Chapter-1

1- What is the probability of an electron state being filled if it is located at the Fermi level?

Solution:

Assume *E* = E_f and *T* > 0K in Equation (1.7.1). *f(E)* becomes ½. Hence, the probability is ½.

2- Given that nitrogen is lighter in weight than oxygen, is N_2 concentration at 10 km more or less that 25% of the sea level N_2 concentration? Assuming a constant temperature of 0 ^oC. $k = 1.38 \times 10^{-23}$ J.K $^{-1}$, N=14, e= 2.718

Solution:

There are fewer oxygen molecules at higher altitudes because the gravitational potential energy of a nitrogen molecule at the higher altitude, E_h, is larger than at sea level, E_0.

$$\frac{N_{10\text{km}}}{N_{\text{sea level}}} = \frac{e^{-E_a/kT}}{e^{-E_b/kT}} = e^{-(E_a - E_b)/kT}$$

$E_b - E_a$ is the potential energy difference, i.e., the energy needed to lift a nitrogen molecule from sea level to 10 km.

$$\begin{aligned}
E_a - E_b &= \text{altitude} \times \text{weight of } N_2 \text{ molecule} \times \text{acceleration of gravity} \\
&= 10^4 m \times N_2 \text{ molecular weight} \times \text{atomic mass unit} \times 9.8\ m \cdot s^{-2} \\
&= 10^4 m \times 28 \times 1.66 \times 10^{-27} kg \times 9.8\ m \cdot s^{-2} \\
&= 4.6 \times 10^{-21} J
\end{aligned}$$

$$\begin{aligned}
\therefore \frac{N_a}{N_b} &= e^{-4.6\times10^{-21} J / 1.38\times10^{-23} J \cdot K^{-1} \times 273 K} \\
&= e^{-1.22} = 0.30
\end{aligned}$$

Since nitrogen is lighter than oxygen, the potential energy difference for nitrogen is smaller, and consequently the exponential term for nitrogen is larger than 0.25 for oxygen. Therefore, the nitrogen concentration at 10 km is more than 25% of the sea level N_2 concentration.

3- The electron concentration in a piece of Si at 300 K is 10^5 cm^{-3}. What is the hole concentration? $n_i = 10^{10}$ cm^{-3}.

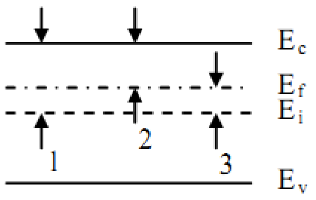
Solution:

$$n \times p = n_i^2 \rightarrow p = n_i^2 / n = \left(10^{10}\right)^2 / 10^5 cm^{-3} = 10^{15} cm^{-3}$$

4- In a silicon sample at T = 300 K, the Fermi level is located at 0.26 eV (10 kT) above the intrinsic Fermi level. What are the hole and electron concentrations? $N_c = 2.8 \times 10^{19}$ cm $^{-3}$, $k = 1.38 \times 10^{-23}$ J.K $^{-1}$, q=1.6x10^{-19} C, $n_i = 10^{10}$ cm^{-3}

Solution:

Since the Fermi level is located 0.26 eV above E_i and closer to E_c, the sample is n-type. If we assume that E_i is located at the mid-bandgap (~ 0.55 eV), then $E_c - E_f = 0.29$ eV.



1: E_c-E_i =0.55 eV
2: E_c-E_f =0.29 eV
3: E_f-E_i =0.26 eV

$$n = N_C\, e^{-(E_C - E_F)/kT} \quad \Longrightarrow \quad n = \frac{N_C}{e^{(E_C - E_F)/kT}}$$

$$n = \frac{2.8 \times 10^{19}\ \text{cm}^{-3}}{2.718^{(0.29\ ev * 1.6\times10^{-19} C)/(1.38\times10^{-23} J.K^{-1} * 300\ K)}}$$

$$n = \frac{2.8 \times 10^{19}\ \text{cm}^{-3}}{2.718^{(4.64\times10^{-20})/(4.14\times10^{-21})}}$$

