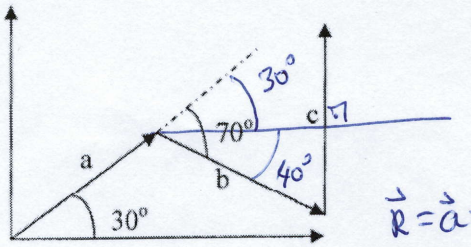
Mechanics I - Question Bank – 4
2018-2019 Fall semester

Full name: ..... KEY .....

Q1. Three vectors a, b, and c have equal magnitudes of 20 m. Find the magnitude and direction of the sum of these vectors.



$\vec{a} = a_x\hat{i} + a_y\hat{j} = 20\cos 30\,\hat{i} + 20\sin 30\,\hat{j} =$

$\vec{b} = b_x\hat{i} + b_y\hat{j} = 20\cos(-40)\,\hat{i} + 20\sin(-40)\,\hat{j} =$

$\vec{c} = c_x\hat{i} + c_y\hat{j} = 20\cos 90\,\hat{i} + 20\sin 90\,\hat{j} =$

$\vec{R} = \vec{a}+\vec{b}+\vec{c} = (a_x+b_x+c_x)\hat{i} + (a_y+b_y+c_y)\hat{j}$ $\quad \vec{R} = \vec{R}_x + \vec{R}_y$

$a_x = 20\cdot\cos 30 = 20\cdot 0{,}87 = 17{,}32$ m

$a_y = 20\cdot\sin 30 = 20\cdot 0{,}5 = 10$ m

$b_x = 20\cdot\cos(-40) = 20\cdot 0{,}77 = 15{,}32$ m

$b_y = 20\cdot\sin(-40) = 20(-0{,}64) = -12{,}86$ m

$c_x = 20\cdot\cos 90 = 20\cdot 0 = 0$

$c_y = 20\cdot\sin 90 = 20\cdot 1 = 20$ m

$R_x = a_x + b_x + c_x$

$R_x = 17{,}32 + 15{,}32 = 32.64$

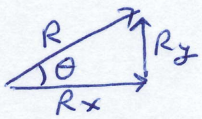
$R_y = 10 - 12{,}86 + 20 = 17.14$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{1359.15}$

$R = 36.87$ m

$\tan\theta = \frac{17{,}14}{32.64}$

$\theta = 27.7°$



Q2. Vectors *a*, *b*, and *c* are related through equations $a + b = c$ and $a - b = 5.0\,c$. If $c = 3.0\,\hat{i} + 4.0\,\hat{j}$, what is the magnitude of vector *a*? (Ans: 15)

$a + b = c$

$a - b = 5c$

$2a = 6c$

$a = 3c$

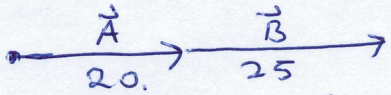
$\vec{a} = 3\vec{c} = 3[3\hat{i} + 4\hat{j}] = 9\hat{i} + 12\hat{j}$

$|\vec{a}| = \sqrt{9^2 + 12^2} = 15$

Q3. A vector *A* of magnitude 20 is added to a vector *B* of magnitude 25. The magnitude of the vector $A + B$ can be: (Ans: 12)

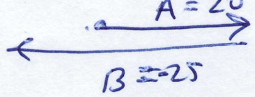
In the question, the direction of $\vec{A}$ and $\vec{B}$ were not given.

So, if they are in the same direction:



$\vec{R} = \vec{A} + \vec{B} = 20 + 25 = 45$ → This is max value.

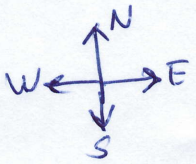
If they are in the opposite direction:

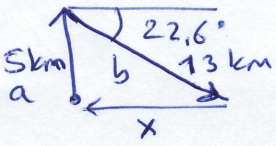


$\vec{R} = \vec{A} + \vec{B} = 20 - 25 = -5$ → This is min value.

∴ Any number btw −5 and 25 can be answer.

