

# Chapter 3

## Motion Along a Straight Line

### 3-1 Position and Displacement

### 3-2 Average Velocity and Average Speed

### 3-3 Instantaneous Velocity and Speed

### 3-4 Acceleration

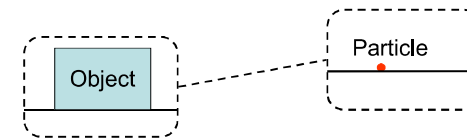
### 3-5 Constant Acceleration

### 3-6 Free-Fall Acceleration

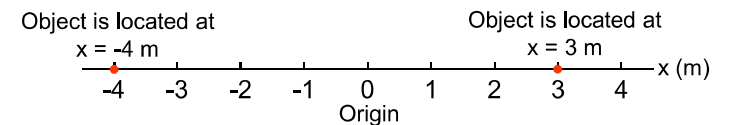
### 3-1 Position and Displacement

#### Position

Motion along a straight line  $\equiv$  One-dimensional motion



An object moves like a particle if every portion of the object moves in the same direction and at the same rate.



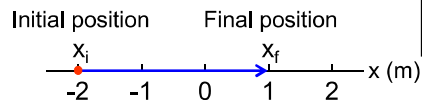
### 3-1 Position and Displacement

#### Displacement

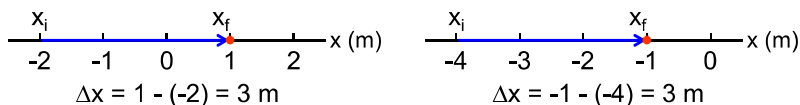
Displacement = final position - initial position

$$\Delta x = x_f - x_i$$

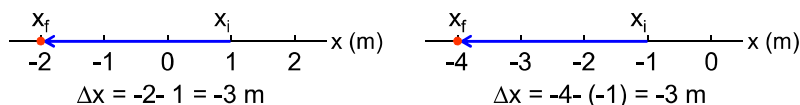
$\Delta$  = uppercase Greek letter delta  
 $\Delta \equiv$  change  
 $\Delta x \equiv$  change in  $x$



Displacement in the positive direction is positive.

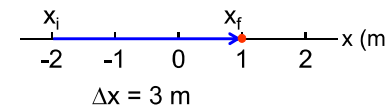


Displacement in the negative direction is negative.

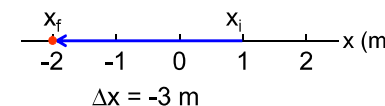


### 3-1 Position and Displacement

#### Displacement is a vector quantity



Displacement  $\Delta x = 3 \text{ m}$  means the object position has changed by 3 m in the positive direction.



Displacement  $\Delta x = -3 \text{ m}$  means the object position has changed by 3 m in the negative direction.

To determine the displacement of an object, you need to specify

- 1- **Magnitude** (The distance between the initial and final positions. Always positive)
  - 2- **Direction** (Negative or positive direction)
- $\rightarrow$  Displacement is a **vector quantity**.

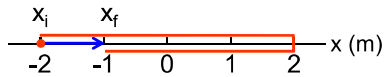
$$\Delta x = -3 \text{ m}$$

Magnitude (Absolute value). Always positive.

Direction.

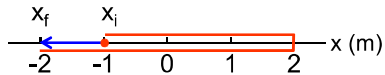
### 3-1 Position and Displacement

#### Total distance



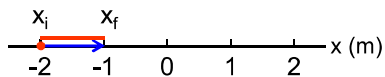
$\Delta x = 1 \text{ m}$ .  
Total distance = 7 m.

Total distance is not always equal to the magnitude of displacement.



$\Delta x = -1 \text{ m}$ .  
Total distance = 7 m.

Total distance is always positive and does not depend on the direction.  
→ Total distance is a **scalar quantity**.



$\Delta x = 1 \text{ m}$ .  
Total distance = 1 m.

Displacement depends only on the initial and final positions.  
Total distance depends on the path.

### 3-1 Position and Displacement

#### Checkpoint 1

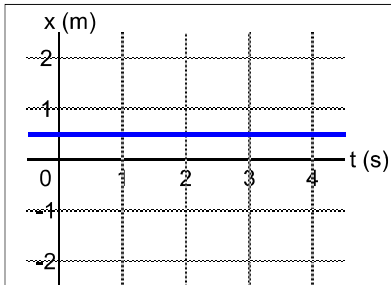
What is the direction of the following displacements?

