Chapter 26
Current and Resistance

26-1 Moving Charges and Electric Currents
26-2 Electric Currents
26-3 Current density
26-4 Resistance and Resistivity
26-5 Ohm's Law
26-6 Power in Electric Circuits

Objective

26-1 Moving Charges and Electric Currents
Charges in motion

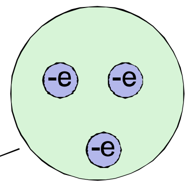
Electrostatics → Charges do not move

Electric current → Charges move

26-1 Moving Charges and Electric Currents
Electric current

Electric current through a surface
is
the flow rate of net charges through the surface.

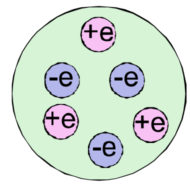
3 electrons moving towards us



Area A

Current ≠ 0

3 electrons and 3 protons moving towards us



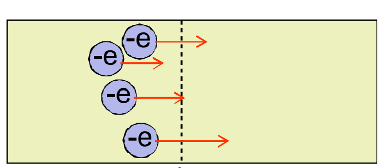
Current = 0

26-1 Moving Charges and Electric Currents
Random motion does not produce current

Electric current through a surface
is
the flow rate of net charges through the surface.

Cross section

4 electrons moving to the right

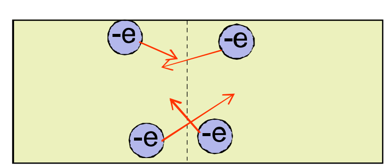


Area A

Current ≠ 0

Cross section

4 electrons moving randomly in all directions



No net transport of charge

Current = 0

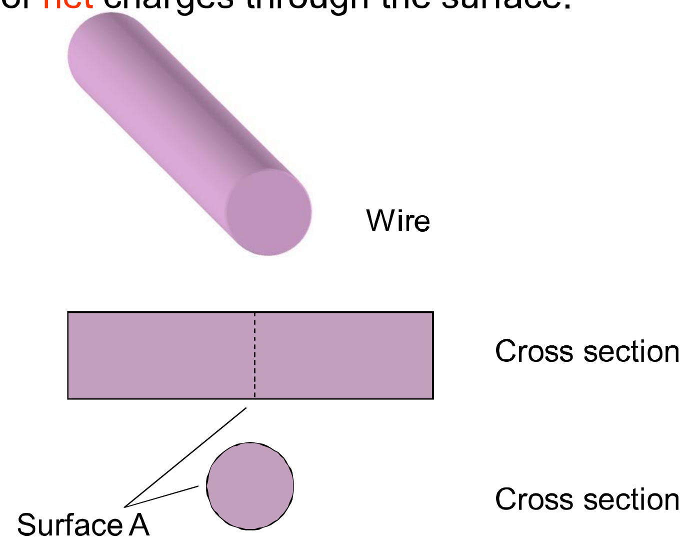
Electrons in isolated conductor
(no external electric field applied)
Random motion - Speed ≈ 10^6 m/s

26-2 Electric Currents
Formula - Electric current

Electric current through a surface
is
the flow rate of net charges through the surface.

Charge dq passes
through surface A
in time dt

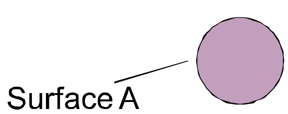
$$i = \frac{dq}{dt}$$



26-2 Electric Currents
Steady-state electric current

$$i = \frac{dq}{dt}$$

Total charge q passes through surface A in time t

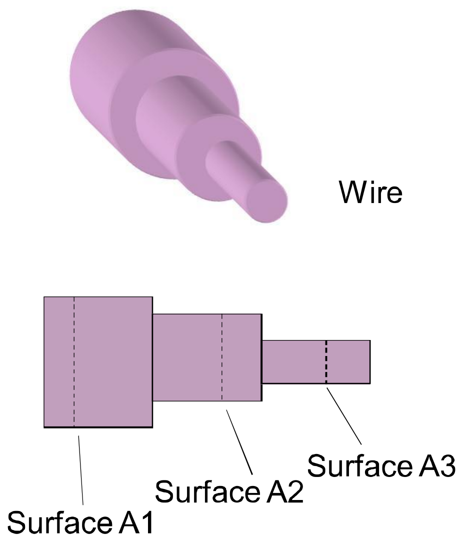


$$q = \int_0^t i \, dt$$

In a steady-state condition,
current does not change with time.

