sides.

All the resistances have the same potential difference.

$$V = V_1 = V_2 = V_3$$

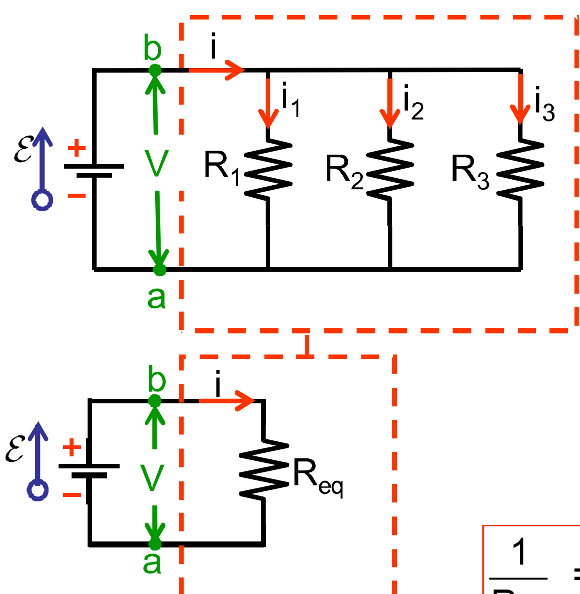
Since the charge is conserved, the total current passing through the resistances is equal to the sum of the current passing through each resistance.

$$i = i_1 + i_2 + i_3$$

For steady flow of charge

27-5 Resistances in Series and in Parallel

Formula - Resistances in parallel



The equivalent resistance R_{eq} has the same total current

$i = i_1 + i_2 + i_3$

and the same potential difference

$V = V_1 = V_2 = V_3$

as the actual resistances

Equivalent resistance

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

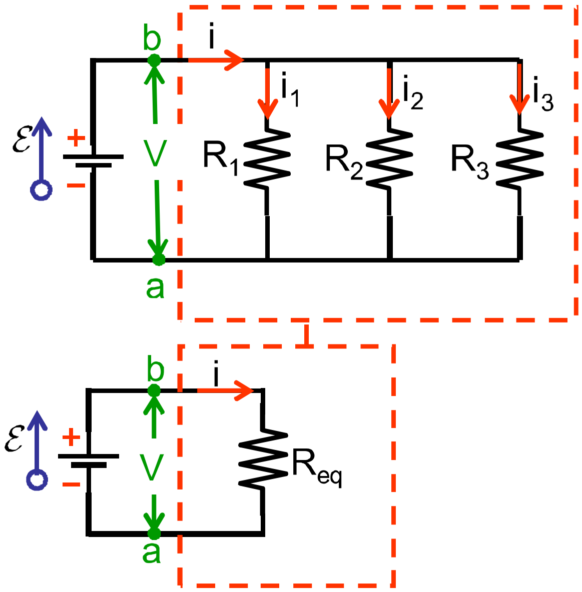
$$\boxed{\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}}$$

For n resistances in parallel

R_{eq} is smaller than any of the actual resistances.

27-5 Resistances in Series and in Parallel
Derivation - Resistances in parallel

Derivation of $\boxed{\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

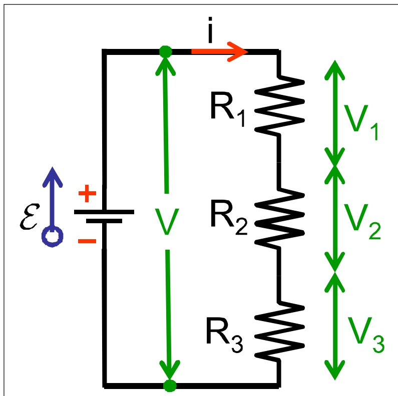


$$i = i_1 + i_2 + i_3$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

27-5 Resistances in Series and in Parallel
Comparison

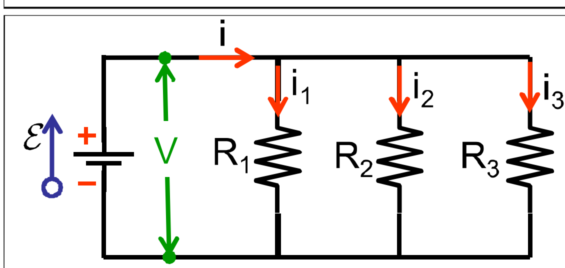


Resistances in series

$i = i_1 = i_2 = i_3$

$V = V_1 + V_2 + V_3$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$



Resistances in parallel

$i = i_1 + i_2 + i_3$

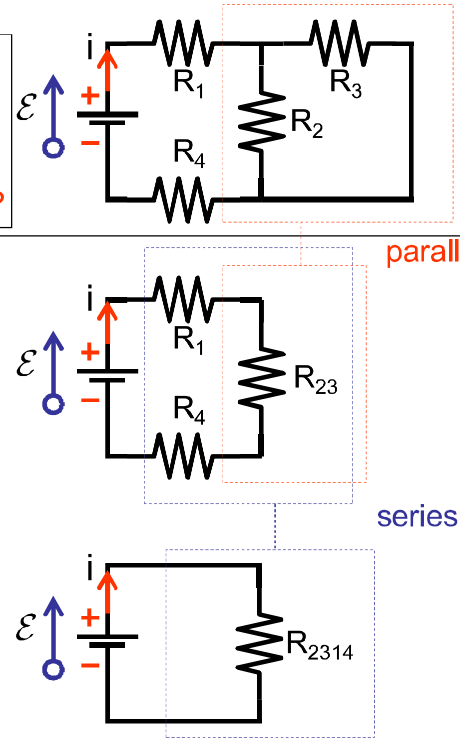
$V = V_1 = V_2 = V_3$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