Q4. A man walks 5.0 km due North, then 13 km 22.6° South of East, and then 12 km due West. The man is finally at:….? (Ans: where he started)





$a^2 + x^2 = b^2$ $\quad x^2 = b^2 - a^2 = 13^2 - 5^2$

$x^2 = 144$ $\quad x = 12$ km

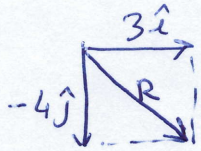
$X = 12$ km, this means that the man is finally at the starting point.

Q5. *A* and *B* are two perpendicular vectors: A=3.0 i and B=2.0 j The magnitude of is: A-2B=? (Ans: 5.0)

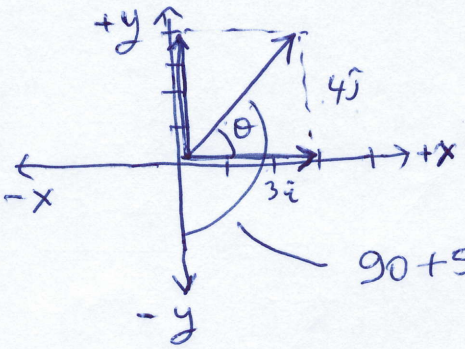
$\vec{R} = \vec{A} - 2\vec{B} = ?$

$\vec{R} = 3\hat{i} - 2(2\hat{j}) = 3\hat{i} - 4\hat{j}$ $\quad R^2 = 3^2 + (-4)^2$ $\quad R = \sqrt{9+16}$

$R = 5$



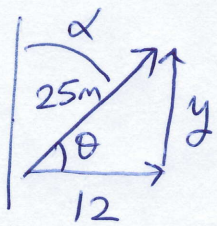
Q6. The angle between vector A=3.00 i+ 4.00j and the negative y-axis is: (A: 143°)



$\tan\theta = \frac{4}{3}$ $\quad \theta = 53$

$90 + 53 = 143°$

Q7. A vector in the xy plane has a magnitude of 25 m and an x component of +12 m and a positive y component. The angle it makes with the positive y axis is: (Ans: 29°)



$\alpha = 90 - \theta$

$\theta = \text{Arccos}\ \frac{12}{25} = 61°$

$\alpha = 90 - 61 = 29°$

Q8. If $A = (2.0\,i - 3.0j)m$ and $B = (1.0i - 2.0j)m$, then $A-2B$ = ? (Ans: (1.0j) m)

$\vec{R} = \vec{A} - 2\vec{B} = [2\hat{i} - 3\hat{j}] - 2[\hat{i} - 2\hat{j}] = 2\hat{i} - 3\hat{j} - 2\hat{i} + 4\hat{j}$

$\vec{R} = 1.\hat{j}$

Q9. Given the vectors A = 3 j + 6 k, B = 15 i + 21 k. Find the magnitude of vector C that satisfies equation 2A + 3C - B = 0. (Ans: 6.16)

$2A + 3C - B = 0 \quad 3C = B - 2A$

$3C = 15\hat{i} + 21\hat{k} - 2(3\hat{j} + 6\hat{k})$

$3C = 15\hat{i} + 21\hat{k} - 6\hat{j} - 12\hat{k}$

$3C = 15\hat{i} + (-6)\hat{j} - 9\hat{k}$

$C = 5\hat{i} + (-2)\hat{j} - 3\hat{k}$

$C = \sqrt{25 + 4 + 9} = \sqrt{38}$

$C = 6.16$

Q10. In Fig 1, A = (12m, 60°) and B = (8m, 300°). Then x component of *(A-B)* is:.....? (Ans: 2 m)

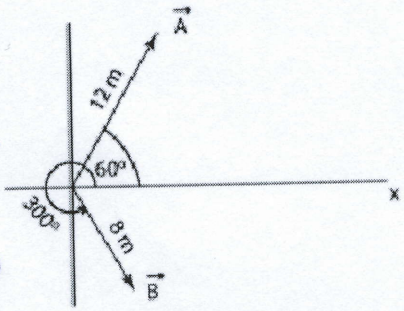


Figure 1

$\vec{A} = A_x\hat{i} + A_y\hat{j} = 12\cos 60\,\hat{i} + 12\sin 60\,\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} = 8\cos 300\,\hat{i} + 8\cos 300\,\hat{j}$