**Solution**

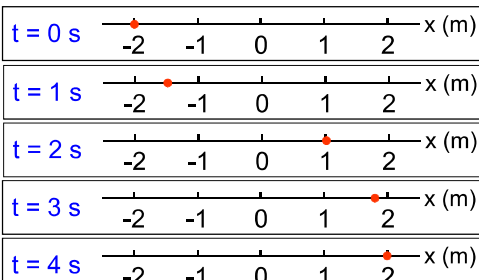
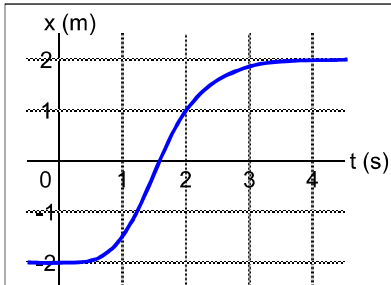
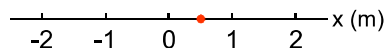
Initial position	Final position	Direction
-4 m	-2 m	Positive
-2 m	-6 m	Negative
3 m	-2 m	Negative

### 3-2 Average Velocity and Average Speed

#### Position-time graph



Stationary particle



### 3-2 Average Velocity and Average Speed

#### Average velocity

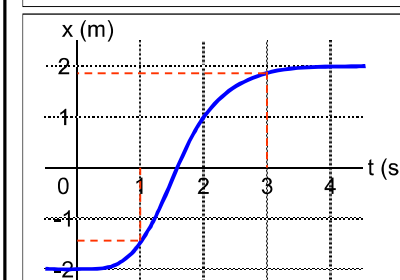
The average velocity  $v_{\text{avg}}$  for a time interval is

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{\text{Final position} - \text{Initial position}}{\text{Final time} - \text{Initial time}}$$

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Since  $\Delta t$  is always positive, average velocity has the same sign as the displacement.

SI unit for average velocity is  $\frac{\text{m}}{\text{s}}$

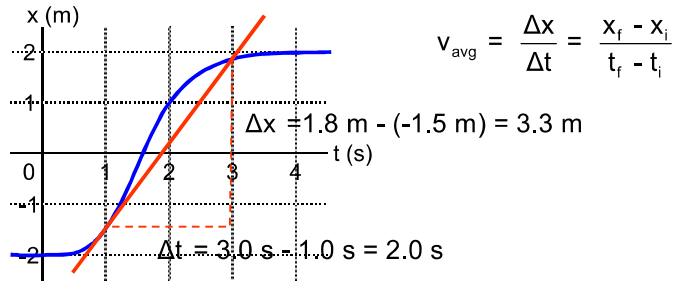


The average velocity for the time interval  $t = 1.0 \text{ s}$  to  $t = 3.0 \text{ s}$  is

$$v_{\text{avg}} = \frac{1.8 - (-1.5) \text{ m}}{3.0 - 1.0 \text{ s}} = \frac{3.3 \text{ m}}{2.0 \text{ s}} = 1.7 \frac{\text{m}}{\text{s}}$$

### 3-2 Average Velocity and Average Speed

#### Average velocity from x-t graph



On the x-t graph, the average velocity  $v_{\text{avg}}$  for a time interval is the **slope of the straight line connecting the initial and final points.**

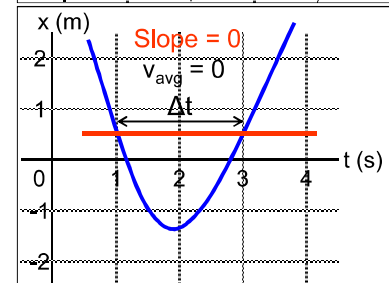
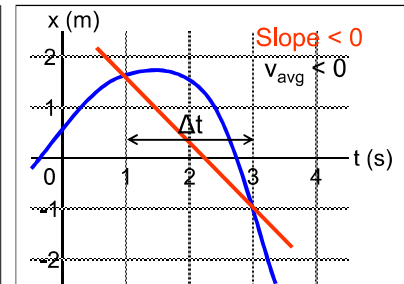
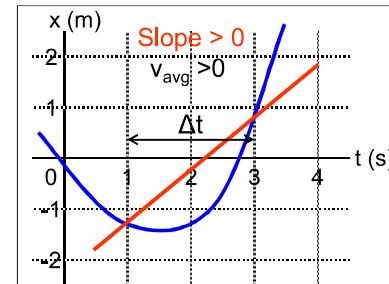
The average velocity for the time interval  $t = 1.0 \text{ s}$  to  $t = 3.0 \text{ s}$  is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{3.3 \text{ m}}{2.0 \text{ s}} = 1.7 \frac{\text{m}}{\text{s}}$$

### 3-2 Average Velocity and Average Speed

#### Average velocity is a vector quantity

The sign of the average velocity for the time interval  $t = 1.0 \text{ s}$  to  $t = 3.0 \text{ s}$



The average velocity has both magnitude and direction.