27-5 Resistances in Series and in Parallel
Example 4

$\mathcal{E} = 12$ V,
$R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$,
$R_3 = 30\ \Omega$, $R_4 = 8.0\ \Omega$,

What is the current through the battery?



Solution

Simplify the circuit

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 12\ \Omega$$

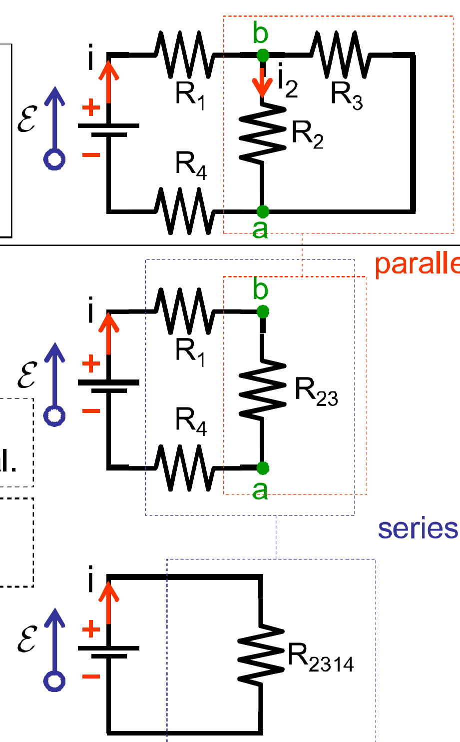
$$R_{2314} = R_1 + R_4 + R_{23} = 40\ \Omega$$

$$i = \frac{\mathcal{E}}{R_{2314}} = 0.3\text{ A}$$

27-5 Resistances in Series and in Parallel
Example 5

$\mathcal{E} = 12$ V,
$R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$,
$R_3 = 30\ \Omega$, $R_4 = 8.0\ \Omega$,
$R_{23} = 12\ \Omega$, $i = 0.3$ A

What is the current i_2 through R_2?



Solution

$$i_2 = \frac{V_{ba}}{R_2}$$

R_2 and R_3 are in parallel, potential differences across R_2 and R_{23} are equal.

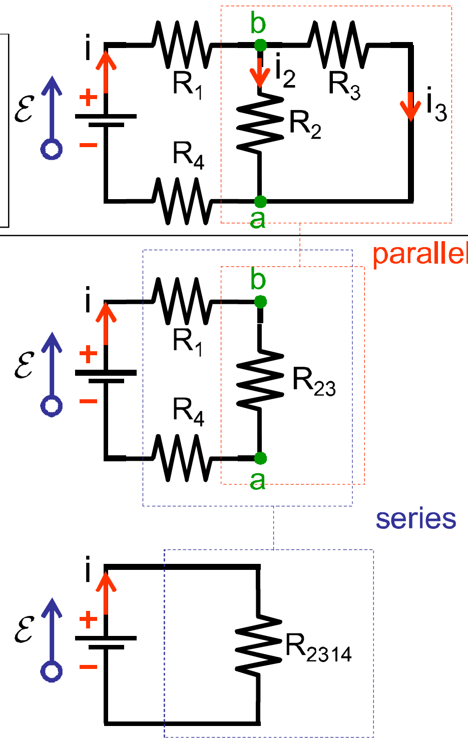
R_1, R_{23} and R_4 are in series, currents thorough R_{2314} and R_{23} are equal.

$$V_{ba} = R_{23}\, i = 3.6\text{ V}$$

$$i_2 = \frac{V_{ba}}{R_2} = 0.18\text{ A}$$

27-5 Resistances in Series and in Parallel
Example 6

$\mathcal{E}$= 12 V,
R_1 = 20 Ω, R_2 = 20 Ω,
R_3 = 30 Ω, R_4 = 8.0 Ω,
R_{23} = 12 Ω, i = 0.3 A, i = 0.18 A, V_{ba} = 3.6 A
What is the current i_3 through R_3?