$$q = i\,t$$

26-2 Electric Currents
Electric current through a wire



In a steady-state condition

Current through A1
= Current through A2
= Current through A3

Since charge is conserved,
any electron passes through A1 should pass through A2 and A3.

26-2 Electric Currents
SI unit for electric current

SI unit for current

Ampere

Symbol

A

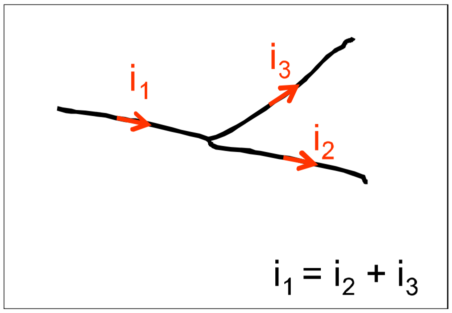
$$i = \frac{dq}{dt}$$

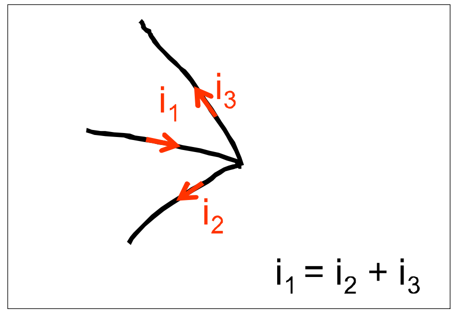
$$1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ Second}}$$

26-2 Electric Currents
Current is a scalar quantity

Current is a scalar quantity

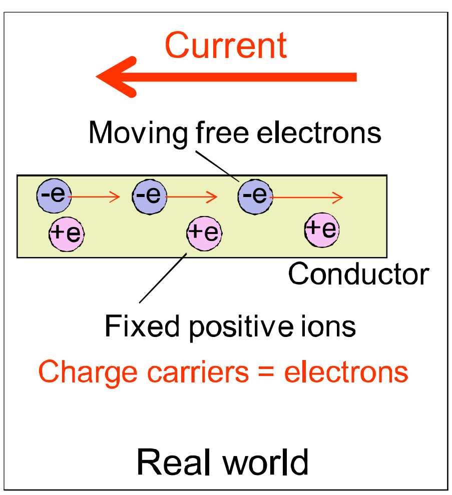
We use arrows to indicate directions of currents in wires.

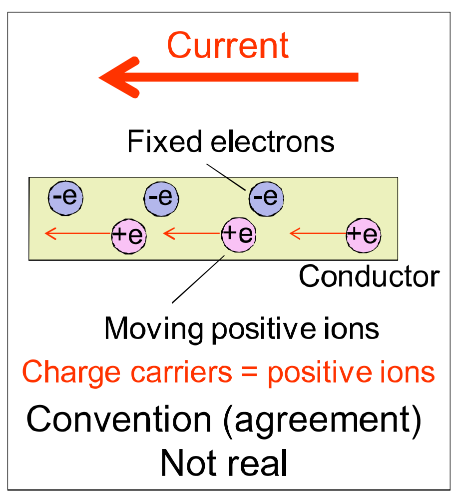




These arrows are not vectors.