$\vec{A} = 6\hat{i} + 10.4\hat{j}$

$\vec{B} = +4\hat{i} - 6.9\hat{j}$

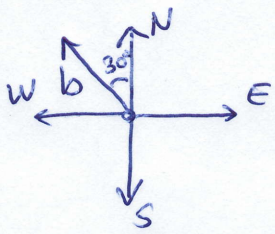
$\vec{R} = \vec{A} - \vec{B}$

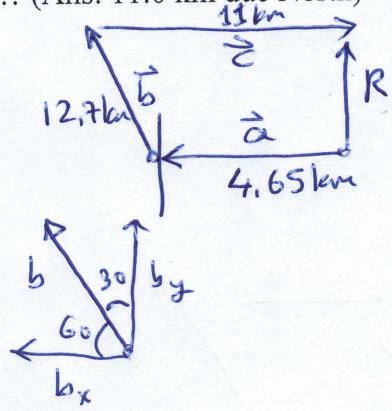
$R = (6\hat{i} + 10.4\hat{j}) - (+4\hat{i} - 6.9\hat{j})$

$= 2\hat{i} + 17.3\hat{j}$

This is x component : 2m

Q11. A man walks 4.65 km West, then 12.7 km in the direction 30° West of North and finally 11.0 km due East. The man is now at ….? (Ans: 11.0 km due North)





$\vec{a} = 4.65(-\hat{i}) + 0\hat{j}$

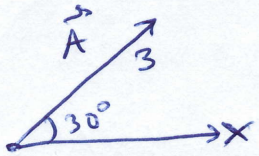
$\vec{b} = 12.7\cos 60(-\hat{i}) + 12.7\sin 60\hat{j}$

$\vec{c} = 0\hat{j} + 11\hat{i}$

$\vec{R} = (-4.65 + 11 - 6.35)\hat{i} + 11\hat{j}$

$\vec{R} = 0\hat{i} + 11\hat{j}$. This result show that the man is now at 11 km due North.

Q12. If vector *A* has the magnitude of 3.0 m and makes an angle 30° with the +x-axis, then the vector $B = -2A$ = ? (Ans:-5.2 i –3.0j)



$\vec{A} = A_x\hat{i} + A_y\hat{j}$

$= A\cos 30\,\hat{i} + A\sin 30\,\hat{j}$

$= 3.(0,87)\,\hat{i} + 3.(0.5)\,\hat{j}$

$\vec{A} = 2,6\,\hat{i} + 1,5\,\hat{j}$

$B = -2(2.6\hat{i} + 1.5\hat{j}) = -5,2\hat{i} - 3\hat{j}$

Q13. Two vectors A and B are shown in Fig 1. Each vector has a magnitude of 5.0 m. Find the magnitude of the resultant vector R = A + B and the angle (theta) between R and the positive x-axis counter clockwise. (Ans: magnitude = 7.1 m, theta = 90 degrees)

$\vec{R} = \vec{A} + \vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} = A_x.\cos 45\,\hat{i} + A_y\sin 45\hat{j} = 5.0,71\,\hat{i} + 5.0,71\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} = B_x\cos 45(-\hat{i}) + A_y\sin 45\hat{j} = 5(0,71)(-\hat{i}) + 5.(0,71)\hat{j}$

$\vec{R} = \vec{A} + \vec{B} = (5\times 0,71 - 5\times 0,71)\hat{i} + (5\times 0,71 + 5\times 0,71)\hat{j} = 0\hat{i} + 7,1\hat{j}$

$R = 7,1\hat{j}$. This means that the resultant vector's magnitude is 7,1 km in north direction.