Solution

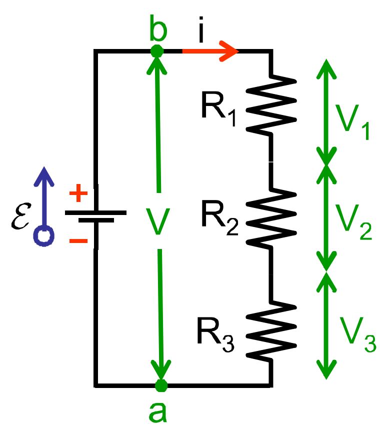
$$i_3 = \frac{V_{ba}}{R_3} = 0.12\text{ A}$$

Another way

$i = i_2 + i_3$

$i_3 = i - i_2 = 0.12$ A

27-5 Resistances in Series and in Parallel
Checkpoint 3



Let $R_1 > R_2 > R_3$

Rank …

the current through the resistances

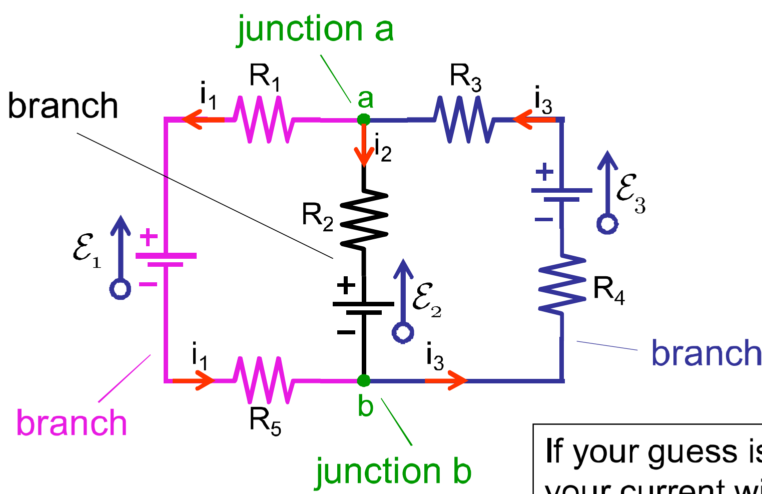
the potential difference across them

Solution

All tie

$R_1 > R_2 > R_3$
$iR_1 > iR_2 > iR_3$
$V_1 > V_2 > V_3$

27-6 Multiloop Circuits
Definitions

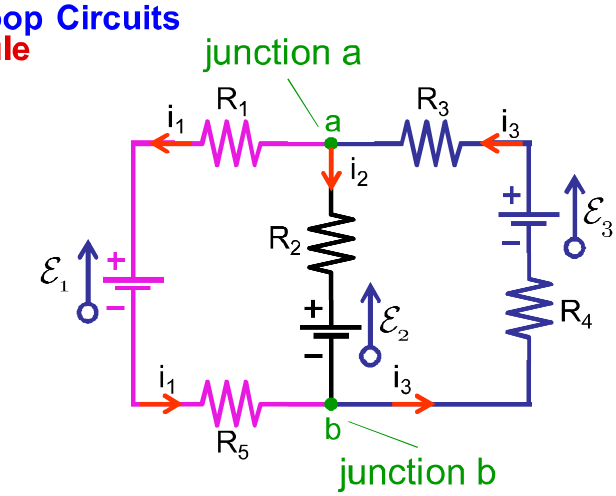


Pick any direction for the currents

If your guess is wrong, your current will be negative. The actual current is moving opposite to your guess.

Any point in a branch has the same current.

27-6 Multiloop Circuits
Junction rule

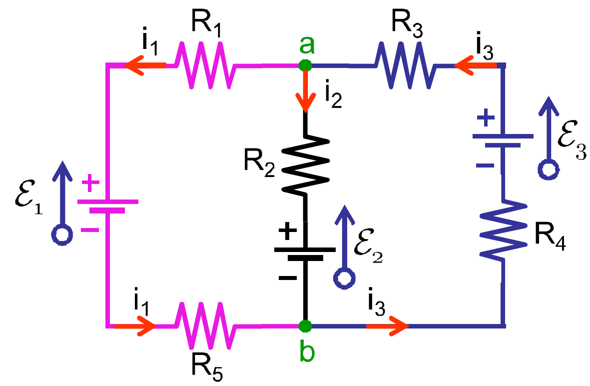


Junction rule (Kirchhoff's junction rule):
The sum of the currents entering any junction must be equal to the sum or the currents leaving that junction.

$$\sum_{in} i = \sum_{out} i$$

Conservation of charge for steady flow of charge

27-6 Multiloop Circuits
Unknowns and independent equations



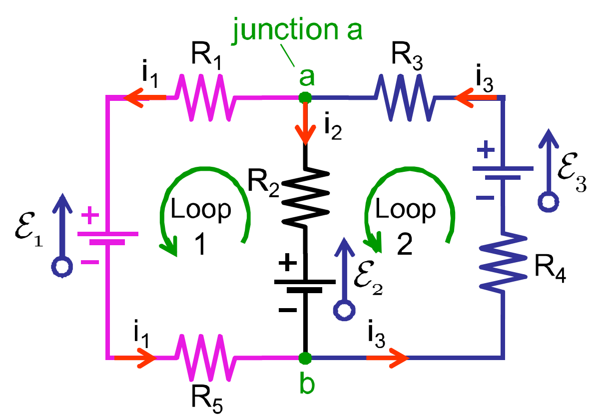
To analyze a circuit, we need to find as many independent equations as the number of unknowns in the circuit.