→ Average velocity is a **vector quantity**.

### 3-2 Average Velocity and Average Speed

#### Average speed

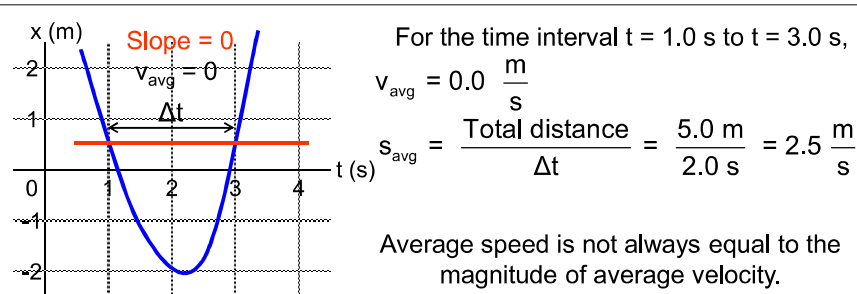
The average speed  $s_{\text{avg}}$  for a time interval is

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Time interval}}$$

$$s_{\text{avg}} = \frac{\text{Total distance}}{\Delta t}$$

Average speed is always positive. It does not include direction.

→ Average speed is a **scalar quantity**.



### 3-2 Average Velocity and Average Speed

#### Example 1

Trip along a straight line



Driving  
10 km  
60 km/h



Walking  
2.0 km  
25 min

What is your average velocity for the whole trip?

**Solution**

Displacement

$$\Delta x = \Delta x_D + \Delta x_W = 10 \text{ km} + 2.0 \text{ km} = 12 \text{ km}$$

D = Driving  
W = Walking

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{12 \text{ km}}{0.59 \text{ h}} = 20 \text{ km/h}$$

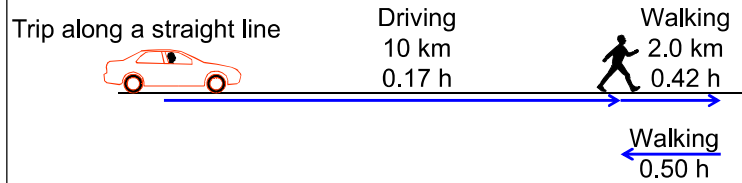
$$\Delta t = \Delta t_D + \Delta t_W = 0.17 \text{ h} + 0.42 \text{ h} = 0.59 \text{ h}$$

$$25 \text{ min} = 25 \text{ min} \left( \frac{1 \text{ h}}{60 \text{ min}} \right) = 0.42 \text{ h}$$

$$v_{\text{avg}, D} = \frac{\Delta x_D}{\Delta t_D} \rightarrow \Delta t_D = \frac{\Delta x_D}{v_{\text{avg}, D}} = \frac{10 \text{ km}}{60 \text{ km/h}} = 0.17 \text{ h}$$

### 3-2 Average Velocity and Average Speed

#### Example 2



What is your average velocity and average speed for the whole trip?

#### Solution

Displacement  $\Delta x = 10 \text{ km} + 2.0 \text{ km} - 2.0 \text{ km} = 10 \text{ km}$  In the negative direction

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ km}}{1.09 \text{ h}} = 9.2 \text{ km/h}$$

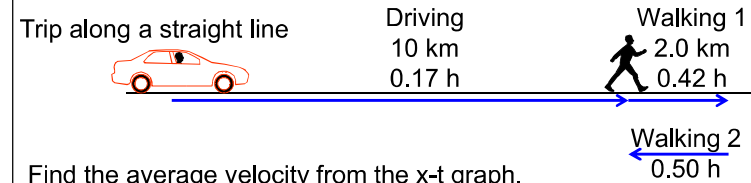
$\Delta t = 0.17 \text{ h} + 0.42 \text{ h} + 0.50 \text{ h} = 1.09 \text{ h}$

Total distance =  $10 \text{ km} + 2.0 \text{ km} + 2.0 \text{ km} = 14 \text{ km}$