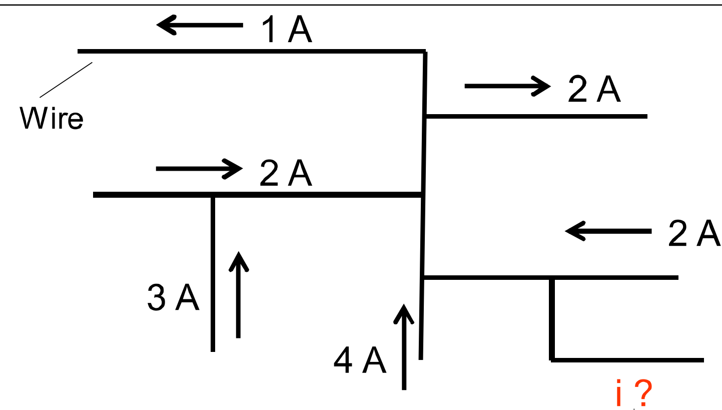
26-2 Electric Currents
Convention - current direction





A current arrow is drawn in the direction in which positive charge would move, even if the actual charge carriers (electrons) are negative and move in the opposite direction.

26-2 Electric Currents
Checkpoint 1



Solution

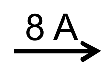
Charge is conserved

current in = current out

Given current in = 3 + 2+ 4 + 2 = 11 A

Given current out = 1 + 2 = 3 A

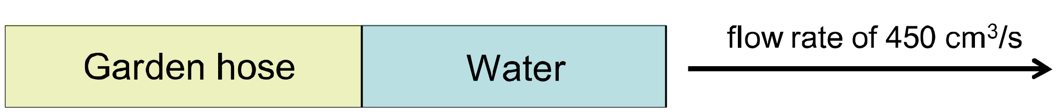
i = 8 A out



26-2 Electric Currents
Example 1

Water flows from a garden hose at a rate of 450 cm^3/s

What is the current of negative charge?



Solution

$$i = \frac{dq}{dt} = \frac{\text{negative charge}}{\text{time}}$$

The current of negative charge = The current of positive charge

There is no net flow of charge (current = 0) through the hose.

26-2 Electric Currents

Example 1

Solution

$$i = \frac{\text{negative charge}}{\text{time}} = \frac{\text{negative charge}}{\text{volume}}\,\frac{\text{volume}}{\text{time}}$$

volume/time = 450 cm^3/s

$$\frac{\text{negative charge}}{\text{volume}} = \frac{\text{negative charge}}{\text{mass}}\,\frac{\text{mass}}{\text{volume}}$$

mass/volume = water density = 1 g/cm^3

$$\frac{\text{negative charge}}{\text{mass}} = \frac{\text{negative charge}}{\text{mole}}\,\frac{\text{moles}}{\text{mass}}$$

moles/mass = 1/molar mass = 1/(18 g/mole)

H_2O

oxygen = 8 protons + 8 neutorns

hydrogen = 1 proton

$$\frac{\text{negative charge}}{\text{mole}} = \frac{\text{negative charge}}{\text{molecule}}\,\frac{\text{molecules}}{\text{mole}}$$

molecules/mole = Avogadro's Number = $6.02\text{X}10^{23}$ Molecules/mole

$$\frac{\text{negative charge}}{\text{molecule}} = \frac{\text{negatvie charge}}{\text{electron}}\,\frac{\text{electrons}}{\text{moleucle}}$$

negatvie charge/electron = e = $1.6\text{X}10^{-19}$ C/electron

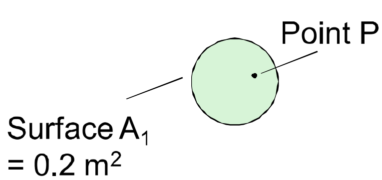
electrons/moleucle = 10 electrons/molecule = 2 from hydrogen atoms and 8 from oxygen atom

$$i = \left(\frac{1.6\text{X}10^{-19}\ \text{C}}{\text{electron}}\right)\left(\frac{10\ \text{electrons}}{\text{molecule}}\right)\left(\frac{6.02\text{X}10^{23}\ \text{molecules}}{\text{mole}}\right)\left(\frac{1\ \text{mole}}{18\ \text{g}}\right)\left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{450\ \text{cm}^3}{\text{s}}\right)$$