Suppose R's and $\mathcal{E}$'s are given. we need to find i_1, i_2, and i_3

Number of unknowns = 3

Number of independent equations = 3

27-6 Multiloop Circuits
Illustration - Unknowns and independent equations



Junction rule at junction a $\qquad i_3 = i_1 + i_2$

Loop rule on loop 1 $\qquad - i_1R_1 - \mathcal{E}_1 - i_1R_5 + \mathcal{E}_2 + i_2R_2 = 0$

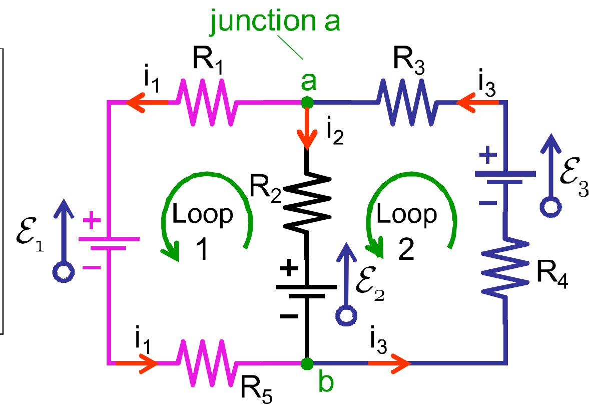
Loop rule on loop 2 $\qquad - i_2R_2 - \mathcal{E}_2 - i_3R_4 + \mathcal{E}_3 - i_3R_3 = 0$

We have three unknowns and three independent equations

27-6 Multiloop Circuits
Example 7

$\mathcal{E}_1 = 3.0$ V,
$\mathcal{E}_2 = \mathcal{E}_3 = 6.0$ V,
$R_1 = R_3 = R_4 = R_5 = 2.0\ \Omega$,
$R_2 = 4.0\ \Omega$.

Find the magnitude and the direction of the current in each branch.



Solution

Junction rule at junction a: $i_3 = i_1 + i_2$

Loop rule on loop 1: $-i_1R_1 - \mathcal{E}_1 - i_1R_5 + \mathcal{E}_2 + i_2R_2 = 0$

$-4\,i_1 + 3 + 4\,i_2 = 0$

Loop rule on loop 2: $-i_2R_2 - \mathcal{E}_2 - i_3R_4 + \mathcal{E}_3 - i_3R_3 = 0$

$-4\,i_2 - 4\,i_3 = 0$

27-6 Multiloop Circuits
Example 7

Solution

$i_3 = i_1 + i_2$

$-4\,i_1 + 3 + 4\,i_2 = 0$

$-4\,i_2 - 4\,i_3 = 0$

Eliminate one variable, say i_3

$-4\,i_1 + 3 + 4\,i_2 = 0$
$-4\,i_2 - 4\,(i_1 + i_2) = 0$

$-4\,i_1 + 3 + 4\,i_2 = 0$
$-8\,i_2 - 4\,i_1 = 0$

$-4\,i_1 + 3 + 4\,i_2 = 0$
$i_2 = -\frac{i_1}{2}$

Eliminate another variable, say i_2

$-4\,i_1 + 3 + 4\left(-\frac{i_1}{2}\right) = 0$

$-6\,i_1 + 3 = 0$

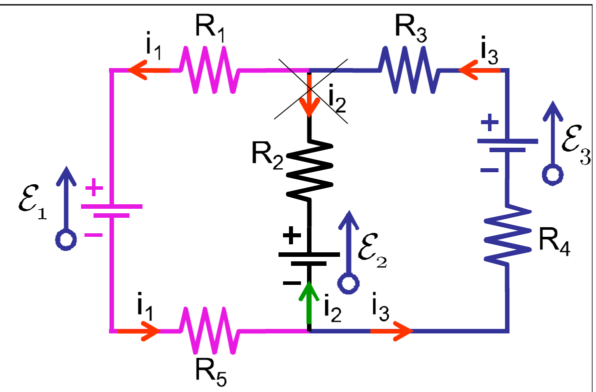
$i_1 = 0.5$ A

$i_2 = -0.25$ A

$i_3 = 0.25$ A

27-6 Multiloop Circuits
Example 7

Solution



$i_1 = 0.5$ A

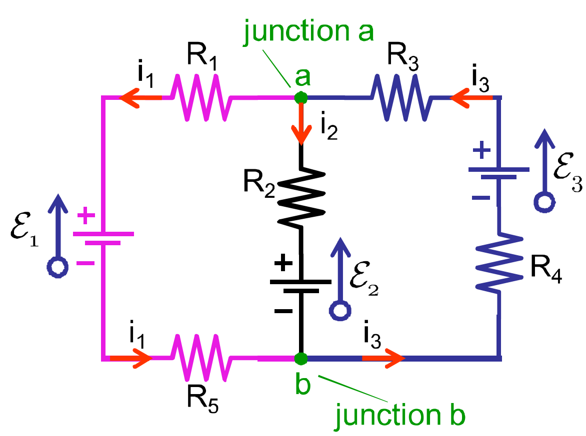
$i_2 = -0.25$ A

$i_3 = 0.25$ A

Our guess for the direction of i_2 is wrong, it should be in the opposite direction.

Only after you finish finding all currents in a circuit, you can correct your guesses about directions.