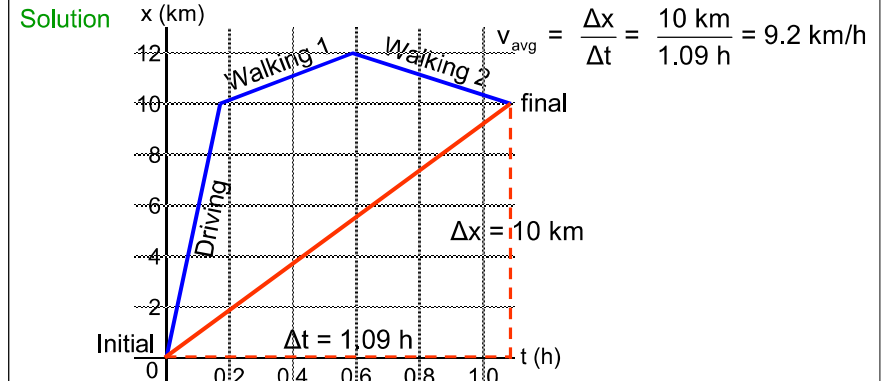
$$S_{\text{avg}} = \frac{\text{Total distance}}{\Delta t} = \frac{14 \text{ km}}{1.09 \text{ h}} = 13 \text{ km/h}$$

### 3-2 Average Velocity and Average Speed

#### Example 3

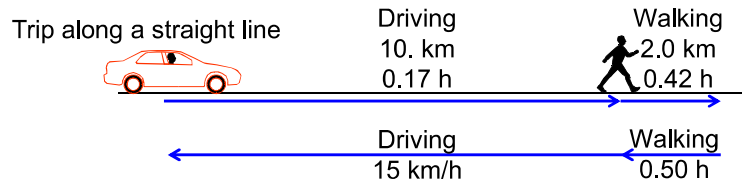


Find the average velocity from the x-t graph.



### 3-2 Average Velocity and Average Speed

#### Checkpoint 2



What is your average velocity?

#### Solution

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = 0 \text{ m/s} \quad \text{Since the displacement is zero.}$$

### 3-3 Instantaneous Velocity and Speed

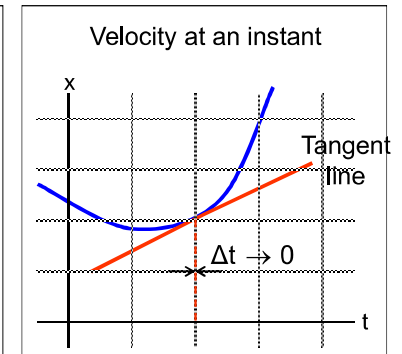
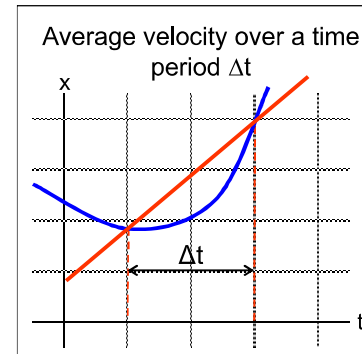
#### Velocity

The instantaneous velocity  $v$  at a given instant is

$$\text{velocity} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

velocity  $\equiv$  instantaneous velocity



### 3-3 Instantaneous Velocity and Speed

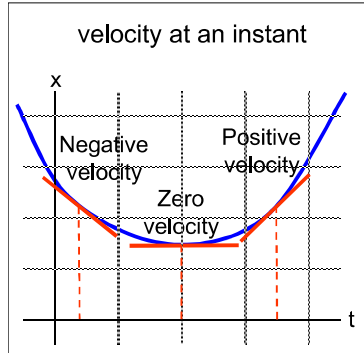
#### Velocity is the slope of x-t curve

The velocity  $v$  at a given instant is

$$v = \frac{dx}{dt} \quad \text{Velocity is the derivative of } x \text{ with respect to } t.$$

Velocity = the slope of the x-versus-t curve.

Velocity = the slope of the line tangent to the x-versus-t curve.



Velocity is a vector quantity.

Velocity is the rate at which the position  $x$  is changing with time.

### 3-3 Instantaneous Velocity and Speed

#### Speed

The speed at a given instant is the magnitude of the velocity.

$$\text{speed} = |v|$$

$$v = -4 \text{ m/s} \quad \rightarrow \text{speed} = 4 \text{ m/s}$$

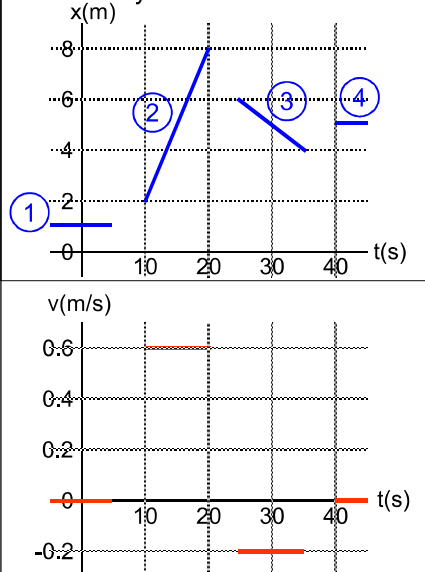
$$v = 4 \text{ m/s} \quad \rightarrow \text{speed} = 4 \text{ m/s}$$

Speed is always positive. Speed is a scalar quantity.