$= 24.1\text{X}10^{6}$ A

26-3 Current density

Definition

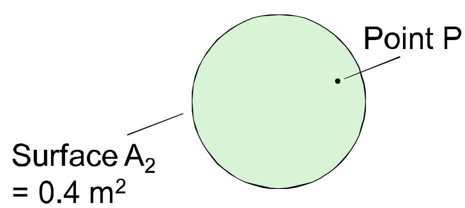


4 coulombs pass per second through surface A_1

Current through A_1 = 4 A

Current density at point P

$$= \frac{\text{current}}{\text{area}} = 20\ \text{A/m}^2$$



4 coulombs pass per second through surface A_2

Current through A_2 = 4 A

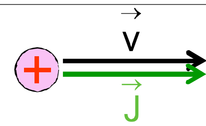
Current density at point P

$$= \frac{\text{current}}{\text{area}} = 10\ \text{A/m}^2$$

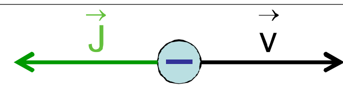
26-3 Current density
Current density is a vector quantity

Current density is a vector quantity

Direction



The same direction as the velocity of the moving positive charges.



Opposite to the direction of the velocity of the moving negative charges.

Magnitude

$$J = \frac{\text{current}}{\text{area normal to the velocity}}$$

The SI unit for the current density J is A/m^2

26-3 Current density
Finding current from current density

$$i = \int \vec{J} \cdot d\vec{A}$$

Area vector

Magnitude	Area of the surface
Direction	Normal to the surface

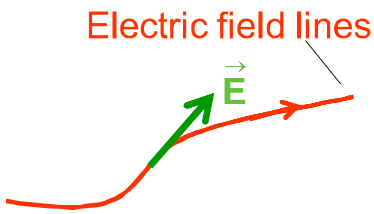
If $\vec{J}$ is uniform and parallel to $d\vec{A}$

$$i = J\,A$$

$$J = \frac{i}{A}$$

26-3 Current density
Streamlines

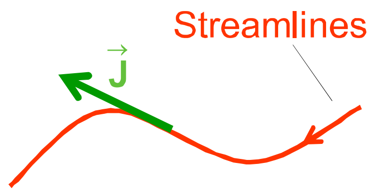
Electric Field $\vec{E}$



At any point, the tangent of an electric field line gives the direction of the electric field.

Number of lines per unit area in a plane perpendicular to the electric field lines is proportional to the magnitude of the electric field.

Current Density $\vec{J}$



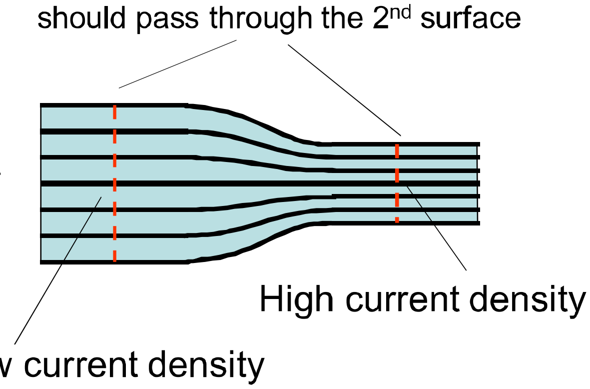
At any point, the tangent of a streamline gives the direction of the current density.

Number of lines per unit area in a plane perpendicular to the streamlines is proportional to the magnitude of the current density.