27-6 Multiloop Circuits
Illustration - Dependent equation from a junction

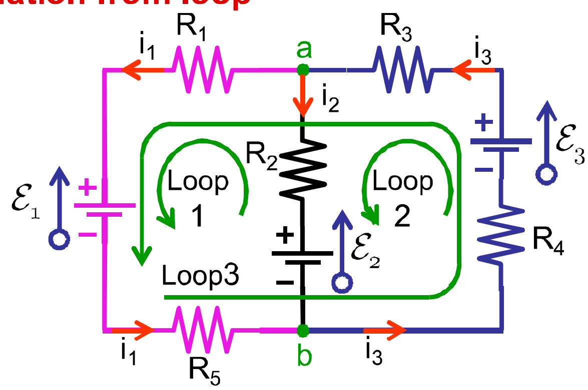


Junction rule at junction a $\quad i_3 = i_1 + i_2$

Junction rule at junction b $\quad i_1 + i_2 = i_3$

Same.
No new information.
These are not independent equations

27-6 Multiloop Circuits
Illustration - Dependent equation from loop



Loop rule on loop 1 $\quad -i_1R_1 - \mathcal{E}_1 - i_1R_5 + \mathcal{E}_2 + i_2R_2 = 0$

Loop rule on loop 2 $\quad -i_2R_2 - \mathcal{E}_2 - i_3R_4 + \mathcal{E}_3 - i_3R_3 = 0$

Add the two equations

$$-i_1R_1 - \mathcal{E}_1 - i_1R_5 - i_3R_4 + \mathcal{E}_3 - i_3R_3 = 0$$

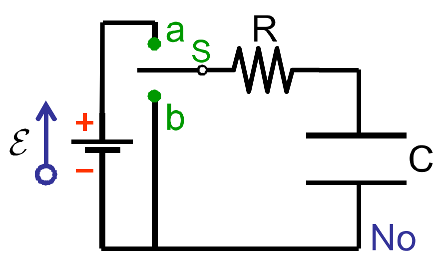
Loop rule on loop 3

Same as loop1 + loop 2.
No new information.

You can only use two of the three loops.

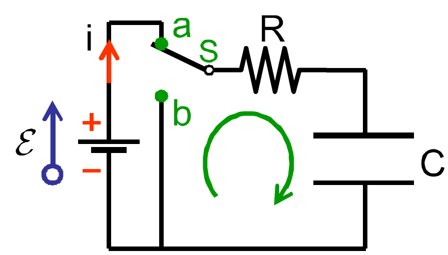
27-7 RC Circuits
Charging a capacitor

Charging a capacitor



No charge

initial



At t = 0, switch to position a

What is the charge on the capacitor as a function of time?

Loop rule

$$\mathcal{E} - iR - \frac{q}{C} = 0$$

$$\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0 \quad \text{Differential equation}$$

At t = 0,	$q = 0$ and $i = \frac{\mathcal{E}}{R}$
At t = ∞,	$q = C\mathcal{E}$ and $i = 0$