### 3-3 Instantaneous Velocity and Speed

#### Example 4

Plot velocity versus time.



**Solution**

$$v = \frac{dx}{dt} = \text{slope}$$

For 1 and 4,  
slope = 0

$$\begin{aligned} \text{For 2,} \\ \text{slope} &= \frac{8.0 - 2.0 \text{ m}}{20 - 10 \text{ s}} \\ &= \frac{6.0 \text{ m}}{10 \text{ s}} = 0.6 \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\begin{aligned} \text{For 3,} \\ \text{slope} &= \frac{4.0 - 6.0 \text{ m}}{35 - 25 \text{ s}} \\ &= \frac{-2.0 \text{ m}}{10 \text{ s}} = -0.2 \frac{\text{m}}{\text{s}} \end{aligned}$$

### 3-3 Instantaneous Velocity and Speed

#### Example 5

The position of a particle moving on an  $x$  axis is given by

$$x = 8.3 + 5.0 t - 3.0 t^3,$$

with  $x$  in meters and  $t$  in seconds.

Find the velocity at  $t = 1.0$  s.

**Solution**

$$\begin{aligned} v &= \frac{dx}{dt} = \frac{d}{dt} (8.3 + 5.0 t - 3.0 t^3) = 5.0 - (3)(3.0) t^2 \\ &= 5.0 - 9.0 t^2 \end{aligned}$$

At  $t = 1.0$  s,

$$v = 5.0 - 9.0 (1.0)^2 = -4.0 \text{ m/s}.$$

At  $t = 1.0$  s, the particle is moving in the negative direction with a speed of 4.0 m/s.

### 3-3 Instantaneous Velocity and Speed

#### Checkpoint 3

The following equations give the position  $x$  of a particle in four situations.  $x$  in meters,  $t$  in seconds, and  $t > 0$ .

$x = 2t - 3$
$x = -3t^2 - 1$
$x = 2/t^2$
$x = -2$

**Solution**

$v = 2$	constant	
$v = -6t$	variable	neagtive
$v = -4/t^3$	variable	negative
$v = 0$	constant	

In which situation is the velocity of the particle constant?  
In which situation is  $v$  in the negative  $x$  direction?

### 3-4 Acceleration

#### Definitions

The average acceleration  $a_{\text{avg}}$  over a time interval  $\Delta t$  is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

The instantaneous acceleration  $a$  is

$$a = \frac{dv}{dt}$$

The acceleration of a particle is the derivative of its velocity with respect to time.

The acceleration of a particle is the rate at which its velocity is changing with time.

acceleration  $\equiv$  instantaneous acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2x}{dt^2}$$

The acceleration of a particle is the second derivative of its position with respect to time.

SI unit for acceleration is  $\frac{m}{s^2}$

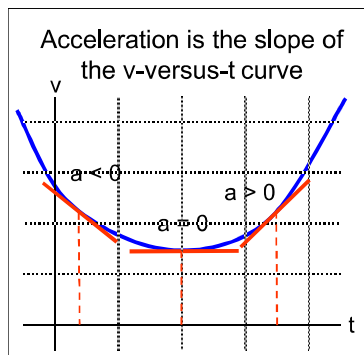
### 3-4 Acceleration

#### Acceleration is the slope of $v$ - $t$ curve

The acceleration  $a$  at an instant is

$$a = \frac{dv}{dt} = \text{the slope of the } v\text{-versus-}t \text{ curve.}$$

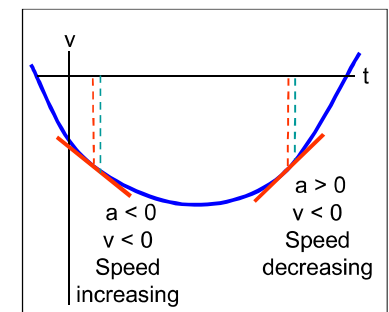
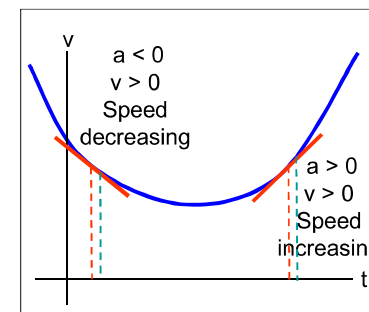
Acceleration is a vector quantity.