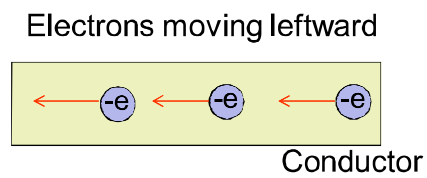
26-3 Current density
Illustration - Streamlines

Same current
Charge is conserved
(Any charge passes though the 1st surface should pass through the 2nd surface

Streamlines representing current density in the flow of charge through a conductor



26-3 Current density
Checkpoint 2



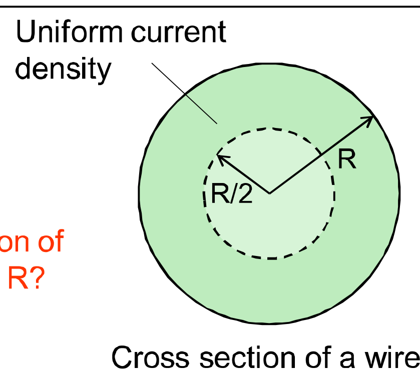
What is the direction of …

What is the direction of …	Solution
Current?	Rightward →
Current density?	Rightward →
Electric Field?	Rightward →

26-3 Current density
Example 2

R = 2.0 mm
$J = 2.0 \times 10^5\ A/m^2$

What is the current through the outer portion of the wire between radial distances R/2 and R?



Solution

$$i = J A$$

$$i = J\left(\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right)$$

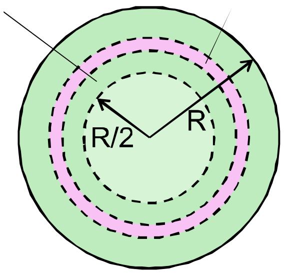
$$i = J\left(\frac{3}{4}\pi R^2\right) = 1.9\ A$$

26-3 Current density
Example 3

R = 2.0 mm
J = a r^2
a = 3.0 X 10^{11} A/m^4
r in meters

Not uniform current density

dA = 2πrdr



Cross section of a wire

What is the current through the outer portion of the wire between radial distances R/2 and R?

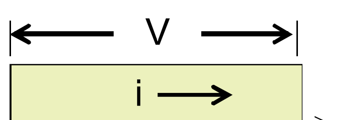
Solution

$$i = \int \vec{J} \cdot d\vec{A} = \int J\, dA = \int_{R/2}^{R} J\, 2\pi r dr = \int_{R/2}^{R} a r^2\, 2\pi r dr$$

$$= 2\pi a \int_{R/2}^{R} r^3 dr = 2\pi a \left[\frac{r^4}{4} \right]_{R/2}^{R} = 2\pi a \left(\frac{R^4}{4} - \frac{R^4}{64} \right)$$

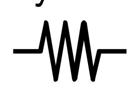
$$= \frac{15}{32} \pi a R^4 = 7.1 \text{ A}$$

26-4 Resistance and Resistivity
Formula - Potential difference and current



An object (Resistor)

Resistor symbol



$$V = R\, i$$

Potential difference — Resistance — Current

26-4 Resistance and Resistivity
SI unit for resistance

SI unit for resistance

Ohm

Symbol

Ω

$V = R\,i$

$$1\ \text{Ohm} = \frac{1\ \text{Volt}}{1\ \text{Ampere}}$$

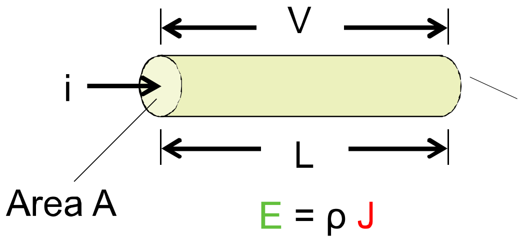
Ω is pronounced omega

26-4 Resistance and Resistivity
Resistivity

Resistance R of an object	Resistivity ρ of a material
Property of the object Depends on the shape	Property of the material Does not depend on the shape
$V = R\,i$	$\vec{E} = \rho\,\vec{J}$
SI unit ohm Ω	SI unit ohm•meter Ω•m