27-7 RC Circuits
Formula - Charge on a capacitor while charging

Differential equation

$$\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0$$

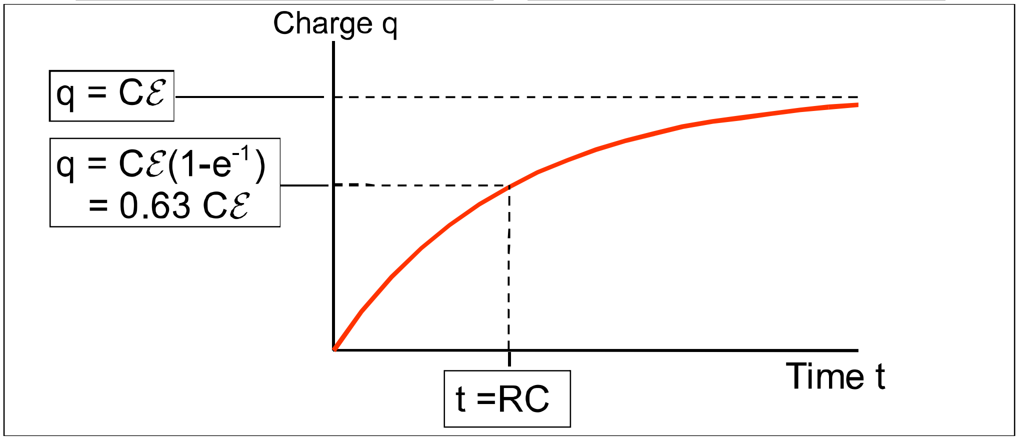
At $t = 0$, $q = 0$

At $t = \infty$, $q = C\mathcal{E}$

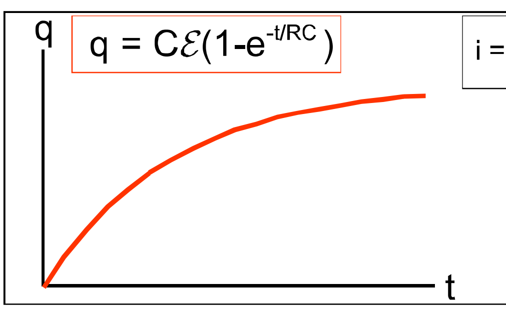
Solution

$$q = C\mathcal{E}(1-e^{-t/RC})$$

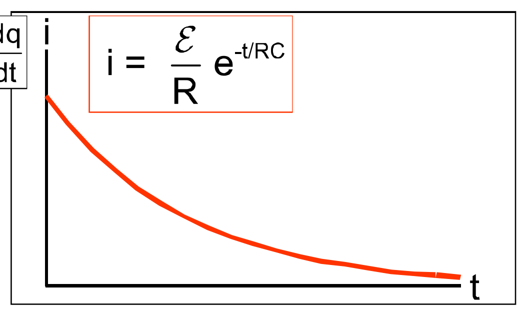
While charging the capacitor

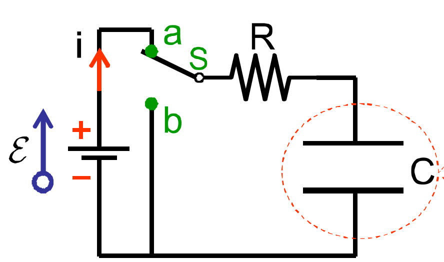


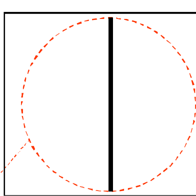
27-7 RC Circuits
Formula - Current through a capacitor while charging



$i = \frac{dq}{dt}$

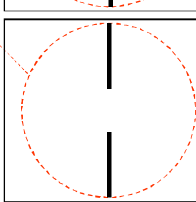






$t = 0$, $i = \frac{\mathcal{E}}{R}$

Initially, the capacitor acts as a connecting wire.

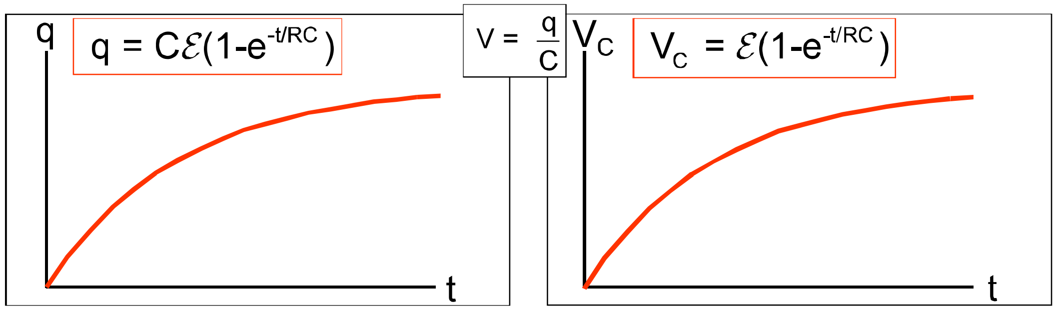


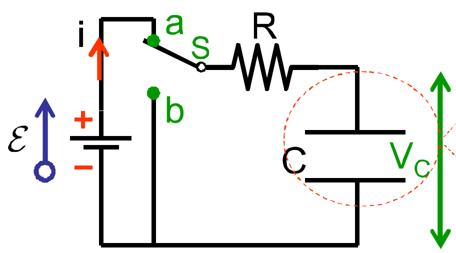
$t = \infty$, $i = 0$

finally, the capacitor acts as a broken wire.

27-7 RC Circuits
Formula - Potential difference across a capacitor while charging

$q = C\mathcal{E}(1-e^{-t/RC})$ $\quad V = \frac{q}{C} \quad$ $V_C = \mathcal{E}(1-e^{-t/RC})$





$t = 0, \quad V_C = 0$

Initially, the capacitor acts as a connecting wire.

$t = \infty, \quad V_C = \mathcal{E}$

finally, the capacitor acts as a broken wire.

27-7 RC Circuits
Time constant

RC has dimension of time

RC = τ

RC = capacitive time constant

RC

Dimension similar to

$$\frac{V}{i}\frac{Q}{V} = \frac{Q}{i} = \frac{Q}{\frac{Q}{t}} = t$$

$$q = C\mathcal{E}(1-e^{-t/RC}) = C\mathcal{E}(1-e^{-t/\tau})$$

After one time constant

$$q = C\mathcal{E}(1-e^{-1}) = 0.63\, C\mathcal{E}$$

$$i = \frac{\mathcal{E}}{R} e^{-1} = 0.37 \frac{\mathcal{E}}{R}$$

$$V_C = \mathcal{E}(1-e^{-1}) = 0.63\, \mathcal{E}$$

τ pronounced tau

27-7 RC Circuits
Derivation - Charge on a capacitor while charging

Show that $\boxed{q = C\mathcal{E}(1-e^{-t/RC})}$ is a solution of $\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0$