### 3-4 Acceleration

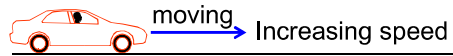
#### Acceleration direction

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases, if the signs are opposite, the speed decreases.



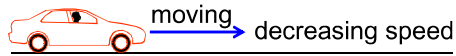
### 3-4 Acceleration Checkpoint 4

What is the sign of acceleration?



**Solution**

Positive  $\rightarrow a$



Negative  $\leftarrow a$



Negative  $\leftarrow a$



Positive  $\rightarrow a$

### 3-4 Acceleration Example 6

The position of a particle moving on the x axis is given by

$$x = 1.0 + 5.0 t - 3.0 t^3$$

with x in meters and t in seconds.

Find the acceleration of the particle as a function of time.

**Solution**

$$v = \frac{dx}{dt} = \frac{d}{dt} (1.0 + 5.0 t - 3.0 t^3) = 5.0 - (3)(3.0) t^2 = 5.0 - 9.0 t^2$$

$$a = \frac{dv}{dt} = \frac{d}{dt} (5.0 - 9.0 t^2) = - (2)(9.0) t = -18 t$$

### 3-5 Constant Acceleration Formulas

Equations for motion with **constant** acceleration

Valid only for  $a = \text{constant}$

$$v = v_0 + a t$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2 a (x - x_0)$$

$$x - x_0 = \frac{1}{2} (v + v_0) t$$

$$x - x_0 = v t - \frac{1}{2} a t^2$$

$$v_{\text{avg}} = \frac{1}{2} (v + v_0)$$

Basic equations

Useful and can be derived from the two basic equations

Notations

$t_i$	$\rightarrow$	0
$t_f$	$\rightarrow$	t
$x_i$	$\rightarrow$	$x_0$
$x_f$	$\rightarrow$	x
$v_i$	$\rightarrow$	$v_0$
$v_f$	$\rightarrow$	v

### 3-5 Constant Acceleration Derivations

$$v = v_0 + a t$$

$$a = \frac{dv}{dt} \rightarrow dv = a dt$$

$$\int_{v_0}^v dv = \int_0^t a dt$$

$$v - v_0 = \int_0^t \text{constant } a dt$$

$$v - v_0 = a \int_0^t dt = a t$$

$$v = v_0 + a t$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v = \frac{dx}{dt} \rightarrow dx = v dt$$

$$\int_{x_0}^x dx = \int_0^t v dt$$

$$x - x_0 = \int_0^t v dt$$

$$x - x_0 = \int_0^t (v_0 + a t) dt$$

$$x - x_0 = \int_0^t v_0 dt + \int_0^t a t dt$$

$$x - x_0 = v_0 \int_0^t dt + a \int_0^t t dt$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

### 3-5 Constant Acceleration Derivations

$$v = v_0 + a t$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

eliminate t

$$v^2 = v_0^2 + 2 a (x - x_0)$$

$$v = v_0 + a t \rightarrow t = \frac{v - v_0}{a}$$

$$x - x_0 = v_0 \left( \frac{v - v_0}{a} \right) + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2$$

$$2 a (x - x_0) = 2 v_0 (v - v_0) + (v - v_0)^2$$

$$2 a (x - x_0) = 2 v_0 v - 2 v_0^2 + v^2 - 2 v v_0 + v_0^2$$

$$2 a (x - x_0) = v^2 - v_0^2$$

$$v^2 = v_0^2 + 2 a (x - x_0)$$

### 3-5 Constant Acceleration Checkpoint 5

The following equations give the position of a particle. To which of these cases do the equations of this section apply?

$$x = 5 t + 2$$

$$x = 2 t^3 + 3 t$$

$$x = \frac{2}{t^2}$$

$$x = 5 t^2 + 4$$

Solution

$$a = 0$$

Yes

$$a = 12 t$$

No

$$a = \frac{12}{t^4}$$

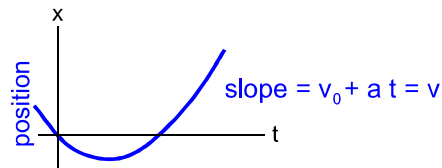
No

$$a = 10$$

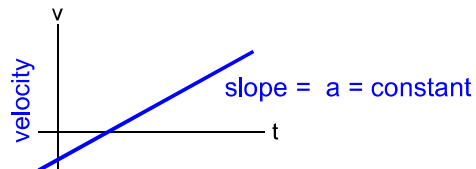
Yes

### 3-5 Constant Acceleration Graphs

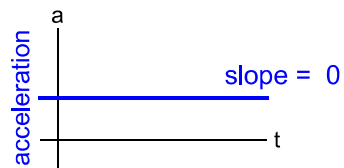
$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$



$$v = v_0 + a t$$



$$a = \text{constant}$$



### 3-5 Constant Acceleration Example 7

Braking at a constant acceleration.