$$n = \frac{2.8 \times 10^{19}\ \text{cm}^{-3}}{2.718^{11.2}}$$

$$n = \frac{2.8 \times 10^{19}\ \text{cm}^{-3}}{7.3 \times 10^{4}} = 0.38 \times 10^{15} \text{cm}^{-3} = 3.8 \times 10^{14} \text{cm}^{-3}$$

$$p = \frac{n_i^2}{n}$$

$$p = \frac{n_i^2}{n} = \frac{(10^{10})^2}{3.8 \times 10^{14} \text{cm}^{-3}} = \frac{10^{20}}{3.8 \times 10^{14}} = 2.63 \times 10^{5} \text{cm}^{-3}$$

5- For a germanium sample at room temperature, it is known that $n_i = 10^{13}$ cm^{-3}, n = 2p, and Na = 0. Determine n and p.

Solution:

$$n = \frac{n_i^2}{p}$$

n = 2p

$$2p^2 = n_i^2 \longrightarrow p = \frac{1}{\sqrt{2}} n_i$$

$$p = \frac{1}{\sqrt{2}} 10^{13} \text{cm}^{-3} = 7.07 \times 10^{12} \text{cm}^{-3}$$

n = 2p

$$n = 1.41 \times 10^{13} \text{cm}^{-3}$$

6- Boron atoms are added to a Si film resulting in an impurity density of 4×10^{16} cm^{-3}.

(a) What is the conductivity type (N-type or P-type) of this film?

(b) Why does the mobile carrier concentration increase at high temperatures?

Solution:

a) B is a group III element. When added to Si (which belongs to Group IV), it acts as an acceptor producing a large number of holes. Hence, this becomes a P-type Si film.

b) At high temperatures, there is enough thermal energy to free more electrons from silicon-silicon bonds, and consequently, the number of intrinsic carriers increases.

Chapter-2

7- For an electron mobility of 500 cm²/V·s, calculate the time between collisions. (Take $m_n = m_0$ in these calculations.) $m_o = 9.1 \times 10^{-31}$ kg, q=1.6x10^{-19} C

Solution:

500 cm^2/Vsec = 0.05 m^2/Vsec

$$\tau_{mn} = \frac{\mu_n m_n}{q} = \frac{(0.05m^2 / V.s)(9.1\times10^{-31}\text{kg})}{1.6\times10^{-19} C} = 2.84\times10^{-13} s$$

8- For an electric field 100 V/cm, calculate the distance an electron travels by drift between collisions. μ_n=500 cm^2/Vsec, $\tau_{mn} = 2.84\times10^{-13} s$.

Solution:

$$v = \mu_n \xi = (500 cm^2 / V.s)(10^2 V / cm) = 5\times10^4 cm / s$$

$$\mathrm{d} \ = \ \tau_{\mathrm{mp}} v = (2.84\times10^{-13} s)(5\times10^4 \mathrm{cm/s}) = 14.2\times10^{-9}\mathrm{cm} = 0.14\times10^{-9}\mathrm{cm} = 0.14\mathrm{nm}$$

9- Derive the equation for electron mobility (μ_n).

Solution:

lose = gain

$$m_n v = q\xi\tau_{mn} \quad \rightarrow \quad v = \frac{q\xi\tau_{mn}}{m_n}$$

$$\mu_n = \frac{-v}{\xi}$$

- **μ_n** is the **electron mobility**, a metric of how mobile the electrons are.

$$\boxed{\mu_n = \frac{q\tau_{mn}}{m_n} (\mathrm{cm}^2 / \mathrm{V{\cdot}s})}$$

The negative sign in equation means that the electrons drift in a direction opposite to the field.

10- Derive the equation for hole mobility (μ_p)..

Solution:

lose = gain

$$m_p v = q\xi\tau_{mp} \quad \rightarrow \quad v = \frac{q\xi\tau_{mp}}{m_p}$$

$$\mu_p = \frac{v}{\xi}$$

- **μ_p** is the **hole mobility**, a metric of how mobile the holes are.

$$\boxed{\mu_p = \frac{q\tau_{mp}}{m_p} (\mathrm{cm}^2 / \mathrm{V{\cdot}s})}$$

11- Derive the equation for concentration of electrons (n).