At t = 0, q = 0

At t = ∞, q = $C\mathcal{E}$

Check the solution

At t = 0, $q = C\mathcal{E}(1-e^{0/RC}) = C\mathcal{E}(1-1) = 0$

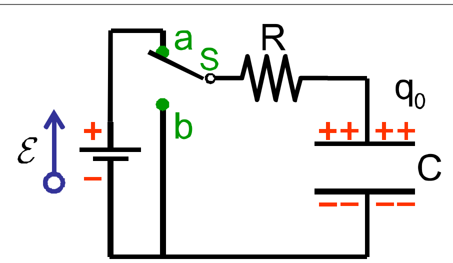
At t = ∞, $q = C\mathcal{E}(1-e^{-\infty/RC}) = C\mathcal{E}(1-0) = C\mathcal{E}$

$$\boxed{\frac{dq}{dt} = C\mathcal{E}\left(\frac{1}{RC}e^{-t/RC}\right) = \frac{\mathcal{E}}{R}e^{-t/RC}}$$

$$\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = \mathcal{E} - \frac{\mathcal{E}}{R}e^{-t/RC}R - \frac{C\mathcal{E}(1-e^{-t/RC})}{C}$$

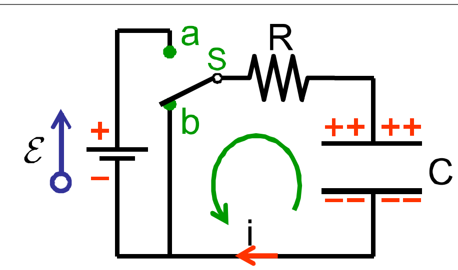
$$= \mathcal{E} - \mathcal{E}e^{-t/RC} - \mathcal{E}(1-e^{-t/RC}) = 0$$

27-7 RC Circuits
Discharging a capacitor



initial

Discharging a capacitor



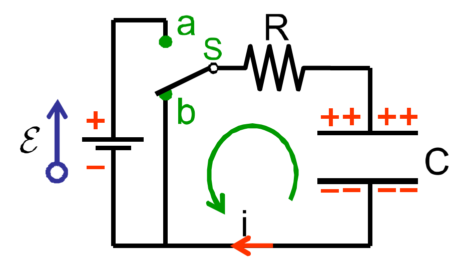
At t = 0, switch to position b

What is the charge on the capacitor as a function of time?

$$\frac{q}{C} + iR = 0$$

$$\frac{q}{C} + \frac{dq}{dt}R = 0 \qquad \text{Differential equation}$$

27-7 RC Circuits
Formula - Charge on a capacitor while discharging



At t = 0, switch to position b

Differential equation

$$\frac{dq}{dt}R + \frac{q}{C} = 0$$

At $t = 0$, $q = q_0$

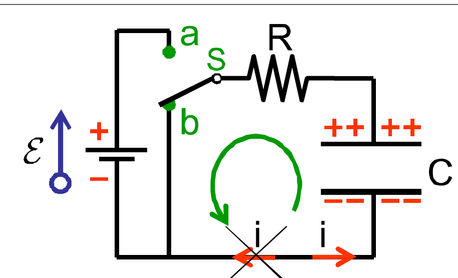
At $t = \infty$, $q = 0$

Solution

$$\boxed{q = q_0 e^{-t/RC}}$$

While discharging the capacitor

27-7 RC Circuits
Formula - Current through a capacitor while discharging



Discharging a capacitor

$$\boxed{q = q_0 e^{-t/RC}}$$

$$i = \frac{dq}{dt}$$

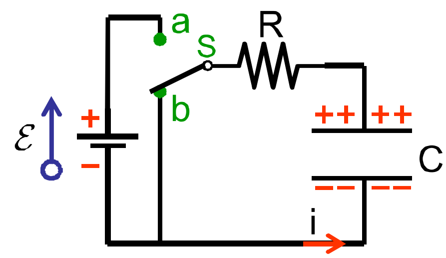
$$\boxed{i = -\frac{q_0}{RC} e^{-t/RC}}$$

Our guess for the direction of the current is wrong

27-7 RC Circuits
Discharging after one time constant

$$q = q_0 e^{-t/RC}$$

$$i = \frac{q_0}{RC} e^{-t/RC}$$



Discharging a capacitor

After one time constant

$$q = q_0 e^{-1} = 0.37\, q_0$$

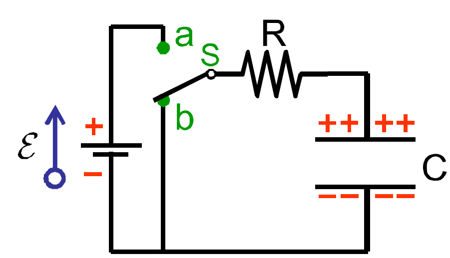
$$i = -\frac{q_0}{RC} e^{-1} = 0.37 \frac{q_0}{RC}$$

$$V_C = \frac{q_0}{C} e^{-1} = 0.37 \frac{q_0}{C}$$

27-7 RC Circuits
Example 8

In terms of RC, when will the charge on the capacitor be half its initial value?