26-4 Resistance and Resistivity
Formula - Resistance and resistivity

Calculating Resistance from Resistivity



A homogenous isotropic conductor of uniform cross section A and length L

$$E = \rho J$$

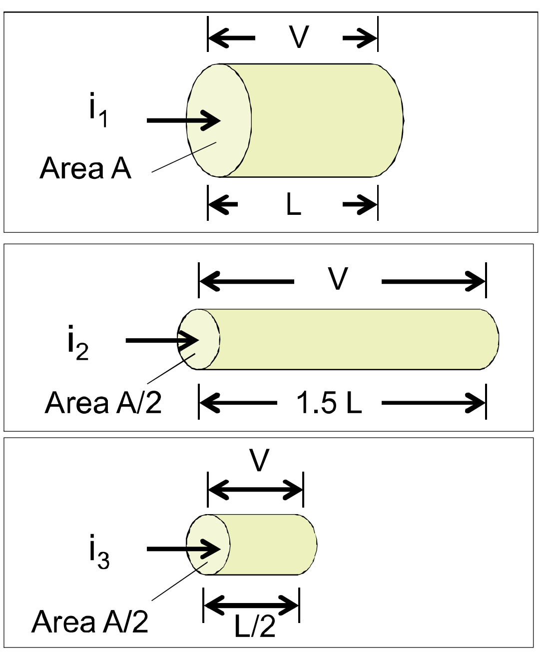
$$\frac{V}{L} = \rho \frac{i}{A}$$

$$\frac{V}{i} = \rho \frac{L}{A}$$

$$\boxed{R = \rho \frac{L}{A}}$$

homogenous
property does not depend on the position
Isotropic
property does not depend on direction

26-4 Resistance and Resistivity
Checkpoint 3



All made of copper

Rank current greatest first.

Solution

$$i = \frac{V}{R}$$

$$R = \rho \frac{L}{A}$$

$$i = \frac{V}{\rho}\frac{A}{L}$$

$$\boxed{i_1 = \frac{V}{\rho}\frac{A}{L}}$$

$$\boxed{i_2 = \frac{V}{\rho}\frac{A/2}{1.5\,L} = \frac{1}{3}\frac{V}{\rho}\frac{A}{L}}$$

$$\boxed{i_3 = \frac{V}{\rho}\frac{A/2}{L/2} = \frac{V}{\rho}\frac{A}{L}}$$

$$i_1 = i_3 > i_2$$

26-4 Resistance and Resistivity
Formula - resistivity and temperature

Variation of resistivity with temperature

$$\rho - \rho_0 = \rho_0 \, \alpha \, (T - T_0)$$

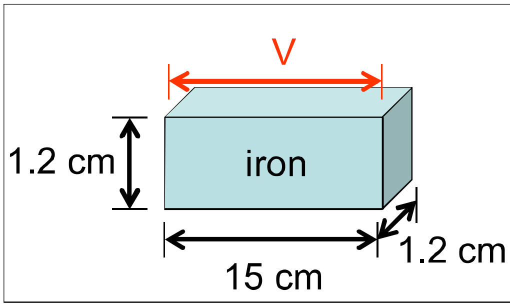
- ρ: Resistivity at T
- ρ_0: Resistivity at T_0
- α: Temperature coefficient of resistivity
- T: Temperature
- T_0: Reference temperature
- Good approximation over a wide temperature range

26-4 Resistance and Resistivity
Example 3

For iron $\rho = 9.68 \times 10^{-8}\ \Omega.m$.

What is the resistance of the block?

