Mechanics I - Question Bank – 5

2018-2019 Fall semester

KEY

Full name: .....

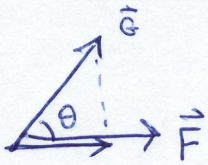
Q1. Three vectors *F*, *v* and *B* are related through $F = 5.0\ (v \times B)$. If vector $v = 3.0\ \hat{i} - 5.0\ j$ and $B = -2.0\ \hat{k}$, then vector *F* is: .....? (Ans:50 $i + 30\ j$)

$v \times B = (3\hat{i} - 5\hat{j}) \times (-2\hat{k}) = -6(-\hat{j}) + 10\hat{i} = +6\hat{j} + 10\hat{i}$

$v \times B = 10\hat{i} + 6\hat{j}$

$5(v \times B) = 50\hat{i} + 30\hat{j}$

Q2. Vectors *F* and *G* are defined as $F = 3.0\ \hat{i} + 4.0\ \hat{j}$, and $G = -\hat{i} + \hat{j}$. Find the component (projection) of vector *G* along the direction of vector *F*. (Ans: 0.20)



$\vec{G}.\cos\theta$

This is projection of G along the direction of vector F.

$\vec{F}.\vec{G} = |\vec{F}|.|\vec{G}|.\cos\theta$

$|\vec{G}|\cos\theta = \frac{\vec{F}.\vec{G}}{|\vec{F}|}$

projection of G, on the vector F.

$|\vec{G}|.\cos\theta = \frac{(3\hat{i}+4\hat{j}).(-\hat{i}+\hat{j})}{|\vec{F}| = \sqrt{3^2+4^2}}$

$|\vec{G}|.\cos\theta = \frac{1}{5} = 0,20$

Q3. The angle between the two vectors A = 2 i + 4 j and B = 4 i - 2 j is: .....? (Ans:90 degrees)

$\vec{A}.\vec{B} = |\vec{A}|.|\vec{B}|\cos\theta$

$(2\hat{i}+4\hat{j})(4\hat{i}-2\hat{j}) = |\vec{A}||\vec{B}|\cos\theta$

$8-8 = |\vec{A}||\vec{B}|.\cos\theta$

$0 = |\vec{A}||\vec{B}|\cos\theta \rightarrow$ This means that $\cos\theta = 0$ then $\theta = 90°$.

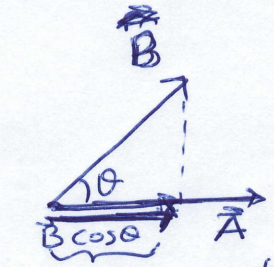
Q4. Three vectors are given as: $A=-3.0i$; $B= -5.0\ k$ *and* $C=2.0\ j$. The value of $A.(B \times C)$ is:....? (Ans -30)

$B \times C = -5\hat{k} \times 2\hat{j} = -10\ \hat{k} \times \hat{j} = -10(-\hat{i}) = 10\hat{i}$

$A.(B \times C) = -3\hat{i}.10\hat{i} = -30$

KEY

Q5. Two vectors and *A and B* have magnitudes of 10 m and 15 m respectively. The angle between them is 65°. The component (projection) of *B* along *A* is: ….? (Ans: 6.3 m)



The component (projection) of B along A.

$A = 10m \quad B = 15m$

$B \cdot \cos\theta = 15 \cdot \cos 65$

$= 15 \cdot 0.43 = 6.3\ m$

Q6. Let A=2.0*i*+ 3.0 *k* and *B*=2.0 *i*+ *k*. The vector *D*=*(A-B)xA* is: ….. ? (Ans: -4.0 *j*)

$A - B = (2\hat{i} + 3k) - (2\hat{i} + k) = 2\hat{i} + 3k - 2\hat{i} - k$

$= 2k$

$(A - B) \times A = 2k \times (2\hat{i} + 3k) = 4(-\hat{j}) + 0$

$= -4\hat{j}$

Q7. Three vectors *A*, *B*, and *C* are such that: *C* =*A*+*B*, *B*=5*i* and *C*=5*j*. Find the angle between *A* and B:…? (Ans: 135°)

$\vec{C} = \vec{A} + \vec{B} \qquad 5\hat{j} = \vec{A} + 5\hat{i} \qquad \vec{A} = -5\hat{i} + 5\hat{j}$

$|\vec{A}| = \sqrt{5^2 + 5^2}$

$|\vec{A}| = \sqrt{50}$

$|\vec{B}| = 5$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \qquad (-5\hat{i} + 5\hat{j}) \cdot (5\hat{i}) = \sqrt{50} \cdot 5 \cdot \cos\theta$