Discharging a capacitor



At t = 0, switch to position b

Solution

$$q = q_0 e^{-t/RC}$$

$$\frac{1}{2} q_0 = q_0 e^{-t/RC}$$

$$\frac{1}{2} = e^{-t/RC}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-t/RC})$$

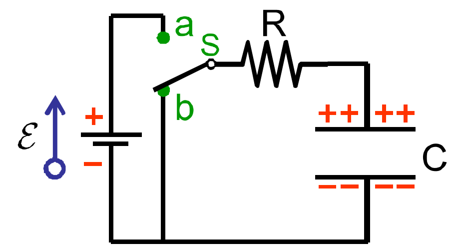
$$\ln(2) = \frac{t}{RC}$$

$$t = \ln(2)RC = 0.69\ RC = 0.69\ \tau$$

27-7 RC Circuits
Example 9

In terms of RC, when will the energy stored in the capacitor be half its initial value?

Discharging a capacitor



At t = 0, switch to position b

Solution

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C}e^{-2t/RC} = U_0 e^{-2t/RC}$$

$$\frac{1}{2}U_0 = U_0 e^{-2t/RC}$$

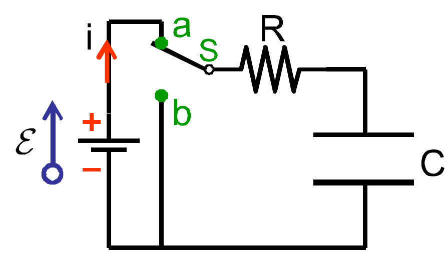
$$\frac{1}{2} = e^{-2t/RC}$$

$$\ln(2) = \frac{2t}{RC}$$

$$t = \frac{\ln(2)}{2}RC = 0.35\ RC = 0.35\ \tau$$

27-7 RC Circuits
Checkpoint 4

Charging a capacitor



At t = 0, switch to position a

Rank the sets according to …
A - initial current
B - time required for the current to decrease to its half its initial value

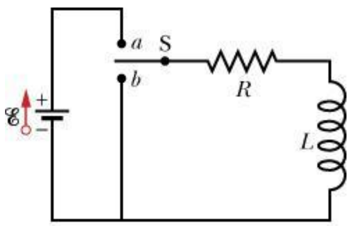
$\mathcal{E}$(V)	R(Ω)	C(μF)	A	B
12	2	3	1	2
12	3	2	2	2
10	10	0.5	4	4
10	5	2	3	1

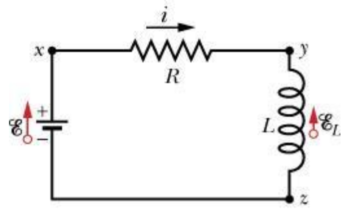
Solution

$\boxed{i = \frac{\mathcal{E}}{R}}$ $\boxed{t = 0.69\ RC}$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R}$$

30 - 9 RL Circuits





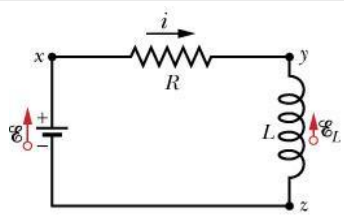
Consider the circuit in the upper figure with the switch S in the middle position. At $t = 0$ the switch is thrown in position a and the equivalent circuit is shown in the lower figure. It contains a battery with emf $\mathcal{E}$, connected in series to a resistor R and an inductor L (thus the name "RL circuit"). Our objective is to calculate the current i as a function of time t. We write Kirchhoff's loop rule starting at point x and moving around the loop in the clockwise direction:

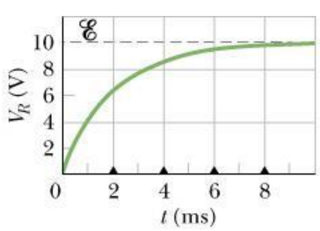
$$-iR - L\frac{di}{dt} + \mathcal{E} = 0 \rightarrow \quad L\frac{di}{dt} + iR = \mathcal{E}$$

The initial condition for this problem is $i(0) = 0$. The solution of the differential equation that satisfies the initial condition is

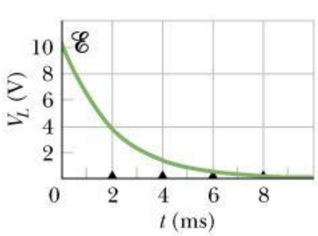
$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$. The constant $\tau = \frac{L}{R}$ is known as the "***time constant***" of the RL circuit.

(30–18)





(a)



(b)

$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$ Here $\tau = \frac{L}{R}$.

The voltage across the resistor $V_R = iR = \mathcal{E}\left(1 - e^{-t/\tau}\right)$.

The voltage across the inductor $V_L = L\frac{di}{dt} = \mathcal{E}e^{-t/\tau}$.

The solution gives $i = 0$ at $t = 0$ as required by the initial condition. The solution gives $i(\infty) = \mathcal{E}/R$.

The circuit time constant $\tau = L/R$ tells us how fast the current approaches its terminal value:

$i(t = \tau) = (0.632)(\mathcal{E}/R)$

$i(t = 3\tau) = (0.950)(\mathcal{E}/R)$

$i(t = 5\tau) = (0.993)(\mathcal{E}/R)$

If we wait only **a few time constants** the current, for all practical purposes, has reached its terminal value $(\mathcal{E}/R)$.

(30–19)