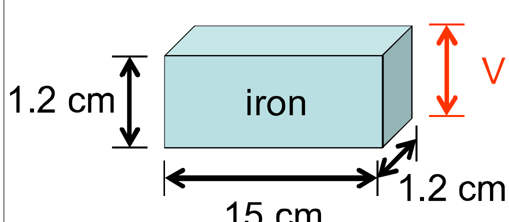
Solution



$$R = \rho \frac{L}{A}$$

$$R = (9.68 \times 10^{-8}) \frac{0.15}{(.012)(0.012)}$$

$$R = 100\ \mu\Omega$$

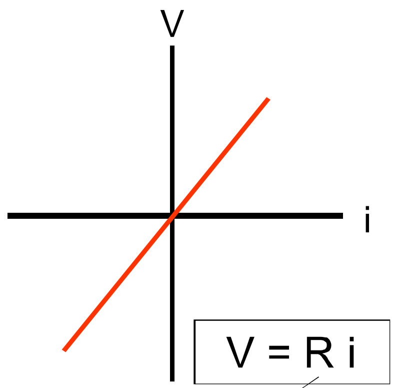


$$R = \rho \frac{L}{A}$$

$$R = (9.68 \times 10^{-8}) \frac{.012}{(0.15)(0.012)}$$

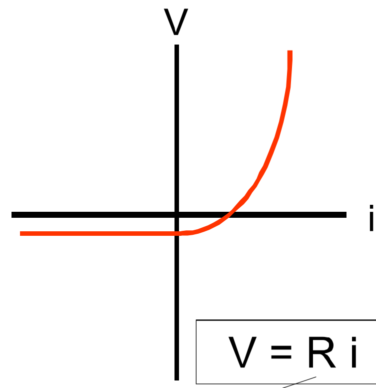
$$R = 0.65\ \mu\Omega$$

26-5 Ohm's Law
Formula - Ohm's law



Constant
Current is directly proportional to the potential difference

Device
obeys Ohm's law



Function of
the potential difference

Device
does not obey Ohm's law

26-5 Ohm's Law
Checkpoint 4

Which device does not obey Ohm's law?

Device 1

V	I
1.0	2.0
2.0	4.0
3.0	6.0

Device 2

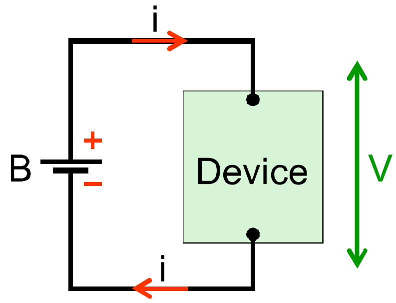
V	I
1.0	1.0
2.0	4.0
3.0	9.0

Solution

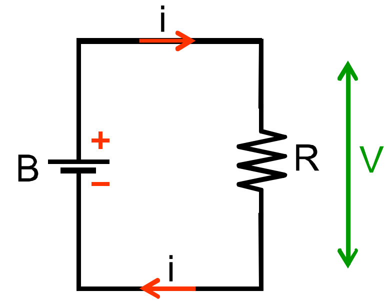
Device 1
obeys Ohm's law

Device 2
does not obey Ohm's law

26-6 Power in Electric Circuits
Formula - Power



Power is the rate of electric energy transfer from the battery to the device.



Power is the rate of electric energy transfer from the battery to the device.

$P = i\,V$

For any device (resistor, motor, capacitor)

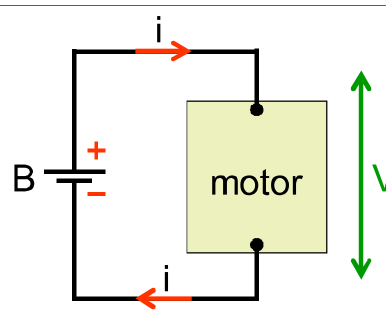
$P = i\,V$

For a resistor $V = R\,i$

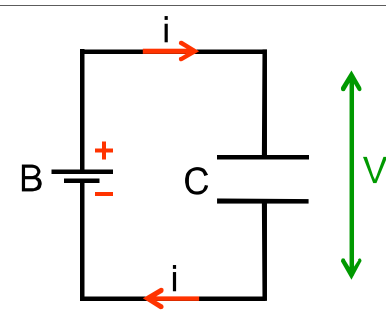
$P = i^2\,R$

$P = \dfrac{V^2}{R}$

26-6 Power in Electric Circuits
Energy rate to rotate motors and charge capacitors

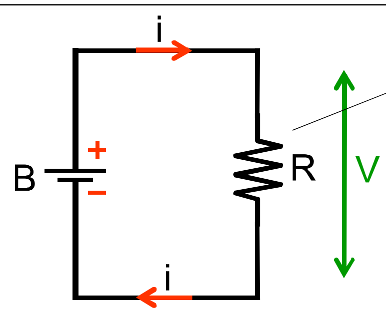


Electric energy from the battery is transferred at a rate of $P = i\,V$ to rotate the motor.