110 km/h

60.0 m

40.0 km/h

What is the acceleration in  $\text{m/s}^2$ ?

Solution

Since we know the displacement  $x - x_0$ , the initial velocity  $v_0$ , and the final velocity  $v$ , we can use the following equation to find  $a$

$$v^2 = v_0^2 + 2 a (x - x_0)$$

$$a = \frac{v^2 - v_0^2}{2 (x - x_0)} = \frac{(40.0 \text{ km/h})^2 - (110.0 \text{ km/h})^2}{2 (60.0 \text{ m})} = \frac{-10500 (\text{km/h})^2}{2 (.0600 \text{ km})} = -8.75 \times 10^4 \text{ km/h}^2$$

$$a = -6.75 \text{ m/s}^2$$

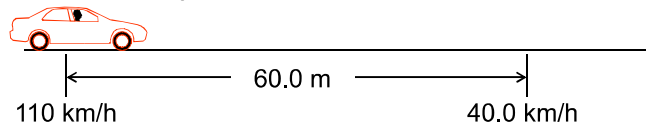
$$= (8.75 \times 10^4 \frac{\text{km}}{\text{h}^2}) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)^2 = 6.75 \text{ m/s}^2$$



### 3-5 Constant Acceleration

#### Example 8

Braking at a constant acceleration.



What is the time in seconds?

#### Solution

Since we know the displacement  $x - x_0$ , the initial velocity  $v_0$ , and the final velocity  $v$ , we can use the following equation to find  $t$

$$x - x_0 = \frac{1}{2} (v + v_0) t$$

$$t = \frac{2(x - x_0)}{(v + v_0)} = \frac{2(0.0600 \text{ km})}{(110.0 \text{ km/h}) + (40.0 \text{ km/h})} = 8.00 \times 10^{-4} \text{ h}$$

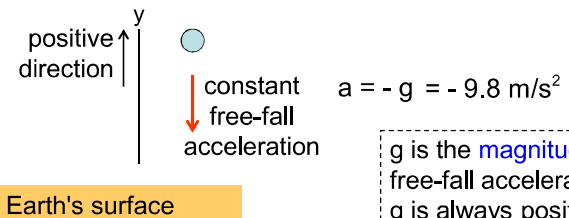
$$t = 2.88 \text{ s}$$

$$\begin{aligned} &\equiv (8.00 \times 10^{-4} \text{ h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) \\ &= 2.88 \text{ s} \end{aligned}$$

### 3-6 Free-Fall Acceleration

#### Free-fall

In the absence of the effects of air, all objects dropped or thrown near Earth's surface have a certain **constant acceleration** toward Earth. This acceleration is called **free-fall acceleration** and it is due to Earth's gravity.



$g$  is the **magnitude** of the free-fall acceleration.  
 $g$  is always positive.  
 $g = 9.8 \text{ m/s}^2$ .

The value of  $g$  varies slightly from place to place on Earth's surface. the value  $g = 9.8 \text{ m/s}^2$  is accurate enough for our purposes in this course.

### 3-6 Free-Fall Acceleration

#### Formulas

Equations for free-fall = Equations for motion with **constant** acceleration

$$\begin{aligned} x &\rightarrow y \\ x_0 &\rightarrow y_0 \\ a &\rightarrow -g \end{aligned}$$

Basic equations

$$v = v_0 - g t$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

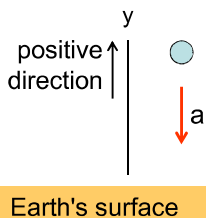
$$v^2 = v_0^2 - 2 g (y - y_0)$$

$$y - y_0 = \frac{1}{2} (v + v_0) t$$

$$y - y_0 = v t + \frac{1}{2} g t^2$$

$$v_{\text{avg}} = \frac{1}{2} (v + v_0)$$

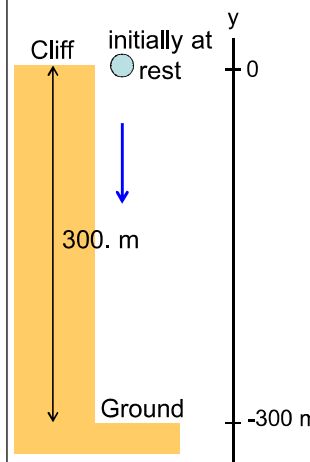
Useful and can be derived from the two basic equations



$$a = -g = -9.8 \text{ m/s}^2$$

### 3-6 Free-Fall Acceleration

#### Example 9



How long does the ball fall to reach the ground?