Solution:

First, we will derive the concentration of electrons in the conduction band, known as the **electron concentration**. Since $D_c(E)\,dE$ is the number of energy states between E and $E + dE$ for each cubic centimeter, the product $f(E)\,D_c(E)\,dE$ is then the number of electrons between E and $E + dE$ per cubic centimeter of the semiconductor. Therefore, the number of electrons per cubic centimeter in the entire conduction band is

$$n = \int_{E_c}^{\text{Top of conduction band}} f(E) D_c(E)\, dE$$

We now substitute Eqs. $f(E)$ and $D_c(E)$ into Eq. n and set the upper limit of integration at infinity.

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c}\; e^{-(E - E_F)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3}\, e^{-(E_c - E_F)/kT} \int_0^{\infty} \sqrt{E - E_c}\; e^{-(E - E_c)/kT} d(E - E_c)$$

Introducing a new variable

$$x = (E - E_c)/kT$$

The integral in this equation is of a form known as a gamma function and is equal to $\sqrt{\pi}/2$.

$$\int_0^{\infty} \sqrt{x}\; e^{-x} dx = \sqrt{\pi}/2$$

$$\int_0^{\infty} \sqrt{x}\, e^{-x} dx = \int_0^{\infty} (x)^{\frac{3}{2}-1} e^{-x} dx = \Gamma(3/2) = \frac{\sqrt{\pi}}{2}. \quad \text{(Gamma function)}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

$$\boxed{n = N_c\, e^{-(E_c - E_F)/kT}}$$

$\mathbf{N_c}$ is called the **effective density of states** (of the conduction band).

12- Derive the equation for the intrinsic carrier concentration (n_i).

Solution:

Since E_F cannot be close to both E_c and E_v, n and p cannot both be large numbers at the same time. When n and p are multiplied together, we obtain

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

This equation states that the np product is a constant for a given semiconductor and T, independent of the dopant concentrations. It is an important relationship and is usually expressed in the following form:

n = p = n_i (in intrinsic semiconductors)

$$\boxed{np = n_i^2}$$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$ **intrinsic carrier concentration**

There are always some electrons and holes present—whether dopants are present or not. If there are no dopants present, the semiconductor is said to be **intrinsic**.

13- Derive the expression for the intrinsic Fermi level (***E_i***).

Solution:

In an intrinsic semiconductor, $n = p$. Therefore $E_c - E_F \approx E_F - E_v$ and the Fermi level is nearly at the middle of the band gap, i.e., $E_F \approx E_c - E_g/2$. This level is called the **intrinsic Fermi level**, E_i. Here we derive a more exact expression for E_i. Rewriting Eq. (1.8.12) for $\ln n_i$, yields

$N_c = N_v$

$$\boxed{n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}}$$

$$\ln n_i = \ln\sqrt{N_c N_v} - E_g/2kT$$ ………equation (1)

$$n_i = N_c e^{-(E_c - E_i)/kT}$$ **For the intrinsic condition where n_i = n**

$$\therefore\ E_i = E_c - kT \ln\frac{N_c}{n_i}$$

$$ln\frac{N_c}{n_i} = \ln N_c - \ln n_i$$

$$= E_c + kT\ln n_i - kT\ln N_c$$ Substitute (**ln n_i**) with the eq. (1).

$$= E_c - \frac{E_g}{2} - kT\ln\frac{N_c}{N_v}$$

$E_i = E_c - E_g/2 - 0$

$E_i = E_c - E_g/2$

We see that E_i would be at the midgap, and it is equal to $\boldsymbol{E_c - E_g/2}$, because $N_c = N_v$ and $ln\frac{N_c}{N_v} = ln\ 1 = 0$. Then for silicon, E_i is very close to the midgap.

14- Describe the effect of adding donor and acceptor impurity atoms to a semiconductor.