Electric energy from the battery is transferred at a rate of $P = i\,V$ to charge the capacitor.

26-6 Power in Electric Circuits
Energy rate to heat up resistors

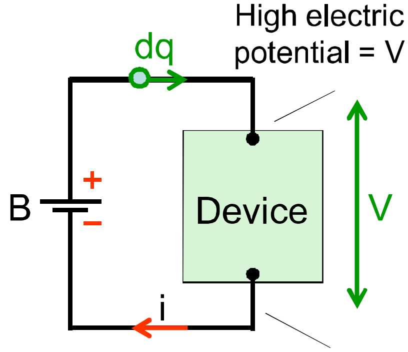


Electric energy from the battery is transferred at a rate of P = i V to heat up the resistor.

When electrons drift through a resistor, they collide with the molecules of the resistor, this increases the random motion of the molecules which is equivalent to higher temperature.

26-6 Power in Electric Circuits
Derivation - Electric power

Derivation of $P = i V$



Suppose a charge of dq moves through the device during time dt

The change in potential energy of dq is

$$dU = dq(0-V) = -dqV$$

The rate at which potential energy changes

$$\frac{dU}{dt} = -\frac{dq\,V}{dt} = -\frac{dq}{dt}V = -iV$$

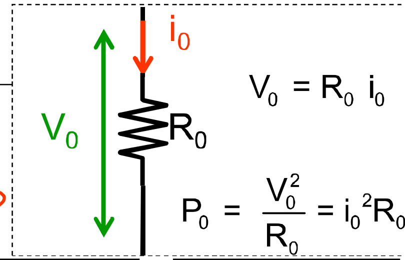
loss

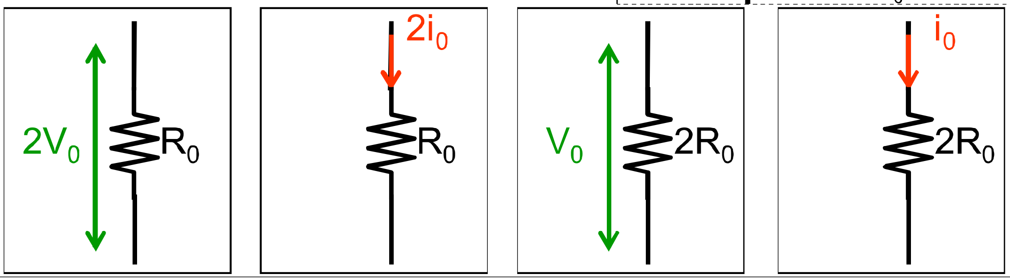
Since the energy is conserved, the rate at which potential energy is lost is the rate at which energy of other form is gained.

$$P = -\frac{dU}{dt} = iV$$

26-6 Power in Electric Circuits
Checkpoint 5

Rank the change in the rate at which electrical energy is converted to thermal energy due to the resistance?



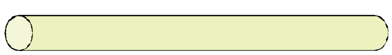


Solution

$$P = \frac{V^2}{R} = i^2R$$

$P = \frac{(2V_0)^2}{R_0}$	$P = (2i_0)^2R_0$	$P = \frac{V_0^2}{2R_0}$	$P = i_0^2\ (2R_0)$
$P = 4P_0$	$P = 4P_0$	$P = \frac{P_0}{2}$	$P = 2\ P_0$

26-6 Power in Electric Circuits
Example 4



Nichrome heating wire
Nickel-chromium-iron alloy

$R = 72\ \Omega$
$V = 120\ V$

At what rate is energy dissipated?

Solution

$$P = \frac{V^2}{R} = 200\ W$$

If the same voltage is applied to a wire that has half the original length, at what rate is energy dissipated ?

Solution

For half the length $R = 36\ \Omega$

$$P = \frac{V^2}{R} = 400\ W$$