#### Solution

Since we know  $y - y_0$  and  $v_0$ , we can use the following equation to find  $t$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

Choose the origin of the  $y$ -axis at the cliff.  
Initially at rest

$$y = -\frac{1}{2} g t^2$$

$$t^2 = -\frac{2 y}{g}$$

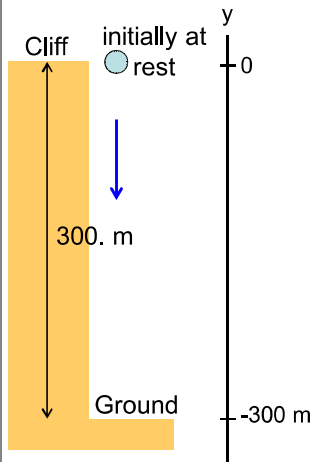
$$t = \pm \sqrt{-\frac{2 y}{g}} = \pm \sqrt{-\frac{2 (-300 \text{ m})}{9.80 \text{ m/s}^2}} = \pm 7.82 \text{ s}$$

Since the ball reaches the ground after  $t = 0$ , the negative answer is not valid.

$$t = 7.82 \text{ s}$$

### 3-6 Free-Fall Acceleration

#### Example 10



What is the ball velocity when it hits the ground?

#### Solution

Since we know  $y - y_0$  and  $v_0$ , we can use the following equation to find  $v$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Choose the origin of the y-axis at the cliff.  
Initially at rest

$$v^2 = -2gy$$

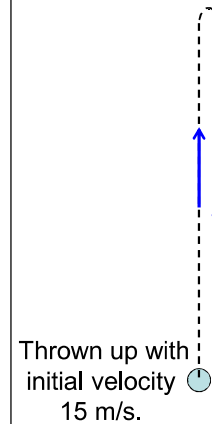
$$v = \pm \sqrt{-2gy} = \pm \sqrt{-2(-300\text{ m})(9.80\text{ m/s}^2)} = \pm 76.7\text{ m/s}$$

Since the ball is moving in the negative direction, the positive answer is not valid.

$$v = -76.7\text{ m/s}$$

### 3-6 Free-Fall Acceleration

#### Example 11



How long does the ball take to reach its maximum height?

#### Solution

Since we know  $v$  and  $v_0$ , we can use the following equation to find  $t$

$$v = v_0 - gt$$

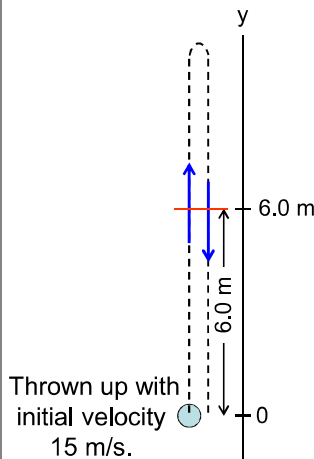
At its maximum height, the ball is at rest  $\rightarrow v = 0$

$$0 = v_0 - gt$$

$$t = \frac{v_0}{g} = \frac{15\text{ m/s}}{9.8\text{ m/s}^2} = 1.5\text{ s}$$

### 3-6 Free-Fall Acceleration

#### Example 12



How long does the ball take to reach a point 6.0 m above its release point?

#### Solution

Since we know  $y - y_0$  and  $v_0$ , we can use the following equation to find  $t$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

Choose the origin of the y-axis at the release point

$$y = v_0 t - \frac{1}{2} g t^2$$

$$g t^2 - 2 v_0 t + 2 y = 0$$

$$t = \frac{2 v_0 \pm \sqrt{4 v_0^2 - 8 g y}}{2 g} = \frac{v_0 \pm \sqrt{v_0^2 - 2 g y}}{g}$$

$$= \frac{15\text{ m/s} \pm \sqrt{(15\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(6\text{ m})}}{(9.8\text{ m/s}^2)}$$

$$= \begin{cases} 0.47\text{ s} & \text{The way up} \\ 2.6\text{ s} & \text{The way down} \end{cases}$$

### 3-6 Free-Fall Acceleration

#### Checkpoint 6



What is the sign of the ball displacement for the ascent, from the release point to the highest point?

What is the sign of the ball displacement for the descent, from the highest point back to the release point?

What is the ball's acceleration at its highest point?

#### Solution

Positive

Negative

$$a = -g = -9.8\text{ m/s}^2$$