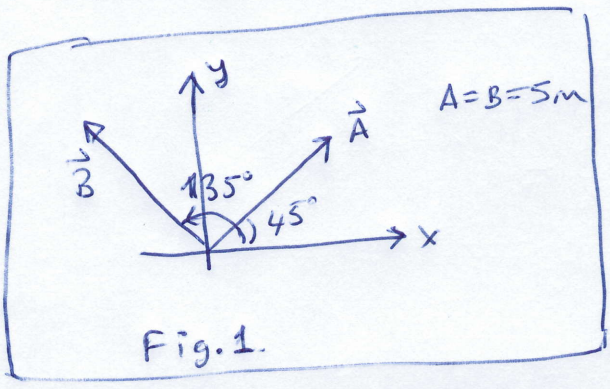
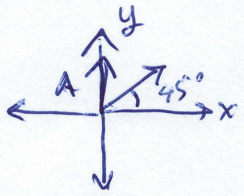
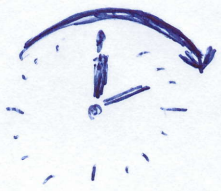


Fig. 1.

Q14. Two displacement vectors A and B have equal magnitudes of 10 m. Vector A is along the +y axis and vector B makes 45 degrees counterclockwise with +x axis. Find the vector C such that B + C = 2A.
(Ans: C = -7 i + 13 j)





clockwise direction (cw)

cw

ccw

counterclockwise (ccw)

$\vec{A} = 10\hat{j}$ $\quad \vec{B} = 10.\cos 45\,\hat{i} + 10\sin 45\,\hat{j}$

$= 7.1\hat{i} + 7.1\hat{j}$

$B + C = 2A \quad C = 2A - B$

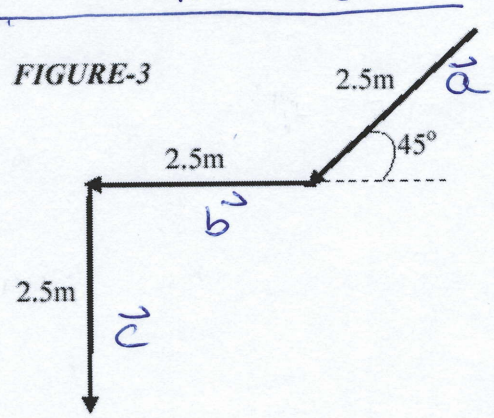
$C = 2.(10\hat{j}) - (7.1\hat{i} + 7.1\hat{j})$

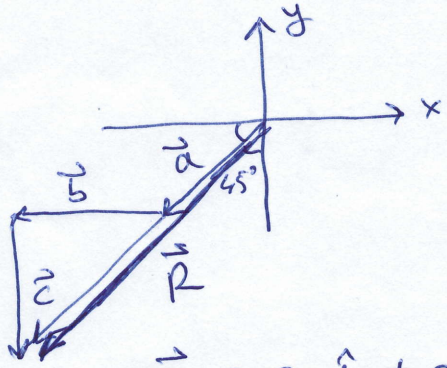
$= 20\hat{j} - 7.1\hat{i} - 7.1\hat{j}$

$= -7.1\hat{i} + 13\hat{j}$

Q15. As shown in Fig. 3, a block moves down on a 45-degree inclined plane of 2.5 m length, then horizontally for another 2.5 m, and then falls down vertically a height of 2.5 m. Find the magnitude and direction of the resultant displacement vector of the block.
(Ans: 6.0 m and 45 degrees below horizontal axis)

FIGURE-3





$\vec{a} = a_x\hat{i} + a_y\hat{j} = 2.5(-\cos 45)\,\hat{i} + 2.5(-\cos 45)\,\hat{j}$

$\vec{a} = -1.77\hat{i} - 1.77\hat{j}$

$\vec{b} = -2.5\hat{i} + 0\hat{j}$

$\vec{c} = 0\hat{i} - 2.5\hat{j}$

$\vec{R} = \vec{a} + \vec{b} + \vec{c} = (-1.77 - 2.5)\,\hat{i} + (-1.77 - 2.5)\,\hat{j}$

$= -4.27\hat{i} - 4.27\hat{j}$

$R = \sqrt{(4.27)^2 + (4.27)^2}$

$R = 6 \qquad \theta = 45^\circ$