# Mechanics I - Question Bank – 6

**2018-2019 Fall Semester**

Full name: ..........KEY..........

**Q1**. The position of a particle moving along the x-axis is given by x(t) = $3t^3-9t^2+18$, where x is in meters and t is in seconds. What is the value of x when the particle's acceleration is zero?

$$x = 3t^3 - 9t^2 + 18$$

$$V = \frac{dx}{dt} = 9t^2 - 18t$$

$$a = \frac{dv}{dx} = \frac{d^2x}{dt^2} = 18t - 18 = 0 \Rightarrow \quad 18t - 18 = 0 \quad t = 1s$$

$$x = 3(1)^3 - 9(1)^2 + 18 = 3 - 9 + 18 = 12m$$

**Q2**. The position of a particle is given by $\vec{r} = (4t - t^2)i + t^3 j$, where $\vec{r}$ is in meters and t in second. Find the average acceleration in m/s$^2$ of the particle in the time interval between t = 2s and t = 4s.

$$V = \frac{dr}{dt} = (4 - 2t)\hat{i} + 3t^2\hat{j}$$

$$V_{t=2} = (4 - 2(2))\hat{i} + 3(2)^2\hat{j} = 12\hat{j}$$

$$V_{t=4} = (4 - 2(4))\hat{i} + 3(4)^2\hat{j} = -4\hat{i} + 48\hat{j}$$

$$\Delta V = V_{t=4} - V_{t=2} = -4\hat{i} + 48\hat{j} - 12\hat{j} = -4\hat{i} + 36\hat{j}$$

$$\Delta t = 4 - 2 = 2 \Rightarrow a = \frac{\Delta V}{\Delta t} = \frac{-4\hat{i} + 36\hat{j}}{2} = -2\hat{i} + 18\hat{j}$$

**Q3**. The coordinate of a particle is given by x(t) = $16t - 3t^3$, where x is in meters and t is in seconds. Find the time when the particle is momentarily at rest?

When the object's velocity is equal to zero (0), then the object is at rest.

$$V = \frac{dx}{dt} = \frac{d}{dt}(16t - 3t^3) = 16 - 9t^2 = 0$$

$$16 = 9t^2 \Rightarrow t^2 = \frac{16}{9}$$

$$t = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3} = 1.3s$$

KEY

**Q7.** The velocity of a truck moving in a straight line is given by $v(t) = t^3 - t^2 - 2t$, where v is in m/s and t is in seconds. Find the velocity of the truck at the instant when its acceleration is 6 $m/s^2$.

$v = t^3 - t^2 - 2t \qquad a = \frac{dv}{dt}$

$a = 3t^2 - 2t - 2 = 6$

$\therefore 3t^2 - 2t - 8 = 0 \qquad (3t+4)(t-2) = 0$

$3t \quad t \qquad -2 \quad 4$

$3t + 4 = 0 \qquad t = \frac{-4}{3}$ X $\qquad t - 2 = 0 \qquad t = 2s$ ✓

Time cannot be negative.

when $t = 2s \quad v = t^3 - t^2 - 2t = 2^3 - 2^2 - 2 \cdot 2 = 0 \Rightarrow v_{t=2} = 0$

**Q8.** A particle is moving along an x-axis with a constant acceleration of -3.0 $m/s^2$. The velocity of the particle is given by the equation $v(t) = 4 - 3t$, where v is in m/s and t is in seconds. Find the displacement of the particle during the time interval t = 0 to t = 2s.

$t_0 = 0 \qquad t_f = 2s \qquad v(2) = 4 - 3 \cdot 2 = -2\ m/s$

$v(0) = 4 - 3 \cdot 0 = 4\ m/s$

$v_f^2 = v_0^2 + 2a\Delta x$

$(-2)^2 = 4^2 + 2 \cdot (-3) \cdot \Delta x$

$4 = 16 + (-6) \cdot \Delta x \qquad 4 - 16 = -6\Delta x$

$-12 = -6\Delta x$

$\Delta x = 2m$

**Q9.** The position of a particle moving along the x-axis is given by: $x = 3t^2 - 2t^3$, where x is in meters and t is in seconds. At what time is its acceleration zero?

$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = 0$

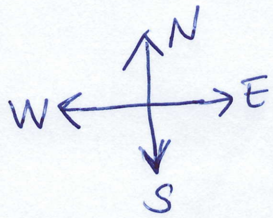
$a = \frac{d^2x}{dt^2} \qquad v = \frac{d}{dt}(3t^2 - 2t^3) = 6t - 6t^2$

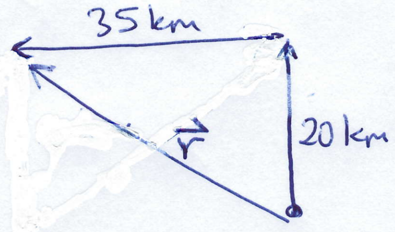
$a = \frac{dv}{dt} = \frac{d}{dt}(6t - 6t^2) = 0$

$a = 6 - 12t = 0 \qquad t = \frac{6}{12} = 0{,}5s$

KEY

**Q4.** A car travels 20 km due north and then 35 km due west. Find the ear's resultant displacement relative to the starting point?

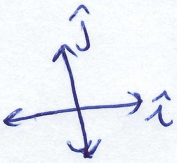



$\Delta\vec{r} = 20\hat{i} - 35\hat{j}$

$|\Delta\vec{r}| = \sqrt{20^2 + (-35)^2}$

$|\Delta\vec{r}| = 40.3$ km




$\theta = \tan^{-1}\left(\frac{20}{-35}\right) = -29.74^\circ \cong -30^\circ$

$90 - 30 = 60^\circ$ west of north.

**Q5.** A particle's position vector is initially $\vec{r} = 10i - 12j + 4k$, and 10s later it is $\vec{r} = -4i + 16j - 4k$, all in meters. In unit vector notation, what is its $\vec{v}_{avg}$ during the 10s?

$\vec{r}_1 = 10\hat{i} - 12\hat{j} + 4\hat{k}$

$\vec{r}_2 = -4\hat{i} + 12\hat{j} - 4\hat{k}$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = -4\hat{i} + 12\hat{j} - 4\hat{k} - [10\hat{i} - 12\hat{j} + 4\hat{k}] = -14\hat{i} + 24\hat{j} - 8\hat{k}$

$\Delta\vec{r} = -4\hat{i} + 12\hat{j} - 4\hat{k} - 10\hat{i} + 12\hat{j} - 4\hat{k}$

$\Delta t = 10$ s $\quad \vec{v} = \frac{\Delta\vec{r}}{dt} = \frac{-14\hat{i} + 24\hat{j} - 8\hat{k}}{10} = \underline{-1.4\hat{i} + 2.4\hat{j} - 0.8\hat{k}}$

**Q6.** The velocity of a train is 80 km/h, due west. One and a half hour later its velocity decreases to 65km/h, due west. What is the train's average acceleration?

$a = \frac{\Delta V}{\Delta t} \qquad a = \frac{V_f - V_i}{t_f - t_i} = \frac{65 - 80}{1.5} = \frac{-15}{1.5} = -10 \frac{km}{h^2}$ west