$-25 = \sqrt{50} \cdot 5 \cdot \cos\theta$

$\cos\theta = \frac{-5}{\sqrt{50}} = -0.71 \qquad \theta = 135°$

Q8. Three vectors are A =1.00i + 2.00 j -3.00 k , B = 3.00 k and C = 6.00 i -7.00 j. Find 2C.(A X B). (Ans: 114)

$A \times B = (\hat{i} + 2\hat{j} - 3\hat{k}) \times (3\hat{k}) = -3\hat{j} + 6\hat{i}$

$2C(A \times B) = 2[(6\hat{i} - 7\hat{j})(-3\hat{j} + 6\hat{i})] = 2[36 + 21] = 2 \cdot 57 = 114$

Q9. Two vectors are given as: A = -3.0 i + 5.0 j + 4.0 k and B = 4.0 i + 5.0 j + 3.0 k, where i,j and k are the unit vectors in the positive x, y and z directions. Find the angle between the vectors A and B. (Ans: 60 degrees)

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

$|\vec{A}| = \sqrt{(-3)^2 + 5^2 + 4^2} = \sqrt{50}$

$|\vec{B}| = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{50}$

$(-3\hat{i} + 5\hat{j} + 4\hat{k})(4\hat{i} + 5\hat{j} + 3\hat{k}) = |\vec{A}||\vec{B}| \cdot \cos\theta$

$(-12 + 25 + 12) = \sqrt{50}\sqrt{50} \cdot \cos\theta$

$25 = 50\cos\theta \qquad \cos\theta = \frac{1}{2}$

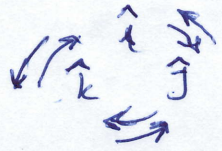
$\theta = 60°$

KEY

Q10. In the cross product F = v x B, take v = 2.0 i, F = 6.0 j and the x component of vector B equals zero. What then is B in unit-vector notation? (Ans: -3.0 k)

$\frac{6}{2}\hat{j} = \frac{2\hat{i}}{2} \times B$

$3\hat{j} = \hat{i} \times B$ ↓ ?



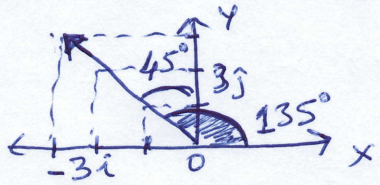
$3\hat{j} = \hat{i} \times B$

↓

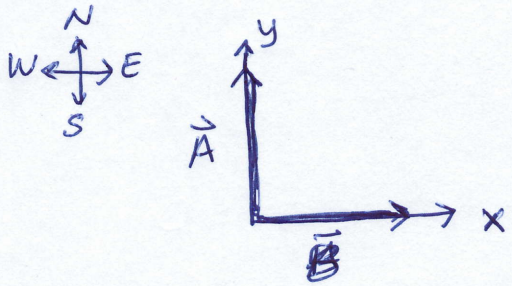
$-3\hat{k}$

Q11. Two vectors are given by: $P = -1.5\hat{i} + 2.0\hat{j}$, $Q = 1.0\hat{j}$. The angle that the vector $2P - Q$ makes with the positive x-axis is:....? (A: 135°)

$2\vec{P} - \vec{Q} = 2(-1.5\hat{i} + 2\hat{j}) - (\hat{j}) = -3\hat{i} + 4\hat{j} - \hat{j} = -3\hat{i} + 3\hat{j}$



Q12. Vector $\vec{A}$ is directed due north. Vector $\vec{B}$ is directed due east. Determine the direction of the vector product, $\vec{A} \times \vec{B}$



$\vec{A} = A_y\hat{j}$ $\vec{B} = B_x\hat{i}$

$\vec{A} \times \vec{B} = A_y\hat{j} \times B_x\hat{i} = A_y \cdot B_x(-\hat{k})$

Q13. Under what conditions does $\vec{A} \times \vec{B} = AB$?

$\vec{A} \times \vec{B} = A \cdot B \cdot \sin\theta$

if $\vec{A} \times \vec{B} = A \cdot B$, then $\sin\theta = 1 \Rightarrow \theta = 90.$

This means that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Q14. Let $A = 2.0i - 3.0k$ and $B = 2.0i + k$. Find the vector $D = (A - B) \times A = ?$

$\vec{A} - \vec{B} = (2\hat{i} - 3\hat{k}) - (2\hat{i} + \hat{k}) = 2\hat{i} - 3\hat{k} - 2\hat{i} - \hat{k} = -4\hat{k}$

$D = (\vec{A} - \vec{B}) \times \vec{A} = -4\hat{k} \times (2\hat{i} - 3\hat{k}) = -8\hat{j} + 12 \cdot 0$

$\vec{D} = -8\hat{j}$